Source Count, Spectral Indices and Angular Sizes of Weak Radio Sources in the 5 C 2 Region

P. Katgert
Institute of Astronomy, Cambridge, England*

Received December 30, 1975

Summary. The results of the third 1415 MHz Westerbork survey are used to derive the angular size distribution and the source count of a sample, part of which has been observed earlier at 408 MHz. About 25% of the sources stronger than about 10 mJy have an angular size larger than 20 seconds of arc, while about 50% is larger than 10 seconds of arc.

The combined effect of resolution, flux density determination procedure and noise on the source count is analyzed. It is found that significant corrections must be applied to account for these effects, especially near the flux density limit of the survey. The corrected 1.4 GHz count converges less strongly than the counts from earlier surveys at the same frequency. Between 0.01 and 0.1 Jy the differential count has a logarithmic slope of $-2.1$ to $-2.0$.

The spectral indices between 0.4 and 1.4 GHz of a source sample selected from the original 0.4 GHz observations are used to study possible changes of the spectral index distribution with flux density. No significant changes are detected from about 10 down to about 0.1 Jy. There is some evidence for an increase in the fraction of flat-spectrum sources below about 0.1 Jy.

Comparison of the 0.4 and 1.4 GHz counts reveals an inconsistency between counts and spectral index distribution at low flux densities. The expected 1.4 GHz count, computed from the observed 0.4 GHz count and spectral index distribution is significantly lower than the observed 1.4 GHz count. It is very unlikely that this discrepancy is due to unrecognized systematic errors in the data. It is suggested that the effect is due to a population of weak flat-spectrum sources present in the 1.4 GHz sample but almost absent from the 0.4 GHz sample. This explanation is consistent with all observational data and confirms a similar effect in the second Westerbork survey.

Key words: radio source counts — spectral index distribution — angular size distribution

I. Introduction

This paper deals with a rather detailed analysis of the results of the third 1415 MHz Westerbork survey (Katgert, 1975), one of the main objectives of which was to produce a reliable source count down to low flux densities. In combination with the original 408 MHz observations, made with the One-Mile Telescope at Cambridge (Pooley and Kenderdine, 1968), this survey also allows one to study the spectra of weak radio sources.

The high angular resolution of the Westerbork Telescope enables us to study the angular size distribution of the present sample. This information is subsequently used in an analysis of the systematic effects due to the partial resolution of extended sources. The results of model calculations, which include the effects of noise, are used to derive a corrected 1415 MHz count, which is compared with earlier counts at the same frequency.

Only for the stronger sources in a sample selected at 408 MHz can the spectral index distribution be determined directly. For the weaker sources, an observational bias against steep-spectrum sources must be corrected for. The spectral index distribution of the present 408 MHz sample is compared with that of a sample of strong sources.

The spectral index distribution is used to compute the expected 1415 MHz count from the observed 408 MHz count. The discrepancy, at low flux densities, between the observed and computed counts is explained in terms of a different spectral composition of the samples selected at the two frequencies. This proposed explanation can be verified experimentally by making additional low frequency observations of the present 1415 MHz sample.

The full potential of deep radio surveys like the present one can only be realized when deep optical identifications become available for large fractions of the radio samples. A mere optical detection (as opposed to a descriptive identification) for a sizeable fraction of such samples would already be very useful for the interpretation of the radio data.

* Now at Laboratorio di Radioastronomia CNR, Bologna, Italia.
II. Flux Density Definitions and Terminology

In Paper I (Katgert, 1975) we have introduced different kinds of flux density, most of which will be used again in this paper. In order to avoid a large number of references to Paper I we give here a summary of the different kinds of flux density and their use.

Because of the non-constant sensitivity in the observations presented in Paper I we must distinguish between real-sky flux densities (denoted by $S$) and attenuated flux densities (denoted by $\tilde{S}$). The ratio of $S$ and $\tilde{S}$ is equal to the position-dependent correction factor for primary-beam attenuation $G(r)$ [obviously $G(r) \geq 1.0$]. A large part of the subsequent analysis will be done in terms of $\tilde{S}$ instead of $S$ because flux density errors and most of the selection effects operate on $\tilde{S}$ rather than on $S$.

In several instances it will be necessary to distinguish explicitly between real (i.e. noise-free) flux densities, which will be marked by asterisks (e.g. $S^*$ and $\tilde{S}^*$) and their noise-affected estimates (without asterisks i.e. $S$ and $\tilde{S}$). When misunderstanding is precluded we will loosely use the term flux density even when flux density estimate is meant.

Of primary interest in statistical work is the total or integrated flux density. With an aperture synthesis telescope it is very difficult to derive total flux density estimates in a consistent manner, because such an instrument is relatively insensitive to large-scale structure. For the stronger sources one can use the individual visibilities at the several baselines to derive the total flux density (and the angular size) on the basis of a specific source model. For the weaker sources, however, one cannot derive the visibility function with the same amount of detail and one must therefore combine the observed visibilities to get meaningful results.

We combined the observed visibilities in two different ways, thereby obtaining two weighted mean visibilities (i.e. flux density estimates) $\tilde{S}_n$ and $\tilde{S}_s$ at effective baselines of about 3500 and 1100 wavelengths respectively. Clearly, $\tilde{S}^*_n$ and $\tilde{S}^*_s$ depend upon the total flux density $\tilde{S}^*$ and the structure of a source. In Paper I we derived the relationship between the three flux densities for equal double sources with an equivalent separation $\Psi_{eq}$ between the two equal point sources.

Before one can use such a relationship to derive $\tilde{S}$ from $\tilde{S}_n$ and $\tilde{S}_s$ one must be reasonably certain that any possible difference between $\tilde{S}_n$ and $\tilde{S}_s$ is not just a result of limited signal to noise ratios but that the observed ($\tilde{S}_n, \tilde{S}_s$)-combination indicates genuine extendedness. Only when the probability for the observed ($\tilde{S}_n, \tilde{S}_s$)-combination to result from a point source was less than about 16 per cent did we consider a source extended. The flux density of such an extended source was derived from $\tilde{S}_n$ and $\tilde{S}_s$ by using the relationship calibrated for equal double sources.

When, with the above criterion, a source had to be considered non-extended its flux density was determined as the weighted average of $\tilde{S}_n$ and $\tilde{S}_s$. When only either $\tilde{S}_n$ or $\tilde{S}_s$ could be determined the flux density was of course set equal to the one available estimate. For a more detailed description of the procedure we refer to Section IVc of Paper I.

This scheme, aiming at optimum use of the available information, has one obvious disadvantage. For all the sources that could not confidently be called extended the catalogued flux density $\tilde{S}$ is, on average, an underestimate of the real total flux density $\tilde{S}^*$. The effects of this underestimation can be estimated only when the angular size distribution is known.

III. Angular Size Distribution

In the preceding section we mentioned the equivalent diameter $\Psi_{eq}$, which was introduced as an operational measure of the angular size of a source, and derived directly from the two flux density estimates [viz. $\Psi_{eq} = 28.7 \left( \ln (\tilde{S}_n/\tilde{S}_s) \right)^{\frac{1}{2}}$]. Only for equal double sources is $\Psi_{eq}$ a true measure of the angular size; for all other sources it generally is an underestimate of the true angular size. For this reason we will attempt to determine the equivalent diameter distribution $g(\Psi_{eq})$ first and from that try to derive the angular size distribution $h(\Psi)$.

In Fig. 1 we have illustrated the way in which $g(\Psi_{eq})$ was determined. For each flux density $\tilde{S}^*$ one can easily compute the value of $\Psi_{eq}$ that separates "extended" from "non-extended" sources. By definition, all extended sources (code 4) and almost all complex sources (code 5) in the source list of Paper I have equivalent diameters that are larger than the limiting value corresponding to their respective flux densities. By computing the fraction of such extended sources in various flux density intervals one effectively measures the integral of the distribution function $g(\Psi_{eq})$ from a variable lower limit to a fixed upper limit. Differentiation of the function so obtained in principle yields $g(\Psi_{eq})$.

It will be clear that this scheme is valid only if $g(\Psi_{eq})$ does not depend on attenuated flux density. Although there is a global dependence of angular size on real-sky flux density [see e.g. Swarup (1975)], it seems reasonable to assume that the convolution over attenuation factor will sufficiently remove any serious dependence of $g(\Psi_{eq})$ on attenuated flux density. Nevertheless, it should be remembered that the information on small equivalent diameters is provided by the strong sources only.

Application of the method also requires consistency between the flux density estimates of extended and non-extended sources. As explained in Section II, this requirement is not satisfied in the present source list because flux densities of non-extended sources have, on average, been underestimated. This problem can, to a large extent, be solved by revising the flux densities of non-extended sources as follows. For this particular
realized that, even though \( g(\Psi_{eq}) \) is given in integral form, the individual values of the integral shown by the data points in Fig. 2 are independent. This result may now be translated into an angular size distribution \( h(\Psi) \) of a population of sources with more realistic structures. In order not to complicate the discussion unduly we will only consider unequal double sources, with component separation \( \Psi \) and component flux density ratio \( f : (1-f) \) (i.e. \( f \) is the fraction of the total flux density contained in the stronger component, hence \( f \geq 0.5 \)). We then need the transformations \( (\Psi_{eq}, f) \rightarrow (\Psi) \) as well as the distribution function of \( f \) to be able to determine the angular size distribution \( h(\Psi) \). Unfortunately, it turns out that a given set of \( (\Psi_{eq}, f) \)-values corresponds to either none or two values of \( \Psi \). Therefore we will have to determine the angular size distribution by model-fitting rather than directly from the data.

The distribution of \( f \)-values is given in Table 1. The second column of this table contains the distribution of \( f \) among the twelve complex sources (with more than 85% of the total flux density in the outer two components) from the source list of Paper I. The third column gives the result for 43 double sources (with sizes less than 80 seconds of arc) from the 3C-samples studied by Macdonald et al. (1968) and Mackay (1969) at a frequency and with a resolution identical to those of the present survey. This distribution does not differ significantly from the one derived by Mackay (1971) for all 6s sources in the 3C-samples “showing some kind of double structure”. The fourth column of Table 1 gives the smoothed \( f \)-distribution that we adopted for the following analysis.

Given the function \( \Psi_{eq}(\Psi, f) \), shown implicitly in Fig. 3, an \( f \)-distribution and an angular size distribution \( h(\Psi) \), one can easily compute the corresponding equivalent diameter distribution \( g(\Psi_{eq}) \). With the above \( f \)-distribution and an exponential angular size distribution, i.e. \( h(\Psi) \propto \exp(-c\Psi) \), we get a very reasonable fit to the observed \( g(\Psi_{eq}) \) with \( c = 0.06 \) (cf. Fig. 2), with an estimated uncertainty in \( c \) of about 0.01. On the basis of this angular size distribution one expects about 25% of complex sources, a value that is in good agreement with the 21% observed in the sample complete down to 31.3 mJy. For this comparison the limiting flux density must be so high as to exclude selection against sources with a large value of \( f \).
of about 1.3. Both the $\Psi_{eq}$ and the $\Psi$-scale are defined in the direction of maximum resolution. As a result, angular sizes must be increased by 15% (assuming a random distribution of position angles) to make them comparable with other data.

### IV. Observed 1415 MHz Source Count

#### IVa. Effects of Confusion and Resolution

It is well-known that the detection limit of a survey may considerably exceed the limit due to receiver noise only. Often this is due to confusion caused by the integrated emission of weak sources in the beam. The theory of confusion is well-known and methods have been developed for estimating source densities at confusion-affected flux density levels [see e.g. Wall and Cooke (1975) for a recent discussion and application].

An obvious remedy against the effects of confusion is to improve the resolution (i.e. decrease the beam solid angle). Drastic improvement of the resolution will, however, lead to the situation where the majority of the sources becomes appreciably resolved. In that case it will become increasingly difficult to avoid systematic underestimation of flux densities and consequent systematic errors in source counts and spectral index distributions.

From the discussion of the angular size distribution it is clear that such resolution effects may be important in the present observations, because the fraction of sources larger than 10 seconds of arc is estimated to be between 50 and 60%. The main effect of resolution is an underestimation of flux densities. Near the (sensitivity-) limit of the survey this causes sources to be dropped from the sample to which, on the basis of their total flux density, they do belong.

We have estimated the effects of resolution on the source count from model calculations of the following kind. Assuming a reasonable source count, together with angular diameter and $f$-distributions, one can compute the apparent source count which would have resulted, taking into account the procedure by which flux densities were determined as well as the effects of noise. In principle, the real source count can then be derived by an iterative procedure; in practice it is sufficient to derive “consistent” correction factors to be applied to the apparent source count.

These correction factors, viz. the ratio between the assumed real source count and the computed apparent source count, clearly depend on the flux density determination procedure used. The factors derived in the following section are therefore not directly applicable to other surveys made with the Westerbork Radio Telescope [see e.g. Katgert et al. (1973), Katgert and Spinrad (1974), Jaffe and Perola (1975) and Willis et al. (1976)]. Yet, the calculations described below can be adapted fairly easily to other flux density determination procedures.
IVb. Resolution Correction Factors for the 3rd Westerbrook Survey

In general, there exists a rather complicated relationship between the catalogue flux density of a source on the one hand, and the intrinsic source parameters (total flux density, angular size, component flux density ratio) on the other hand. First there is the survey-, reduction- and analysis-dependent relation between the source properties and the theoretical catalogue flux density. Second, there is the noise contribution which changes this into a flux density estimate (this noise contribution need not operate directly on the catalogue flux density, because this last one may be formed from more than one intermediate flux density, e.g. by extrapolation from a visibility function).

In the present survey, the attenuated catalogue flux density was derived from two flux density estimates, $\tilde{S}_k$ and $\tilde{S}_i$ (see Section II). It was therefore necessary to determine the functions $\tilde{S}_k^*(\tilde{S}^*, \Psi^*, f^*)$ and $\tilde{S}_i^*(\tilde{S}^*, \Psi^*, f^*)$. These functions, displayed in Fig. 3, were derived from model calculations which follow in detail the iterative least squares fitting procedure in the source finding programme. For large values of $\Psi^*$, both $\tilde{S}_k^*$ and $\tilde{S}_i^*$ approach a constant value, namely $f^* \tilde{S}^*$. The slight depression in the $\tilde{S}_k^*/\tilde{S}^*$-curves around 30 seconds of arc is due to the superposition of the maximum of the stronger component and the first negative sidelobe of the weaker component.

Using these functions, we can compute for any $(\tilde{S}^*, \Psi^*, f^*)$-combination the corresponding $(\tilde{S}_k^*, \tilde{S}_i^*)$-combination. If we then assume distribution functions for $\tilde{S}^*, \Psi^*$ and $f^*$ (i.e. a source count, an angular size distribution and an $f^*$-distribution) it is no very difficult to compute the corresponding distribution in the $(\tilde{S}_k^*, \tilde{S}_i^*)$-plane. The effect of noise (which operates almost independently on $\tilde{S}_k^*$ and $\tilde{S}_i^*$) can thereafter be simulated by convolving this two-dimensional distribution function with a two-dimensional gaussian error distribution with appropriate dispersions.

The resulting distribution function (this time with respect to the flux density estimates $\tilde{S}_k$ and $\tilde{S}_i$) can then be used to compute the apparent source count [i.e. $n(\tilde{S}_i)]$ by applying the scheme for determining flux densities, outlined in Section II. These calculations yield the ratio of the number of sources with $\tilde{S}_i = A$ and the number of sources with $\tilde{S}^* = A$, as a function of $A$, for the particular distribution functions used in the calculations. The inverse of this ratio can be considered as a correction factor, to be applied to the observed number of sources with $\tilde{S}_i = A$ in order to get an unbiased estimate of the number of sources with $\tilde{S}^* = A$.

In the actual calculations we used distribution functions of the following type: a power-law source count $n(\tilde{S}^*) \propto \tilde{S}^{x_k}$, an exponential angular size distribution $h(\Psi^*) \propto \exp(-c\Psi^*)$, while the $f^*$-distribution was assumed to be identical to the $f$-distribution given in the last column of Table 1. Such a factorization of the general tri-variate distribution function seems justified because a) there is no evidence for a correlation between $f^*$ and $\tilde{S}^*$ or $\Psi^*$; b) the implied absence of a correlation between $\Psi^*$ and $\tilde{S}^*$ is consistent with the assumptions made in Section III in deriving the angular size distribution.

The dispersions of the gaussian were taken to be $\sigma_k = 1.25$ and $\sigma_\Psi = 2.15$ mJy (see Appendix). As in Paper I the limit in $\tilde{S}_i$ was 6.25 mJy, but the limit in $\tilde{S}_i$ was increased from 10.75 to 12.5 mJy (both in the model and in the observations). The reason for this is that the noise in $\tilde{S}_i$ is for a large part caused by grating noise. Consequently, the reality of some of the six sources for which only $\tilde{S}_i$ could be determined may be questioned. Because these sources all have $10.75 < \tilde{S}_i < 12.5$ mJy we decided to raise the limit in $\tilde{S}_i$ to 12.5 mJy, thereby removing six sources from the sample and revising the flux density of 33 sources. This revision should result in an increased reliability of the data.

The result of the model calculations is shown in Fig. 4, in the form of the ratio $n(\tilde{S}^* = A)/n(\tilde{S}_i = A)$. Circles apply to the model with $k = -2.0$ and $c = 0.06$. The variations in the computed ratios due to variations in $k$ and $c$ are indicated by “error bars”, which display minimum and maximum values in the range $k = -1.8$ to $-2.2$ and $c = 0.05$ to 0.07. It is clear that these correction factors differ significantly from unity below about 16 mJy. More important even is the fact that below about 14 mJy the correction factors also exceed $1 + n^{-1/2}$, i.e. the upper value of the fractional uncertainty due to limited statistics, at the flux density resolution.
employed. This means that, for the present sample at least, the resolution effect is a significant systematic effect, well in excess of the random statistical uncertainties.

It is rather difficult to estimate the dependence of the derived correction factors on the global characteristics of the model distributions, such as the assumed forms of the count and the angular size distribution. From varying k and c it was found that the correction factors do not depend very strongly on the relative proportions of sources with different flux densities and sizes. On the contrary, it looks as if the result of the model calculations is determined primarily by the scheme used for converting $\bar{S}_c$ and $\bar{S}_i$ into $\bar{S}_c$.

Actually, it is fairly easy to understand qualitatively the dependence of the correction factor on flux density. Above, say, 20 mJy the average underestimation of the flux density appears to be compensated by the average upward migration of sources due to noise. Below about 15 mJy the flux density underestimation becomes increasingly severe (to a large extent because of strongly decreasing availability of $\bar{S}_i$), resulting in a serious underestimation of the number of sources around 11 mJy. Near the flux density limit the deficit is much less serious, because of the combined effects of a general downward migration of sources below about 15 mJy and the spill-over, due to noise, of weak sources below the flux density limit.

IVc. Resulting Source Count; Comparison with Earlier Counts

The effects of resolution and noise have been described in terms of a statistical correction to the number of sources and not as a correction to the individual flux densities. Therefore, the correction can be incorporated fairly easily into the procedure for deriving the real-sky flux density count. To illustrate this we have plotted in Fig. 5 G (i.e. the attenuation factor) versus $\bar{S}_c$ (the attenuated flux density after raising the limit in $\bar{S}_i$ to 12.5 mJy) for all sources in the present sample. For a given value $\bar{S}_c = A$ each plotted point represents $W(A)$ sources, where $W(A)$ is the correction factor $n(\bar{S}^* = A)/n(\bar{S}^*_c = A)$ shown in Fig. 4.

However, by applying these $\bar{S}_c$-dependent weights one effectively transforms the $\bar{S}_c$-scale into an $\bar{S}^*$-scale. For this reason the real-sky flux densities $S$ (i.e. $G \bar{S}_c$) need no longer be treated as estimates, at least not in a statistical sense. In other words: a source with $\bar{S}_c = A$ and attenuation factor $G$ represents $W(A)$ sources with real-sky flux densities $S^* = GA$.

Obviously, it has been implicitly assumed throughout that $W(A)$ does not depend on $G$. Basically this means that we assumed the source properties (i.e. the $S^*$-distribution, the $\Psi^*$-distribution and the $f^*$-distribution) as well as the transformation $(S^*/G, \Psi^*, f^*) \rightarrow (\bar{S}^*_c, \bar{S}^*_i)$ to be independent of attenuation (i.e. position).

Fig. 5. The distribution of the sources with respect to $G$, the primary beam attenuation factor, and $\bar{S}_c$, the attenuated catalogue flux density. Lines of constant real-sky flux density have a "minus-one" slope in this double logarithmic diagram. Each observed source represents $W(\bar{S}_i)/\Omega$ sources, where $W(\bar{S}_i)$ is the correction factor shown in Fig. 4 and $\Omega$ is the solid angle over which the source would have remained observable.
This last assumption may not be completely justified for sources in which only a small part of the radiation is concentrated in two point-source components. Especially for large values of G it becomes more and more difficult to detect extended regions of low surface brightness. For this reason we decided to exclude from the sample the 19 sources with attenuation factors larger than 4.0.

It is now easy to derive \( n(S^*) \), the differential count with respect to real-sky flux density. For every source one computes the attenuation at which it would have been dropped from the sample \( (G_{\text{max}} = S^*/6.25 \text{ or } 4.0, \text{ whichever is the smaller}) \) and from this the solid angle \( \Omega \) over which it would have been visible. The quotient \( W(S^*)/\Omega \) then gives the surface density of sources with flux density \( S^* \) implied by the detection of this one source.

The resulting count in the flux density range 6.25 to 200 mJy is given in Table 2, which also contains values normalized with respect to a count of 225 \( S^{-5/2} \text{ sterad}^{-1} \text{ Jy}^{-1} \). The quoted errors include the uncertainty in the correction factors as given in Fig. 4. It should be noted that the flux density intervals do not all have the same logarithmic width: between 6.25 and 100 mJy the intervals have a “2\(1/2\)-width” but we doubled the width of the last interval to improve counting statistics.

Near the flux density limit the values of \( 1/\Omega \) can become very large indeed. Therefore, the count in the lowest interval depends rather strongly on the exact distribution of the few available sources in the lower left-hand corner of the \((G, S^*)\)-plane (cf. Fig. 5). For example, the weakest of the fifteen sources in the lowest interval (6.25 to 8.84 mJy) accounts for 23% of the count in that interval. A small change in its flux density would considerably influence the count in the lowest interval. This discretization effect which is important near the flux density limit only, may, to some extent, have affected the 6.25–8.84 mJy count to which one should therefore not attach too much weight. Above 8.84 mJy the effect is completely negligible and does not influence the count.

In Fig. 6 the major 1.4 GHz counts presently available are compared, again in normalized differential form. Above \( \sim 0.2 \text{ Jy} \) the improved 1400 MHz count based on the GA, GB and BDFL catalogues (Fomalont et al., 1974) is shown. Below \( \sim 0.2 \text{ Jy} \) the counts are based on surveys made with aperture synthesis telescopes. They are: the 1421 MHz count obtained with the Cambridge Half-Mile Telescope (Gillespie, 1975), the 1407 MHz

---

### Table 2: The 1415 MHz source count in the SC2 region

<table>
<thead>
<tr>
<th>( S ) (mJy)</th>
<th>( n(S) ) (sterad(^{-1} ) Jy(^{-1} ))</th>
<th>Normalized*</th>
<th>n</th>
<th>Res. corr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.25–8.84</td>
<td>1.62 (0.53) ( 10^7 )</td>
<td>0.34 (0.11)</td>
<td>15</td>
<td>1.28</td>
</tr>
<tr>
<td>8.84–12.5</td>
<td>6.52 (1.26) ( 10^6 )</td>
<td>0.33 (0.06)</td>
<td>28</td>
<td>1.39</td>
</tr>
<tr>
<td>12.5–17.7</td>
<td>3.53 (0.62) ( 10^6 )</td>
<td>0.42 (0.07)</td>
<td>35</td>
<td>1.45</td>
</tr>
<tr>
<td>17.7–25.0</td>
<td>1.86 (0.32) ( 10^6 )</td>
<td>0.53 (0.09)</td>
<td>37</td>
<td>1.36</td>
</tr>
<tr>
<td>25.0–35.4</td>
<td>8.68 (1.67) ( 10^5 )</td>
<td>0.58 (0.11)</td>
<td>29</td>
<td>1.27</td>
</tr>
<tr>
<td>35.4–50.0</td>
<td>4.10 (0.87) ( 10^4 )</td>
<td>0.66 (0.14)</td>
<td>23</td>
<td>1.07</td>
</tr>
<tr>
<td>50.0–70.7</td>
<td>1.62 (0.43) ( 10^4 )</td>
<td>0.62 (0.17)</td>
<td>14</td>
<td>0.98</td>
</tr>
<tr>
<td>70.7–100</td>
<td>1.00 (0.29) ( 10^3 )</td>
<td>0.91 (0.26)</td>
<td>12</td>
<td>1.00</td>
</tr>
<tr>
<td>100–200</td>
<td>3.67 (0.95) ( 10^2 )</td>
<td>1.23 (0.32)</td>
<td>15</td>
<td>1.00</td>
</tr>
</tbody>
</table>

* Normalizing count: 225 \( S^{-5/2} \) sterad\(^{-1} \) Jy\(^{-1} \).

---

Fig. 6. The main 1.4 GHz counts in differential form, normalized with respect to a count of 225 \( S^{-5/2} \) sterad\(^{-1} \) Jy\(^{-1} \). The two estimates at 5.5 and 9.0 mJy are derived from the noise analysis (see Appendix)
count obtained with the Cambridge One-Mile Telescope (Pearson, 1975) and the counts from the first and third 1415 MHz Westerbork surveys (Katgert et al., 1973; Katgert, 1975). The count from the second Westerbork survey (Katgert and Spinrad, 1974) has not been included because it essentially confirms the result from the first survey but from a smaller sample. The discrepancy around 0.2 Jy between the original 1400 MHz count (Bridle et al., 1972b) and the count from the first Westerbork survey has largely been resolved by the addition of the GB data and by the revision of the GA flux densities. In fact, the agreement between the present count and the Green Bank count in the region of overlap is very good. Below 0.1 Jy there seems to be some evidence for systematic differences between the four available counts. Such differences are not unexpected because only the present count has been corrected for resolution effects.

Resolution effects are expected to be smallest in the Half-Mile count which should therefore be more or less comparable to the present count. Yet, the Half-Mile count seems to be systematically lower (and flatter) than the present count, at least below 40 mJy. Comparison of the flux densities of the 25 sources common to both surveys shows a non-linear relationship between the two flux density scales. The ratio \( S_{1421}/S_{1415} \) appears to decrease from about 0.9 around 100 mJy to about 0.8 around 10 mJy. The reason for this discrepancy is not clear, but it is possible that there is a difference in the calibration of about 10% combined with a small resolution effect in the 1421 MHz data.

The difference between the present count and that from the first Westerbork survey can be completely explained by the absence of a resolution correction in the former. On the contrary, it is not so easy to understand the systematically low count from the 1407 MHz One-Mile survey, although resolution effects certainly must account for part of the difference. It should also be remembered that all these surveys pertain to different, rather small regions (all less than \( 10^{-2} \) sterad) and the possibility of intrinsic differences can therefore not be ruled out completely.

The noise analysis (see Appendix) provides additional, though not completely independent support for the present count. Two estimates of the source density at 5.5 and 9.0 mJy derived from that analysis (see also Fig. A1) have been indicated in Fig. 6. The uncertainty in these estimates is rather large, partly because the transformation from attenuated to real-sky flux density count requires the slope of the source count. Nevertheless, the estimates seem to support the present corrected count with a slope \( \langle \log n / \log S \rangle \) between \(-2.0\) and \(-2.1\) rather than the flatter and lower counts from the other surveys.

Finally, it is interesting to determine the temperature of the isotropic background radiation due to the discrete extragalactic sources with \( S_{1415} > 6.25 \text{ mJy} \). Using the count from the Green Bank survey together with that from the present survey gives \( T_{\nu}(|S| > 6.25 \text{ mJy}) = 0.06 \text{ K} \). The total background temperature due to discrete sources cannot be measured directly because of the much stronger universal 3 K background. However, using Bridle's (1967) canonical value of 30 ± 7 K for the temperature at 178 MHz we may estimate the total 1.4 GHz temperature to be 0.10 ± 0.03 K. Apparently, more than half of the total background is accounted for by sources with \( S > 6.25 \text{ mJy} \). A continuous power-law count with constant logarithmic slope \( k (> -2.0) \) below this limit will contribute an additional 0.012/(\( k + 2 \)) K. Stronger convergence of the count below the present limit is therefore implied by these data.

V. Observed Spectral Index Distribution

In the following we will use two-point spectral indices defined by: \( \alpha_{1,2} = \log(S_1/S_2)/\log(v_2/v_1) \). Unless stated otherwise, the term “spectral index” and the symbol \( \alpha \) will denote a spectral index between 0.4 and 1.4 GHz. In this section frequencies will be expressed in GHz and flux densities in Jy, unless explicitly stated otherwise. Because spectral index distributions may depend on selection frequency and flux density limit we will, following van der Laan (1969), denote by \( g_\alpha(S) \) the normalized spectral index distribution of a sample of sources selected at frequency \( v \) and complete to flux density \( S \). This definition arises naturally in a discussion of the transformation of an integral source count from one frequency to another. For the transformation of a differential count one needs, at least in principle, the differential spectral index distribution \( \chi_\alpha(S) \) of all sources with flux density equal to \( S \) (see also Section VI). The difference between the two types of spectral index distribution is very small, unless \( \chi_\alpha(S) \) [and therefore also \( g_\alpha(S) \)] depends strongly on flux density.

By reobserving the 5C2 area at 1.4 GHz we obviously aimed to determine \( g_\alpha(\alpha) \), which requires the definition of a complete sample at 0.4 GHz from the original 5C2 catalogue (Pooley and Kenderdine, 1968). A synthesis survey, complete to an attenuated flux density \( \bar{S} \) and with a maximum attenuation \( G_{\max} \), is complete for real-sky flux densities larger than \( \bar{S} G_{\max} \). Of course such a survey contains also sources below this limit but it becomes increasingly incomplete with decreasing flux density. Because the 5C2 survey is complete to \( \bar{S}_{0.4} = 12 \text{ mJy} \), we may define a sample complete to \( \bar{S}_{0.4} = 60 \text{ mJy} \) by including only sources with \( G_{0.4} < 5.0 \).

From the sample so defined we excluded the sources with a 1.4 GHz attenuation larger than 5.0. This leaves us with 137 sources, for which we have plotted in Fig. 7 \( S_{0.4} \) versus \( \alpha_{0.4,1.4} \) (filled circles) or the lower limit to this index based on \( \bar{S}_{1.4} \leq 6.25 \text{ mJy} \) (open circles). Of the 63 sources with \( S_{0.4} > 60 \text{ mJy} \) only 2 have not been detected at 1.4 GHz. However, both these sources were already noted as extended in the original 5C2 observa-

© European Southern Observatory • Provided by the NASA Astrophysics Data System
Fig. 7. The distribution of $S_{0.4}$ and $\alpha_{0.4}$ for the sample defined at 0.4 GHz. Filled circles indicate sources detected at 1.4 GHz, while open circles represent lower limits to $\alpha$ for sources that were not detected at 1.4 GHz. Above 0.06 Jy the sample is complete with respect to $S_{0.4}$.

Fig. 8. The spectral index distribution $\alpha_{0.4}(\gamma)$ for various values of $S$. There are small differences between the zero-points of the four spectral index scales (see text). The apparent spectral difference between extended and non-extended sources is due to a slight underestimation of the 1.4 GHz flux densities of non-extended sources.

Because of the non-uniform flux density determination at 1.4 GHz (see Section II), the spectral index estimates are probably not unbiased. Separate spectral index distributions were therefore determined for the 30 "extended" and the 31 "non-extended" sources; the two distributions are shown in Figs. 8b and 8c respectively. It is not unexpected that the extended sources, on average, have smaller spectral indices because their 1.4 GHz flux densities are less likely to have been underestimated than those of the non-extended sources. Formally, the spectral index difference corresponds to an average difference of about 10% in the 1.4 GHz flux densities, which is not at all unreasonable.

One may wonder what spectral index information may be obtained from the 74 sources with $S_{0.4} < 60$ mJy, of which only 40 were found to have $S_{1.4} > 6.25$ mJy. Due to the incompleteness at both frequencies the observed distribution of points in the $(S_{0.4}, \alpha)$-plane is a very biased estimate of the true distribution. The main reason for this is that, given a minimum value for $S_{1.4}$, there exists for each value of $S_{0.4}$ an absolute maximum to the spectral index beyond which no 1.4 GHz detection is possible. This limit is indicated in Fig. 7 by the inclined line which corresponds to $S_{1.4} = 6.25$ mJy. In addition to this there is a strong bias against sources with steep spectra because, at each position in the survey area, the actual limit to $S_{1.4}$ is larger than 6.25 mJy, due to the local 1.4 GHz attenuation, and can be as large as 31.25 mJy.

It is possible to correct for this flux density and spectral index dependent bias in the manner described by Katgert and Spinrad (1974). For every detected source a weight is determined that is inversely proportional to the solid angle over which the source would have remained detectable at both frequencies. In other words: the solid angle consisting of all positions $r$ for which $S_{0.4}/G_{0.4}(r) > 12$ mJy and $\alpha_{\text{max}}(r) > \alpha$. By applying these weights one makes up for all the sources which have remained undetected at the second frequency solely because of the local attenuation factor. This procedure does not account for the sources that remained undetected because their spectral index exceeds the maximum value corresponding to their $S_{0.4}$.

From Fig. 7 it appears that very few sources above 60 mJy have spectral indices larger than about 1.25. The above correction procedure therefore should be applicable to the sample with $S_{0.4} > 30$ mJy, for which $\alpha_{\text{max}}$ is larger than 1.25. The corrected distribution
\( \frac{g_{0.03}^{0.4}(z)}{} \) is shown in Fig. 8d. Because this distribution is based on sources between 30 and 60 mJy it is not a genuine integral distribution and should perhaps more appropriately be referred to as \( \frac{g_{0.04}^{0.4}(z)}{} \). On the basis of the applied corrections we expect 24 ± 3 upper limits to \( \alpha \) between 30 and 60 mJy. This is in reasonable agreement with the 19 "observed" upper limits.

The spectral index distributions in Figs. 8b to 8d are all based on the 5C2 sample. In addition we present in Fig. 8a the distribution \( g_{0.4}^{0.4}(z) \) derived from a sample complete down to 5.5 Jy taken from the Bologna 408 MHz survey (Colla et al., 1970; Colla et al., 1972; Colla et al., 1973; Fant et al., 1974), covering about 1.7 steradians. For the majority of the 69 sources in this sample the 1.4 GHz flux density is given by Bridle et al. (1972a), while for the remainder the 1.4 GHz flux density was taken from Kellermann et al. (1969).

Before one can compare the distributions in Fig. 8, one must determine the relationship between the spectral index scale in Fig. 8a on the one hand and that in Figs. 8b to 8d. This relationship depends on the relative flux density calibrations of the different surveys. The synthesis surveys have been calibrated by postulating for the source 3C48 flux densities of 35.0 and 15.7 Jy at 408 and 1415 MHz, respectively. However, the flux density of a typical source in these surveys is about three orders of magnitude smaller than that of the calibrator source. The calibration therefore relies rather heavily on the linearity of the receivers, but deviations from linearity are generally accepted to be less than a few percent.

The Bologna and Green Bank flux density scales can only be related to those of the Cambridge and Westerbork surveys if we assume that the flux densities of 3C48 in the former are, within the quoted errors, representative of the respective flux density scales. In this way we find that the spectral indices in Fig. 8a should be reduced by 0.08 ± 0.05 in order to make them comparable to those in Figs. 8b to 8d, or vice versa. There may be a small additional effect due to the fact that the Bologna catalogue gives peak flux densities only. Combining the structural information at 1.4 GHz given by Bridle et al. (1972a) with the calibration for resolution given by Colla et al. (1970), we estimate an average underestimation of the Bologna flux densities of 3 to 4%. The resulting total correction to be applied to the indices in Fig. 8a to make them comparable with those in Figs. 8b to 8d is -0.05 ± 0.05.

Errors in the attenuation functions may have given rise to systematic effects in Figs. 8b to 8d. Condon and Jauncey (1973) reported some evidence for systematic overestimation of the original 0.4 GHz flux densities in the 5C1 and 5C2 surveys, which they ascribed to errors in the attenuation factors. Recently, Pearson (1975) made new observations of part of the original 5C1 survey region and concluded that the original 5C1 flux densities are too low by, on average, 20%.

### Table 3. Fraction of sources with \( \alpha \leq \alpha_s \)

<table>
<thead>
<tr>
<th>( S_{0.4} (\text{Jy}) )</th>
<th>( \alpha_s )</th>
<th>Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; 5.5</td>
<td>0.55</td>
<td>0.12 ± 0.04</td>
</tr>
<tr>
<td>&gt; 0.06 (ext)</td>
<td>0.50</td>
<td>0.07 ± 0.05</td>
</tr>
<tr>
<td>&gt; 0.06 (non-ext)</td>
<td>0.38</td>
<td>0.06 ± 0.04</td>
</tr>
<tr>
<td>0.03-0.06</td>
<td>0.58</td>
<td>0.18 ± 0.04</td>
</tr>
</tbody>
</table>

Unfortunately, no such direct information is available on the 5C2 flux densities. All we can say at the present moment is that there does not seem to be any correlation between spectral index and 0.4 GHz attenuation factor, which would be expected if the attenuation factor was not correct. In what follows we will assume the change in spectral index from Fig. 8b to 8c to be due completely to the 1.4 GHz flux density determination procedure. Figures 8c and 8d should be directly comparable because all but five of the sources in Fig. 8d are non-extended.

Comparing Figs. 8a and 8b, and taking into account all the afore-mentioned effects, there appears to be an increase in the average spectral index of 0.09 ± 0.07 on lowering the completeness limit from 5.5 to 0.06 Jy. This apparent steepening is of course not significant as it represents only a 1.3σ result. Earlier work by Pauliny-Toth et al. (1972), based on observations at 0.4 and 5.0 GHz, did not show a significant dependence of \( g_{0.4}^{0.4}(\alpha_{0.4, 5.0}) \) on \( S \) down to 0.5 Jy. Only a very small increase of the fraction of flat-spectrum sources with decreasing flux density seemed indicated by their data.

Evidence for the steepening of source spectra with decreasing flux density has been reported earlier [see e.g. Condon and Jauncey (1974a, 1974b)] but those results pertain to source samples selected at high frequencies. Such steepening is generally explained in terms of an induced correlation between high-frequency radio power and spectral index. However, recently Balonek et al. (1975) did not find a continued steepening below \( S_{2.7} \sim 0.5 \) Jy.

Returning to the present sample, it appears from Figs. 8c and 8d that below 0.06 Jy the average spectral index decreases by 0.10 ± 0.06. In itself this is not a significant result but it seems as if the distribution in Fig. 8d is significantly wider than that in Fig. 8c due to a larger fraction of sources with \( \alpha \leq 0.6 \). It might be thought that the appearance of these flatter spectra is due to systematic errors in the weights applied to correct for the bias against steep-spectrum sources. However, from the number of observed and expected upper limits to \( \alpha \), it would appear that, if anything, this bias has been slightly overcorrected.

In Table 3 we give the fraction of sources with "flat" spectra, for each of the four distributions in Fig. 8. The limiting spectral index \( \alpha_s \), separating flat and steep spectra, has been changed slightly from one
sample to another to account for the systematic differences between the $x$-scales. Apart from illustrating the change of $\bar{\alpha}$ with $S$, Table 3 also indicates that the increase in the fraction of “flat-spectrum” sources below about 60 mJy may be marginally significant. This apparent flattening at low flux densities and at low frequencies obviously needs to be confirmed. The models introduced by Fanaroff and Longair (1973) to account for the variation of spectral index distribution with sample depth and frequency do not seem to predict such flattening at low flux densities at low frequencies. This must be due partly to their assumption that the combined luminosity and spectral index function is separable at low frequencies.

VI. Consistency of the Counts and the Spectral Index Distribution

It is obviously very interesting to investigate whether the observed counts and the spectral index distribution are consistent. For this purpose we have redetermined the Cambridge 0.4 GHz count, assuming the 5C2 catalogue to be complete to $S_{0.4} = 12$ mJy and using an attenuation function derived from data supplied by Dr. Pooley. This count, shown in Fig. 9, is in good agreement with the independent count from the 5C5 survey (Pearson, 1975) which in its turn is in good agreement with the original 5C2 count. Also shown in Fig. 9 are the 0.4 GHz Bologna count (Colla et al., 1973) as well as the Green Bank and Westerbork 1.4 GHz counts. Notice the difference between the left- and right-hand side scales. Using the 0.4 GHz counts and the observed spectral index distribution we have computed a 1.4 GHz count (indicated by open circles in Fig. 9) as follows:

$$n_{1.4}(S) = \int_{-\infty}^{\infty} dx x_{1.4}^{0.4}(\alpha) \int_{0.4}^{1.4} d\alpha n_{0.4}((1.4/0.4)\alpha \cdot S).$$

In the actual calculations we replaced $x_{0.4}^{0.4}(\alpha)$ by $x_{0.4}^{0.4}(\alpha)$ because the difference is probably small and $x_{0.4}^{0.4}(\alpha)$ is hard to determine. The open circles in Fig. 9 are based on the spectral index distribution from Fig. 8a. The excellent agreement between the observed and computed 1.4 GHz counts above ~0.1 Jy indicates that this spectral index distribution is applicable down to, at least, $S_{0.4} = 0.3$ Jy.

Below 0.2 Jy we have still used the same spectral index distribution, without having corrected the spectral indices by 0.05 ± 0.05 (see Section V). The slight discrepancy between the observed and computed 1.4 GHz counts around 70 mJy is probably caused by this. The larger discrepancy below about 50 mJy—where the ratio between the observed and computed count increases to about 1.7—must have a different origin. Another way to illustrate this apparent inconsistency at low flux density levels is to use the observed counts to derive an effective spectral index $\alpha_{\text{eff}}$ defined by a simplified version of the above equation, viz.:

$$n_{1.4}((0.4/1.4)\alpha \cdot S) = \frac{(1.4)^{\alpha_{\text{eff}}}}{0.4} n_{0.4}(S).$$

The solution of this equation, for $S_{0.4}$ corresponding to $n_{0.4} = 10^2, 10^3, \ldots 10^6$ stered $^{-1}$ Jy $^{-1}$, is given in Table 4. Even though the uncertainty in $\alpha_{\text{eff}}$ is of the order of 0.2, the data seem to imply a drastic flattening of the spectra towards lower flux densities, much more pronounced than in the direct analysis of the previous section.
Table 4. Effective spectral index

<table>
<thead>
<tr>
<th>$S_{0.4}$ (Jy)</th>
<th>$\alpha_{\text{eff}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.8</td>
<td>0.8</td>
</tr>
<tr>
<td>1.1</td>
<td>0.8</td>
</tr>
<tr>
<td>0.39</td>
<td>0.7</td>
</tr>
<tr>
<td>0.12</td>
<td>0.5</td>
</tr>
<tr>
<td>0.032</td>
<td>0.2</td>
</tr>
</tbody>
</table>

It is very improbable that this discrepancy has to be explained by systematic errors in the data. For example, it is improbable that the difference in the slopes of the two counts (which implies the drastic flattening of the spectra) has to be explained away by a strongly nonlinear relationship between the two flux density scales. A large systematic overestimation of the spectral indices of sources with $S_{0.4} \leq 0.1$ Jy, due to unrecognized resolution effects in the 1.4 GHz observations, also seems very unlikely. Leaving out the 1.4 GHz resolution correction (see Section IV and Table 2) would partly remove the discrepancy. It is, however, very unlikely that the resolution correction can be dispensed with, considering the angular size distribution and the flux density determination procedure.

The fact that the two samples do not cover exactly the same solid angle may be partly responsible for the observed discrepancy. However, a much more likely explanation for this discrepancy at low flux densities presents itself when one considers the composition of the 1.4 GHz sample. The 1.4 GHz subsample consisting of those sources for which the 0.4 GHz attenuation is, or would have been, less than 5.0 contains 121 sources. Of these, only 95 were detected at 0.4 GHz. More than half (14 out of 26) of the remaining sources have upper limits to the spectral index less than 0.6. In the case of selection at 0.4 GHz there is a rather strong selection against flat-spectrum sources. It is therefore likely that a sizeable fraction of the weak 1.4 GHz sources is not represented in the 0.4 GHz sample, where flat-spectrum sources are rare. This conclusion is corroborated by the results of Katgert and Spinrad (1974) who found that the spectral index distribution $g_{\alpha}^{1.4}$ ($\alpha_{1.4, 0.4}$) becomes significantly wider and flatter with decreasing flux density.

The transformation of a differential count from one frequency to another is possible only if all classes of sources are represented at both frequencies. Because the weak flat-spectrum sources in the 1.4 GHz sample are not represented in the 0.4 GHz sample, their contribution to the 1.4 GHz count cannot be predicted from the 0.4 GHz count and the 0.4 GHz spectral index distribution. On the other hand, because the 1.4 GHz sample contains all classes of sources present in the 0.4 GHz sample, it should be possible to transform the 1.4 GHz count to 0.4 GHz.

This explanation is consistent with the observed counts and spectral index distribution (cf. Table 3). It also supports the results by Katgert and Spinrad in an indirect way. Quantitative verification requires $g_{\alpha}^{1.4}$ ($\alpha_{1.4, 0.4}$). The present 0.4 GHz observations are not sensitive enough to determine this distribution reliably for low values of $\alpha$; additional observations at a low frequency of the present 1.4 GHz sample are therefore required.

Very little is known about the nature of the weak flat-spectrum sources in the 1.4 GHz sample. Katgert and Spinrad found a weak indication that these sources may be associated with faint galaxies mostly beyond the limit of the Palomar Schmidt Sky Survey plates.

Appendix

On several occasions we needed reasonably accurate estimates for the total noise in the high- and low-resolution maps. Instead of relying on theoretical values, we tried to determine the actual noise in the maps. The distribution function of deflections in the almost empty subtract maps in principle contains combined information on the noise distribution and the source count. Rather than determine the distribution of individual deflections we used the lists of possible sources below the completeness limits, provided automatically by the source finding program.

This distribution of possible sources with respect to flux density is the sum of the noise distribution and the source count. In the transformation from noise deflections to "sources" the normalization of the noise distribution is no longer very obvious and we determined the normalization from the observations. The observed distribution functions are shown in Fig. A1 as differential counts with respect to attenuated flux density, between 3.0 and 6.0 mJy (high-resolution maps) and between 4.5 and 9.5 mJy (low-resolution maps). These distributions are averages from 24 fields and represent the number of sources in the central $\pi 0.6^2$ square degrees of each field. Differences between the noise distributions in the individual fields have consequently been smoothed out.

Decomposition of the observed distributions into two noise distributions and one source count is only possible if we make an assumption about the form of the noise distributions and if we require continuity of the resulting count. We assumed the noise distributions to be gaussians with dispersions $\sigma_{\alpha}$ and $\sigma_{\nu}$. Reasonably consistent, though by no means unique solutions, with $\sigma_{\nu}$ between 1.23 and 1.25 mJy and $\sigma_{\alpha}$ between 2.15 and 2.25 mJy have been indicated in Fig. A1. It is found that the continuity requirement for the resulting count provides the most important constraint to the range of possible models. The logarithmic slope of the resulting count is about $-2.0$, in good agreement with the directly determined count. Around 5.5 and 9.0 mJy
Fig. A1. The observed noise distributions in the flux density intervals 3.0–6.0 mJy (high-resolution maps) and 4.5–9.5 mJy (low-resolution maps). The solid lines represent Gaussian error distributions with dispersions of 1.24 and 2.20 mJy. The residual count at 5.5 and 9.0 mJy is included in Fig. 6, after having been transformed to a real-sky flux density count.

(i.e. where the amplitude of the error distributions is smallest) the count can be determined with some degree of reliability. We find values for $n(S)$ of 1.1 (0.4) $10^7$ and 3.9 (1.4) $10^6$ sterad$^{-1}$ Jy$^{-1}$ respectively. Conversion to $n(S)$, i.e. the count with respect to real-sky flux density, is simple in the case of a power-law count. We then have the relation:

$$n(S = A) = n(S = A) \left[ \int_0^1 dx \times 2 \times \frac{G_A^k}{r_{\text{max}}} \right]^{-1}$$

where $G(r)$ is the attenuation function, $k$ is the logarithmic slope of the count and $r_{\text{max}}$ was taken to be 0.6 degrees. For $k = -2.0 \pm 0.3$ the second factor in the above equation is 2.8 $\pm$ 0.7 and we find therefore $n(S = 5.5 \text{ mJy}) = 3.1 \times 10^7$ and $n(S = 9.5 \text{ mJy}) = 1.1 \times 10^7$ sterad$^{-1}$ Jy$^{-1}$, in good agreement with the directly determined count (see also Fig. 6).

The integral of the noise distribution from the completeness limit upwards in principle yields the number of “noise sources” in the catalogue of Paper I. The expected number of such non-existent sources is 0.5, but the actual number may be higher because, due to grating ellipse noise, the noise distributions may be appreciably higher than the assumed Gaussians at larger values of $\bar{S}$. It is, however, unlikely that the actual number is larger than, say, 3.

Acknowledgement. The main part of this work was done while I was staying at the Institute of Astronomy at Cambridge. This stay was made possible by a Leverhulme Visiting Fellowship of the Royal Astronomical Society.

References


Katgert, P. 1975, Astron. & Astrophys. 38, 87

P. Katgert
Laboratorio di Radioastronomia CNR
Via Irnerio 46, Bologna, Italia