INTERNAL MOTIONS AND DENSITY DISTRIBUTION
IN A GLOBULAR CLUSTER

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Sommaire. — Les auteurs décrivent un modèle de l'amas globulaire Messier 3. La vitesse moyenne des étoiles serait de l'ordre de 2,9 km/sec. Ce sont des vitesses radiales de cet ordre qu'il faudrait essayer de mesurer par les méthodes nouvelles pour vérifier la théorie.

Abstract. — The authors describe a model of the globular cluster Messier 3. The mean velocity of the stars should be of the order of 2.9 km/sec. One should attempt to measure such radial velocities with the new techniques in order to test the theory.

Резюме. — Авторы описывают модель шарового скопления Messier 3. Средняя скорость звезд, повидимому, порядка 2.9 км/сек. Для проверки теории желательно было бы измерить новыми методами лучевые скорости этого порядка.

An attempt has been made to understand the observed structure of Messier 3. In the region up to about 8 pc from the centre encounters of stars must have set up an approximately Maxwellian velocity distribution as well as an approach to equipartition of energy. The cluster extends to at least 50 and perhaps to 100 parsec. Between 10 and 100 pc the effects of encounters must have been almost negligible. But the fact that the density distribution in this outer region fits so smoothly to that in the nucleus suggests that there has been an important interchange of stars between inner and outer regions. It is shown that a satisfactory dynamical representation of the entire cluster can be obtained by assuming a velocity distribution of the type

$$\varphi_r(u,v) = L_0 v^{-\frac{3}{2}} e^{-\frac{k^2}{h^2}r^2},$$

in which

$$\frac{k^2}{h^2} = 1 - \left( \frac{k^2 (15)}{h^2} - 1 \right) \left( \frac{r}{15} \right)^2.$$

At small distances from the centre this approaches the Maxwellian velocity distribution, for larger distances it becomes elongated in the direction of the centre. A suitable choice of the parameter $k^2 (15)/h^2$ ensures that nearly all stars far from the centre have orbits passing through the central region where the velocity distribution is regulated by encounters. Most of the calculations in this article were made with a value of 7 for the above parameter for stars above the main sequence.

The evolution of the cluster might tentatively be pictured as follows. It may
initially have been a compact group of considerably smaller dimension than at present. The heavier stars would have been most concentrated towards the centre. In their transition to the white-dwarf stage these stars must have expelled mass. It is estimated that by this process the total mass of the cluster will have been reduced to something like 40% of its initial value. The expulsion of mass in this way should by itself have caused roughly a doubling of the radius of the cluster. Stellar encounters in the nuclear region will have brought part of the stars into strongly elongated orbits, thus populating the outer regions. This caused a considerable shrinking of the nucleus, which may approximately have counterbalanced the expansion due to gas ejection. The encounters may have been sufficient to approach a semi-equilibrium between the motions of the outer stars and those in the nuclear part. This semi-equilibrium is likely to be of the nature indicated by the above formulae.

The investigation was based on star counts down to the 22nd photovisual magnitude, made by Sandage. The densities were extended to larger distances with the aid of earlier counts by von Zeipel. In this way direct data for the density distribution of about half the total mass in the cluster were obtained. In order to obtain estimates for the distribution of the remaining mass various assumptions were made concerning the luminosity curve and the number of white dwarfs. To some extent these may be tested by comparing the observed distribution of stars down to 19.2 with that computed from the mass distribution in these various models and the assumed velocity distribution.

The calculated average random velocity in radial direction ranges from 2.6 to 4.9 km/sec for the different models. The ratio of total mass to total light (expressed in the sun as unit) varies from 0.19 to 0.62. In the most probable model of the cluster the total mass would be about 150 000 solar masses, the mass-to-light ratio about 0.25 and the mean internal radial velocity 2.9 km/sec. The velocity distribution has always been cut off at the velocity of escape.

An investigation has been made of possible differences between the distribution of RR Lyrae variables and general stars in four clusters that are rich in variable stars. It is found that such differences might be explained as a result of loss of mass prior to the RR Lyrae stage and subsequent re-adjustment of equipartition. The mass loss would have to be of the order of 20%. Finally, we made a rudimentary discussion of structural differences between clusters depending on the fraction of the cluster in which a Maxwellian velocity distribution has been established. It is shown that on this basis the observed differences between clusters can be explained, at least in a qualitative manner.

The most essential desiderata for a verification of the cluster model are radial velocities of stars at different distances from the centre. Because the internal motions are small a very high accuracy would be required.
Discussion

Dr. Evans. — The photoelectric colour-magnitude diagrams for globular clusters usually refer to regions fairly far from the cluster centres because of crowding of the fields. The dynamical hypothesis proposed suggests a segregation of masses of different sizes between the centre and the outer parts of the clusters. Is there any insurance that the colour-magnitude diagram at the centre of the cluster is not seriously different from the diagram in the outer parts?

Dr. Oort. — On the basis of the evolutionary picture at present adopted by most investigators the stars brighter than the so-called break-off point on the main sequence would all have approximately the same mass. As a consequence it was not expected that there would be important differences between the colour-magnitude diagram in the centre of the cluster and that in the outer parts, except mainly for the relative proportion of main-sequence stars fainter than about $+3.5$ visual magnitude. Sandage’s observations appear to corroborate this. There is the possibility that the stars lose mass while they pass through the upper parts of the colour-magnitude diagram and that their distribution would become to some extent re-adjusted to the reduced mass. Indications of such an effect may be exhibited by a wider distribution of RR Lyrae variables, such as has been suggested by several astronomers.

Blauuw. — How does one deal with the invisible stars in these computations? For these the time of relaxation is so long as to throw doubt on whether equipartition of energy has been reached — yet an assumption has to be made about the velocity distribution.

Oort. — In the central part of the cluster there will be an approximation to equipartition of energy. Because of the higher velocity attained by the smaller masses the strongly elongated orbits will be filled more completely by these stars than by those of the larger mass.

That there are differences in distribution between stars of different mass is indicated by the counts made by Dr. Sandage. The differences found in the regions between roughly 8 and 25 parsecs from the center correspond approximately to what would have been expected in the case of equipartition of energy.

Münch. — In regard to the anisotropy of the velocity distribution, I would like to ask whether the value 7 for the factor $k^2/h^2$ was derived from the statistics of orbits of stars arising from the Maxwellian core and moving in a mass point field or in a more general one?

Oort. — The value of 7 adopted for $k^2/h^2$ is to a large extent arbitrary.
Neither the present theory nor the observations are adequate to establish the exact value of this parameter. It is only clear that it must be much higher than 1.

The gravitational field used in these computations was not that of a mass point, but the more complicated field corresponding to the observed and computed density distribution in the cluster.

HECKMANN. — I should like to make a highly theoretical statement: surely an exact thermodynamical equilibrium of a gravitational system is impossible in free space. But it can be shown that the consideration of the integrals of the conservation of angular momentum besides that of energy will create a tendency to make orbits elongated in the outer parts of a cluster.

(The method is mentioned by POINCARE in his "Leçons sur les hypothèses cosmologiques".)