Cygnus A: determination of the physical parameters

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Summary. From an accurate determination of the synchrotron spectrum of Cygnus A hot spots (Muxlow et al., 1988), we show how it is possible to describe Cygnus A using only classical hydrodynamics and we present a self-consistent model to determine the physical parameters of the plasma inside hot spots and of the jet powering them. As a consequence of the low frequency turnover observed, it is shown that it corresponds to the low energy cut-off of the ultra-relativistic electrons. Inside hot spots the ratio $n_e/n_2$ is about $10^{-3}$, where $n_e$ is the density of the ultra-relativistic electrons and $n_2$ is the density of the thermal electrons. In hot spots, the ultra-relativistic electrons lose their energy via inverse Compton emission on the synchrotron photon gas. The inverse Compton emission begins at about 7 eV in the UV range. From the knowledge of the width of the extended lobe and the existence of an equilibrium between the pressure of the radio lobe and the pressure of the external medium, the velocity of the VLBI jet is found to be $v_{	ext{VLBI}} \approx 0.35 c$. We have investigated the physical properties of the VLBI jet and of the extended lobes. It is shown that the correlation between VLBI jets and large scale structure radio jets is a natural consequence of the presence of ultra-relativistic electrons in the VLBI jet; these electrons have an energy greater than the threshold energy and then can be reaccelerated by the weak oblique shock waves along the large scale radio jet. Assuming a Mach number of 10, the pressure inside the large scale structure jet is found to be twice the pressure of the extended lobe, and intersections of weak oblique shock due to the interaction of the jet with the extended lobe can produce the focusing of the jet. Finally, from the knowledge of the high energy cut-off of the ultra-relativistic electrons, we deduce the value of the diffusion coefficient and verify that the shock is an adiabatic shock and estimate the three scale lengths characteristic of this kind of mixed shock.

Key words: radiogalaxies – jets – hot spots

1. Introduction

In a former article (Muxlow et al., 1988), we have shown that we can a priori describe Cygnus A hot spots by classical hydrodynamics, i.e. $v < c/\sqrt{3}$. In this article, we present a self consistent model of Cygnus A hot spots using non-relativistic hydrodynamics and an electron-proton plasma. In a first step we will suppose that extended radio lobes of Cygnus A contain only one hot spot.

In the next section we will present the self consistent model. In Sect. 3, we will study consequences of the model and finally in Sect. 4 we will estimate the diffusion coefficient and the structure of the mixed shock and we will develop further consequences from the knowledge of the synchrotron spectrum of the hot spots.

2. The self-consistent model

In the following we will use the frame of reference associated with the shock wave, i.e. indices 1 and 2 will correspond to upstream and downstream quantities.

The model will be fitted using data concerning hot spot D, i.e. the brightest one (Hargrave and Ryle, 1974). References and notations can be found in the previous article (Muxlow et al., 1988).

Assuming the magnetic field quasi perpendicular in jets powering Cygnus A hot spots, it has been shown that it is reasonable to start adopting for the Alfvénic Mach number the value $M_A \approx 6$ (Muxlow et al., 1988). From the knowledge of the spectral index, after the shock we have $\theta \approx 6$, i.e. in hot spots the downstream pressure is dominated by the pressure of a classical component, the proton gas. Thus the compression ratio is $r \approx 4$ and $\beta_2 \approx 2.4$. Calling $\chi$ the ratio between the pressure of the relativistic protons and the pressure of the relativistic electrons, we must have $\chi < 0.3$, and we will adopt for numerical applications the value $\chi \approx 0.2$.

The pressure inside hot spots is given by

$$p_2 = p_{\text{max}} \approx 0.75 n_{\text{ext}} n_p v_{\text{sep}}^2,$$

where $n_{\text{ext}}$ is the density of the intracluster gas surrounding the hot spot and $v_{\text{sep}}$ the advance speed of the hot spot in the intergalactic medium. Then the pressure of the classical component is

$$p_{c2} \approx 0.75 \frac{\beta}{1 + \beta} \frac{\theta}{1 + \theta} n_{\text{ext}} n_p v_{\text{sep}}^2 \approx 4.6 \times 10^{-7} v_{\text{sep}}^2.$$

From (2), the definition of $\beta_2$, i.e. $\beta_2 = (p_{c2} + p_{m2})/p_{m2}$ and $p_{m2}$, the magnetic field within hot spots is given by

$$B_2 \approx \left( \frac{3}{4} \right)^{1/2} \frac{8\pi}{\rho_{\text{ext}} + \beta_2} v_{\text{sep}} \approx 2.4 \times 10^{-13} v_{\text{sep}}.$$

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Then, with \( v_{\text{esc}} \approx 0.05 \, c \), the pressure of the thermal classical component (2) is \( p_{\text{esc}} \approx 10^{-8} \, \text{erg cm}^{-3} \) and the magnetic field (3) is \( B \approx 3.6 \, 10^{-4} \, \text{G} \).

From the definition of the Alfvénic Mach number it is possible to obtain the product \( n_1 v_1 \). Indeed, from \( M_\Lambda = v_1 / V_{\Lambda1} \), we have with \( B_2 = B_1 / r \)

\[
n_1 v_1^2 = \frac{M_\Lambda^2 B_1^2}{4 \pi n_p m_p} \approx 1.4 \, 10^{16}.
\] (5)

The radio spectrum begins to steepen at frequencies greater than \( v \approx 2 \, \text{GHz} \). From the knowledge of the steepening frequency we can obtain the velocity of the jet. The energy of an electron radiating at \( v \) is \( E_e / m c^2 \approx 2.1 \, 10^3 \), and the synchrotron life time of this electron is \( \tau_{\text{syn}}(v_e) \approx 1.8 \, 10^3 \) s (Ginzburg, 1978). Thus, from \( v_2 \approx \lambda_2 / \tau_{\text{syn}}(v_2) \), we deduce \( v_2 \approx 0.06 \, c \) and \( v_1 \approx 0.24 \, c \).

We obtain the density in the hot spot from relation (5). It is \( n_2 \approx 4 \, n_1 \approx 1.1 \, 10^{-3} \, \text{e}^{-} \text{cm}^{-3} \).

The mean energy per particle of the thermal classical gas component, the classical proton gas, is obtained from

\[
n_2 E_{\text{th}, p} \approx (3/2) n_2 p_2,
\] (6)

and is \( E_{\text{th}, p} \approx 1.4 \, 10^{-5} \approx 8.5 \, \text{MeV} \).

The low energy cut-off of the power law spectrum of the relativistic electrons is given by

\[
E_0 \approx a \, E_{\text{th}, 3} \approx a \, n_p V_{\Lambda, 2} c,
\] (8)

with \( a > 1 \). Indeed, the strong shock creates a population of relativistic electrons and only those which have an energy greater than the threshold energy to be scattered by MHD turbulence on both sides of the shock are accelerated and thus contribute to the power law. So the low energy cut-off of the power law spectrum is close but greater than the threshold energy \( E_{\text{th}, e} \), i.e. \( a > 1 \).

An electron is sensitive to MHD perturbations of scale length \( \lambda \), if its gyro radius \( r_g = c/\omega_{\text{ce}} \) \((E/mc^2)\) is comparable to \( \lambda \). Since the minimum MHD scale is \( \lambda_0 = \lambda_0 / \omega_{\text{ce}} \) and because the threshold energy in the downstream flow is greater than the threshold energy in the upstream flow, we have \( E_{\text{th}, e} \approx a n_p V_{\Lambda, 2} c \) (Lacombe, 1977 and Pellletier and Roland, 1986). The low energy cut-off of the power law spectrum is therefore \( E_0 \approx a E_{\text{th}, e} \).

From relation (8) we have \( E_0 \approx a \, 75 \, \text{MeV} \) and the corresponding radiative frequency is \( v_0 \approx a^2 10 \, \text{MHz} \). From the radio spectrum (Muxlow et al., 1988) the power law spectrum begins at \( v_0 \approx 200 \, \text{MHz} \) and thus \( a \approx 4.5 \). The determination of the parameter \( a \) is a fundamental consequence of low frequency observations made with MERLIN with a resolution of few arc sec. Further observations at low frequencies with high resolution will allow the detection of the energy cut-off in other sources containing hot spots and the value obtained for Cygnus A, i.e. \( E_0 \approx 4.5 \, n_p V_{\Lambda, 2} c \) represents probably a general value for radio sources produced by shock waves. Of course the numerical value of \( V_{\Lambda, 2} \) has to be determined for each radio source. So we can write

\[
E_0 \approx 4.5 \, n_p V_{\Lambda, 2} c \approx 330 \, \text{MeV}.
\] (9)

Note that the low frequency cut-off observed at \( v_0 \approx 200 \, \text{MHz} \) in Cygnus A hot spots is due to the beginning of the power law spectrum of the ultra-relativistic electrons and not to the synchrotron self absorption, which explains the lack of scintillation of the hot spots at 8.15 MHz (Tien and Duffet-Smith, 1982) and is consistent with the no VLBI detection of Cygnus A hot spots. Moreover, with a flux density \( S_e \approx 150 \, \text{Jy} \), a magnetic field \( B \approx 3.6 \, 10^{-4} \, \text{G} \) and a mean angular size \( \theta \approx 1.1' \), the frequency corresponding to the synchrotron self absorption is \( v_s \approx 45 \, \text{MHz} \) and is not consistent with the observed frequency \( v_0 \approx 200 \, \text{MHz} \).

Let us note that the high value of ratio \( E_{\text{th}, e} / E_{\text{th}, p} \approx 9 \) justifies the low value of the ratio \( \chi \). When the separation velocity increases, the ratio \( E_{\text{th}, e} / E_{\text{th}, p} \) diminishes and \( \chi \) increases. It is when \( v_1 \approx c / \sqrt{3} \) that \( \chi \) has its highest value.

The energy density of the relativistic electrons is (Ginzburg, 1978)

\[
W_{\text{re}} \approx K_0 \frac{E_h}{E_0} \frac{E_0^{1/2}}{K_1} \approx 2.1 \, 10^{-9} \, \text{erg cm}^{-3},
\] (10)

where \( E_0 \) and \( E_h \) are the energy of the ultra-relativistic electrons radiating at \( v_0 \approx 200 \, \text{MHz} \) and \( v_h \approx 5 \, \text{GHz} \).

Remark, that independently of the knowledge of \( S_e \) and \( V \), the value of \( W_{\text{re}} \) is bound by \( W_{\text{re}} \approx 3 p_e \approx 3 p_e / \theta \approx 5 \, 10^{-3} \, \text{erg cm}^{-3} \).

We can estimate the energy density of the thermal electrons, i.e. \( W_{\text{th}, e} \), given by

\[
W_{\text{th}, e} \approx \frac{3 p_e}{\theta (1 + \chi)} - W_{\text{re}} \approx 2.1 \, 10^{-9} \, \text{erg cm}^{-3},
\] (11)

and thus the mean energy per thermal electron is \( E_{\text{th}, e} \approx 1.2 \, \text{MeV} \), i.e. they constitute a relativistic gas. The mean energy per thermal electron is bounded by \( E_{\text{th}, e} \approx 2.1 \, 10^{-9} \, \text{e}^{-} \text{cm}^{-3} \). This result agrees with the low internal depolarisation observed for Cygnus A because the contribution of these thermal electrons to the medium birefringence varies as the inverse square of their relativistic mass (Jones and O’Dell, 1977). Thus a thermal electron gas with a density \( n \approx 1 \times 10^{-3} \, \text{e}^{-} \text{cm}^{-3} \) and a mean energy per particle \( E_{\text{th}, e} \approx 1.2 \, \text{MeV} \), contributes to the medium birefringence as an effective cold gas of density \( n_{\text{eff}} \approx 2 \, 10^{-4} \, \text{e}^{-} \text{cm}^{-3} \). This value agrees with the depolarisation observations which give \( n_{\text{eff}} \approx 4 \, 10^{-4} \, \text{e}^{-} \text{cm}^{-3} \) (Dreher et al., 1987).

There are several physical reasons for not postulating a density as low as \( n_1 \approx 10^{-5} \, 10^{-6} \, \text{e}^{-} \text{cm}^{-3} \). The advance speed of the hot spots given by \( v_{\text{esc}} \approx v_1 (n_1 / n_0)^{1/2} \) is then smaller than 0.01 c, a value too low to fit radio observations (see Muxlow et al., 1988). Furthermore the ratio \( n_1 / n_2 \), where \( n_1 \) and \( n_2 \) are the densities of the ultra-relativistic and of the thermal electrons, will be found otherwise to be greater than unity, but as explained in Sect. 4.4, it has to be of order of 0.001.

Given the fact that the total pressure of the relativistic gas is bound by \( p_{\perp} \approx p_{\parallel} / \theta \), regardless of the knowledge of the flux density \( S_e \) and the volume \( V \) of hot spots, it is the relative values of \( W_{\text{th}, e} \), \( W_{\text{re}} \) and \( W_{\text{th}, p} \) which are more uncertain than their sum.

The results of the self consistent model for Cygnus A hot spots and for the jet powering them are summarized in Table 1.

3. Consequences of the model

3.1. The inverse Compton emission from hot spots

Inside hot spots, the ultra-relativistic electrons lose their energy via synchrotron and inverse Compton radiations. The energy density of the synchrotron photon gas, namely \( W_{\text{ph}, s} \), is

\[
W_{\text{ph}, s} \approx \frac{L_s}{\pi (R_{\text{ph}} / c)} \approx 4.4 \times 10^{-10} \, \text{erg cm}^{-3},
\] (12)

where \( (R_{\text{ph}}) \approx 0.55'' \approx 2.6 \times 10^{14} \, \text{cm} \), is the mean radius of hot spots and \( L_s \) their synchrotron luminosity.
### Table 1. Physical parameters of the plasma of Cygnus A hot spots and of the jets powering them

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jet (index 1)</td>
<td></td>
</tr>
<tr>
<td>Alfvénic Mach number</td>
<td>$M_A \approx 6$</td>
</tr>
<tr>
<td>Jet density</td>
<td>$n_i \approx 2.7 \times 10^{-4} \text{ cm}^{-3}$</td>
</tr>
<tr>
<td>Jet velocity</td>
<td>$v_1 \approx 0.24 c$</td>
</tr>
<tr>
<td>Magnetic field in the jet</td>
<td>$B_j \approx 0.9 \times 10^{-4} \text{ G}$</td>
</tr>
<tr>
<td>Strong shock approximation</td>
<td>$\rho_1 v_1^2 \approx 70 (B_j^2 / 8\pi)$</td>
</tr>
<tr>
<td>Hot spot (index 2)</td>
<td></td>
</tr>
<tr>
<td>Condition of classical hydrodynamics</td>
<td>$v_{sep} \leq 0.088 c$ adopted $v_{sep} \approx 0.05 c$</td>
</tr>
<tr>
<td>Heating ratio in hot spots</td>
<td>$\theta \approx 6$</td>
</tr>
<tr>
<td>Compression ratio</td>
<td>$r \approx 4$</td>
</tr>
<tr>
<td>Parameter $\beta = (p_{r2} + p_{r3})/p_{m2}$</td>
<td>$\beta_2 \approx 2.4$</td>
</tr>
<tr>
<td>Thermal density</td>
<td>$n_2 \approx 1.1 \times 10^{-3} \text{ cm}^{-3}$</td>
</tr>
<tr>
<td>Mean energy per thermal proton</td>
<td>$E_{th,p} \approx 8.5 \text{ MeV}$</td>
</tr>
<tr>
<td>Mean energy per thermal electron</td>
<td>$E_{th,e} \approx 1.2 \text{ MeV}$</td>
</tr>
<tr>
<td>Magnetic field</td>
<td>$B_2 \approx 3.6 \times 10^{-4} \text{ G}$</td>
</tr>
<tr>
<td>Alfvén velocity</td>
<td>$V_{A2} \approx 0.078 c$</td>
</tr>
<tr>
<td>Low energy cut-off of the ultra relativistic electrons</td>
<td>$E_0 \approx 4.5 \text{ E$_{thres}$} \approx 330 \text{ MeV}$</td>
</tr>
<tr>
<td>Ratio $\chi = p_{re}/p_{re}$</td>
<td>$\chi &lt; 0.3$ adopted $\chi \approx 0.2$</td>
</tr>
<tr>
<td>Energy density of the thermal classical protons</td>
<td>$W_{c2} \approx 1.5 \times 10^{-8} \text{ erg cm}^{-3}$</td>
</tr>
<tr>
<td>Magnetic energy density</td>
<td>$W_{m2} \approx W_{c2}/2.9$</td>
</tr>
<tr>
<td>Energy density of the thermal electrons</td>
<td>$W_{th,e} \approx W_{c2}/7.1$</td>
</tr>
<tr>
<td>Energy density of the ultrarelativistic electrons</td>
<td>$W_{re} \approx W_{c2}/18$</td>
</tr>
<tr>
<td>Energy density of relativistic protons</td>
<td></td>
</tr>
</tbody>
</table>

This implies that inside hot spots, ultra-relativistic electrons lose their energy via inverse Compton radiation on the synchrotron photon gas rather than on the 3 K photon gas.

Inverse Compton losses are similar to synchrotron losses in a mean magnetic field $B_{com}$, such that $B_{com}^2 / 8\pi \approx W_{ph,s}$ and then we get $B_{com} \approx 10^{-4} \text{ G}$. The inverse Compton luminosity of hot spots $L_{IC}$, is related to their synchrotron luminosity $L_s$, by

$$L_{IC} \approx (10^{-4}/B_2)^4 L_s \approx 2.2 \times 10^{43} \text{ erg s}^{-1},$$

with $B_2 \approx 3.6 \times 10^{-4} \text{ G}$, $L_s \approx 2.8 \times 10^{44} \text{ erg s}^{-1}$ and the spectral index of the synchrotron radiation $\alpha \approx 1.0$.

The detection of the inverse Compton emission is essential because it will allow the determination of the mean magnetic field in hot spots. Consequently from the knowledge of the magnetic field we will deduce the advance speed of the hot spots (see Eq. 3) independently of radio observations. Assuming for sake of simplicity a synchrotron spectral index $\alpha \approx 1.0$ between $\nu_0 \approx 200$ MHz and $\nu_m \approx 2 \times 10^{14}$ Hz, the corresponding mean energy of a synchrotron photon is $\epsilon_{ph,1} \approx h\nu_0 \log (\nu_m/\nu_0)^{\alpha} \approx 10^{-5} \text{ eV}$. Because the energy $\epsilon_{ph,2}$ of a diffused photon is (Ginzburg, 1978)

$$\epsilon_{ph,2} \approx (4/3) \epsilon_{ph,1} (E/mc)^2,$$

the inverse Compton emission corresponding to the low energy cut-off of the ultra-relativistic electrons is about 7 eV, i.e. the inverse Compton emission begins in the UV range.

3.2. The mass ejected by the nucleus

We can easily estimate the mass ejected by the nucleus per year to power the two hot spots. It is

$$M \approx 2\pi R_{HS}^2 n_i m_p 3 \times 10^7 v_1 \approx 5 M_\odot \text{ yr}^{-1}.$$ (15)

Estimating the age of Cygnus A to about $6.6 \times 10^6 \text{ yr}$, the total mass ejected by the nucleus is of order of $3.3 \times 10^7 M_\odot$.

3.3. The kinetic power of the jet and the luminosity of the hot spot

Just before entering the hot spot, the kinetic power of the jet is

$$P_k = \frac{1}{2} n_i m_p \pi R_{HS}^2 v_1^3 \approx 6.2 \times 10^{45} \text{ erg s}^{-1}$$ (16)

As the luminosity of one spot is $L_{HS} \approx 2.8 \times 10^{44} \text{ erg s}^{-1}$, the luminosity is about 4.5% of the kinetic power of the jet.

3.4. The density of the ultrarelativistic electrons inside hot spots

The density of the ultra-relativistic electrons inside the hot spots is given by

$$n_e \approx K_0 \left( \frac{1}{E_0} - \frac{1}{E_0} \right) + \frac{K_1}{2E_0^2} \approx 1.4 \times 10^{-6} \text{ cm}^{-3}. $$ (17)

Thus we have $n_e/n_2 \approx 1.3 \times 10^{-3}$ and we arrive to the conclusion that about $10^{-3}$ of the electrons are ultrarelativistic inside Cygnus A hot spots.

More generally, let us derive the ratio $n_e/n_2$ when the mean distribution of the ultrarelativistic electrons is $\langle \rho(E) \rangle = KE^{-2.6}$. Inside hot spots, we have

$$W_{re} \approx W_{r0} \frac{B_2^2}{20\pi} = 1 \times 5 \times n_e m_p v_1^2 \times 0.1.$$ (18)

With $E_0 \approx 4.5 \text{ E$_{thres}$}$, we have

$$W_{re} = \int KE^{-1.6} dE \approx K \frac{E_0^{0.6}}{0.6} \approx 12 n_e E_{thres}. $$ (19)
Then using the definition of \( E_{\text{thres}} \), namely \( E_{\text{thres}} = m_p V_{A2} c \), we obtain
\[
\frac{n_1}{n_2} \approx \frac{1}{60} \frac{V_{A2}}{c} \approx 1.3 \times 10^{-3}.
\] (20)
And so, for hot spots similar to those in Cygnus A the ratio \( n_1/n_2 \) has probably an general value of about \( n_1/n_2 \approx 0.001 \).

3.5. Characteristics of the VLBI jet

Although it is possible that VLBI jets can be constituted by a fraction of electrons and positrons, we will suppose in the following that the VLBI jet is an electron-proton jet.

3.5.1. Introduction

As pointed out by radio observations, the radio luminosity of the jet is very small compare to the luminosity of the hot spot. Thus from the conservation of the kinetic power of the jet and the ejected mass per year, we conclude that the velocity of the jet is constant along the jet. Just before the hot spot the velocity of the jet is \( v_j \approx 0.29 \, c \). However as indicated in the previous article (Muxlow et al., 1988) the jet powering hot spot D is bent before entering the hot spot. Assuming a maximum deflection of 37\(^\circ\) of the jet, due to an oblique shock wave, the velocity of the jet before the deflection is twice greater, i.e. \( v_j \approx 0.58 \, c \). Thus the VLBI jet has a velocity \( v_{j, \text{VLBI}} \leq 0.58 \, c \).

Remark that
i) the description of extended lobes of one of the most powerful radio source, Cygnus A, can be done using classical hydrodynamics because the advance speed of hot spots is smaller than 0.088 \( c \);

ii) extended lobes of radio sources similar to Cygnus A can be described with classical jet velocities if the advance speed of their hot spots is smaller than: \( 2.1 \times 10^8 \, n_{\text{ext}}^{1/2} \, \text{cm s}^{-1} \), i.e. \( v_{\text{adv}} \leq 0.22 \, c \) when the density of the external medium is \( n_{\text{ext}} \leq 10^{-3} \, \text{e}^{-} \, \text{cm}^{-3} \);

iii) the linear size and consequently the separation velocity of extended double radio sources associated with elliptical galaxies is a decreasing function of the redshift (Oort, 1987).

We conclude that the great majority of extended radio sources can be described by classical hydrodynamics.

3.5.2. The velocity of the VLBI jet

We have seen that the VLBI jet has a velocity smaller than 0.58 \( c \), moreover from radio observations of the extended lobes it is possible to obtain the velocity of the VLBI jet.

The plasma expands from an initial volume say, \( V_i \), to a final volume say, \( V_f \), when the flow goes from the nucleus to the extended lobe. In this section, indice \( f \) will refer to extended lobes.

From 151 MHz map (Leahy et al., 1988), the width of the extended lobe is constant, which indicates that an equilibrium between the pressure of the radio plasma and the pressure of the external medium as what happens for tails of head-tail radio galaxies.

The volume of the extended lobe is \( V_i \simeq \pi (20^\circ)^2 120^\circ/ \cos 20^\circ \approx 1.6 \times 10^{70} \, \text{cm}^3 \).

The equivalent internal energy of the extended lobe is
\[
U_i \approx \rho_j \, V_{j, \text{VLBI}}^2 \pi R_j^2 V_{j, \text{VLBI}} T_j,
\] (21)
where \( T_j \approx 6.6 \, \text{yr} \) is the age of the radio source.

As the nucleus ejects 5 \( M_{\odot} \) per year to power the extended lobes, we have
\[
\rho_j \, V_{j, \text{VLBI}} \approx \frac{5 \, M_{\odot}}{3 \times 10^7 \pi R_j^2 V_{j, \text{VLBI}}^2}.
\] (22)
and thus we obtain from (21) and (22)
\[
U_i \approx 1.6 \times 10^{-7} \, M_{\odot} \, V_{j, \text{VLBI}}^2 T_j.
\] (23)

The extended lobe expands until the internal pressure is equal to the external pressure. Because \( V_f \gg V_i \) and we assume that the external pressure is constant, the internal energy after the expansion is
\[
U_i \approx U_i - p_{\text{ext}} V_f - LT,
\] (24)
where \( L = 2.5 \times 10^{-5} \, \text{erg s}^{-1} \) is the total luminosity of the source. The mean density around the extended lobe is \( n_{\text{ext}} \approx 10^{-2} \, \text{e}^{-} \, \text{cm}^{-3} \) (see Fig. 3 of Arnaud et al., 1984) and we get \( p_{\text{ext}} \approx 1.4 \times 10^{-10} \, \text{erg cm}^{-3} \) with a gas temperature \( T_j \approx 4.9 \times 10^7 \, K \) (Arnaud et al., 1987).

In the extended lobe we have
\[
\frac{U_i}{V_f} \approx W_{e, f} + W_{r, f} + W_{m, f},
\] (25)
which can be written
\[
\frac{U_i}{V_f} \approx \left( \frac{3}{2} \left( 2 + \theta \right) + \frac{1}{\beta_f} \right) \rho_{e, f}.
\] (26)

Estimating the internal pressure, we can deduce the velocity of the VLBI jet such that the pressure inside the extended lobe is equal to the external pressure.

Writing
\[
p_{\text{ext}} \approx p_{e, f} + p_{r, f} + p_{m, f} \approx \frac{1}{\beta_f} \left( 1 + \theta \right) \rho_{e, f},
\] (27)
and using (24) and (26), we get
\[
U_i \approx a(\theta, \beta_f) p_{\text{ext}} V_f + LT,
\] (28)
with
\[
a(\theta, \beta_f) \approx \frac{3}{2} \left( \frac{\beta_f}{2} + 2 + \theta \right) + \frac{2 + \beta_f}{1 + \beta_f}.
\] (29)

From (23), the velocity of the VLBI jet is
\[
v_{j, \text{VLBI}} \approx \left( \frac{a(\theta, \beta_f) \rho_{\text{ext}} V_f + LT}{1.6 \times 10^{-7} \, M_{\odot} T_j} \right)^{1/2}.
\] (30)

The function \( a(\theta, \beta_f) \) is always smaller than 4 and we obtain in this case \( v_{j, \text{VLBI}} \leq 0.35 \, c \).

3.5.3. Physical parameters of the VLBI jet

Let us go further with the physical parameters of the VLBI jet. With a velocity \( v_{j, \text{VLBI}} \approx 0.35 \, c \), a jet radius \( R_j \approx 1 \, \text{pc} \), and an ejection rate \( M \approx 2.5 \, M_{\odot} \, \text{yr}^{-1} \), its density is \( \rho_{j, \text{VLBI}} \approx 330 \, \text{e}^{-} \, \text{cm}^{-3} \).

Assuming the ultra-relativistic electrons in the VLBI jet are created in the nucleus and that no situ reacceleration occurs,
from 5 GHz VLBI observations, we can estimate the magnetic field in the jet, writing \( \tau_{\text{syn}} \approx \frac{1}{v_{\text{s}}} \), with \( \frac{1}{v_{\text{s}}} \approx 0.0099 \approx 0.1 \text{ pc} \) and \( v = 5 \text{ GHz} \), we obtain \( B_{v, \text{VLBI}} \approx 1.7 \times 10^{-2} \text{ G} \).

Note that, if before the hot spot we have \( \rho_\perp \sqrt{2} \approx 70 B_z / \sqrt{2} \pi \), see Table 1, in the VLBI jet we have \( \rho_\perp B_{\text{VLBI}} \sqrt{2} \approx 5500 (B_{\text{VLBI}} / \sqrt{2} \pi) \).

The kinetic power of the VLBI jet is \( P_K \approx 9.3 \times 10^{45} \text{ erg s}^{-1} \).

Knowing the magnetic field, the size and the flux of the VLBI jet at 5 GHz, we can estimate the low frequency turnover of the jet due to synchrotron self-absorption, it is (Kellerman and Pauliny-Toth, 1981)

\[
v_s \approx \frac{8 B^{1.5} \theta^{2.5}}{\theta^{4.3} \text{ GHz}}.
\]

With \( \theta \approx 1 \text{ Jy} \), \( \langle \theta \rangle \approx 3 \text{ mas} \) and \( B \approx 1.7 \times 10^{-2} \text{ G} \) we obtain \( v_s \approx 1.5 \text{ GHz} \) in good agreement with the observed value (Kafatos et al., 1980).

The Alfvén speed in the VLBI jet is \( V_{A, \text{VLBI}} \approx 2 \times 10^8 \times 0.0067 \text{ c} \) and thus the Alfvénic Mach number in the VLBI jet is \( M_{A, \text{VLBI}} \approx 50 \).

Assuming a mean distribution of the relativistic electrons \( \langle \rho(E) \rangle \approx K E^{-2.6} \), we can calculate the density of the ultra-relativistic electrons in the VLBI jet, it is \( n_e, B_{\text{VLBI}} \approx 1.3 \times 10^{-4} \text{ cm}^{-3} \) and we deduce \( n_e, B_{\text{VLBI}} \approx 3.8 \times 10^{-4} \text{ cm}^{-3} \). The energy density of the ultra-relativistic electrons is \( W_{e, B_{\text{VLBI}}} \approx 2.1 \times 10^{-3} \text{ erg cm}^{-3} \), and we have \( W_{e, B_{\text{VLBI}}} \approx 1.9 \text{ W}_{B_{\text{VLBI}}} \).

Let us remark that the synchrotron losses are negligible as compared to the kinetic power of the VLBI jet.

In the VLBI jet we can verify that the dominant energy is in the thermal range. Indeed, assuming a Mach number \( M \approx 10 \), the energy density of the thermal component is

\[
W_{\text{th, VLBI}} \approx \frac{n_e e^2 m_p}{M^2} \approx 6.2 \times 10^{-4} \text{ erg cm}^{-3} \approx 30 W_{e, B_{\text{VLBI}}},
\]

and the mean energy per thermal particle is \( E_{\text{th, VLBI}} \approx 1 \text{ MeV} \), thus the thermal electrons are probably relativistic.

The magnetic field in the compact core has been estimated by Kellerman et al. (1981) to be \( B_z \approx 10^{-2} \text{ G} \).

The results of the VLBI jet parameters are summarized in Table 2 and we will finish this section noting the importance of a self consistent model of extended radio lobes: it allows the determination of the physical parameters of the jet. Also using VLBI observations it is possible to obtain the characteristics of the jet ejected by the nucleus.

### 3.6. Complementary remarks on the jets powering hot spots

#### 3.6.1. Focusing of the jet

Let us estimate the pressure inside the jet before it bends and powers hot spots. Assuming a Mach number of 10, the internal energy density of the jet is

\[
W_{\text{th, j}} \approx \frac{n_e e^2 m_p}{M^2} \approx 4.7 \times 10^{-10} \text{ erg cm}^{-3},
\]

with \( n_e \approx 2.5 \times 10^{-4} \text{ c} \) and \( v_s \approx 0.35 \text{ c} \). It is easy to verify that the thermal energy is the dominant one in the jet. In first approximation, if we neglect the magnetic energy, the pressure of the thermal component is \( p_{\text{th, j}} \approx 3.1 \times 10^{-10} \text{ erg cm}^{-3} \), twice greater than the pressure in the extended lobe, which is \( p_{\text{hot, i}} \approx 1.4 \times 10^{-10} \text{ erg cm}^{-3} \). Consequently the jet is not far from being confined by the pressure of the extended lobe. However, numerical simulations (Norman et al., 1982) show that the interaction of the jet with the cocoon leads to weak oblique shock waves in the jet. Existence of these oblique shocks produces intersection of oblique shocks which induce a focusing of the jet. Details about intersection of oblique shocks can be found in Sect. 110 of Landau and Lifshitz (1987). Thus we suggest that jets powering hot spots similar to those of Cygnus A can be focused by intersections of weak oblique shock waves due to the interaction of the jet with the cocoon.

#### 3.6.2. Large scale radio jets and VLBI jets

As observed for Cygnus A, the large scale radio jets are observed on the same sides that the VLBI jets. This general property can be understood as follows. The VLBI jets contain ultrarelativistic electrons which lose their energy through radiation. The corresponding large scale jets then contain a population of relativistic electrons which have an energy greater than the threshold energy in the jet, i.e. they can be reaccelerated by the weak oblique shock waves. Consequently the large structure jets associated with the VLBI radio jets are the bright radio jets.

#### 3.7. Determination of the physical parameters in extended lobes

Inside hot spots we have seen the different ratios of energy densities are \( W_e / W_T \approx 3 \) and \( W_e / W_M \approx 3 \) (see Table 1).

### Table 2. Characteristics of the VLBI jet

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity</td>
<td>( v_{A, \text{VLBI}} \approx 0.35 \text{ c} )</td>
</tr>
<tr>
<td>Density</td>
<td>( n_{e, \text{VLBI}} \approx 330 \text{ cm}^{-3} )</td>
</tr>
<tr>
<td>Kinetic power of the jet</td>
<td>( P_K, \text{VLBI} \approx 9.3 \times 10^{45} \text{ erg s}^{-1} )</td>
</tr>
<tr>
<td>Magnetic field</td>
<td>( B_{B_{\text{VLBI}}} \approx 1.7 \times 10^{-2} \text{ G} )</td>
</tr>
<tr>
<td>Hydrodynamical assumption</td>
<td>( \rho_\perp \sqrt{2} \approx 5500 (B_{\text{VLBI}} / \sqrt{2} \pi) )</td>
</tr>
<tr>
<td>Synchrotron reabsorption</td>
<td>( v_s \approx 1.5 \text{ GHz} )</td>
</tr>
<tr>
<td>Alfvén velocity</td>
<td>( V_{A, \text{VLBI}} \approx 0.0067 \text{ c} )</td>
</tr>
<tr>
<td>Alfvénic Mach number</td>
<td>( M_{A, \text{VLBI}} \approx 50 )</td>
</tr>
<tr>
<td>Energy density of ultrarelativistic e–</td>
<td>( W_{e, B_{\text{VLBI}}} \approx 1.9 \text{ W}<em>{B</em>{\text{VLBI}}} \approx 2.1 \times 10^{-5} \text{ erg cm}^{-3} )</td>
</tr>
<tr>
<td>Density of ultrarelativistic electrons</td>
<td>( n_e, B_{\text{VLBI}} \approx 1.3 \times 10^{-4} \text{ cm}^{-3} )</td>
</tr>
<tr>
<td>Mach number</td>
<td>Adopted ( M \approx 10 )</td>
</tr>
<tr>
<td>Thermal energy density</td>
<td>( W_{\text{th, VLBI}} \approx 30 W_{e, \text{VLBI}} \approx 6.2 \times 10^{-4} \text{ erg cm}^{-3} )</td>
</tr>
<tr>
<td>Mean energy/thermal particle</td>
<td>( E_{\text{th, VLBI}} \approx 1.2 \text{ MeV} )</td>
</tr>
</tbody>
</table>
These ratios change when the flow expands from hot spots to extended lobes.

Because the ultrarelativistic electrons lose their energy in extended lobes and the ultrarelativistic protons can only increase their energy, the ratio $\gamma_t$ is unknown and with published observations it is difficult to obtain an unambiguous determination of the physical parameters of the radio plasma in extended lobes.

Estimating the total mass ejected by the nucleus to $M \approx 3.3 \times 10^7 M_\odot$, the mean density in the extended lobe is $n_t \approx 2.4 \times 10^{-6} \text{ e}^{-} \text{ cm}^{-3}$.

Although, the expansion of the flow from hot spots to extended lobes is not adiabatic, from the difference between $\gamma_e = 5/3$ and $\gamma_i = 4/3$, it is possible if the expansion is great enough that the heating ratio becomes smaller than unity, i.e. $\theta_t \leq 1$.

The radio spectrum of the extended lobes shows a low frequency turnover at $v_0 \approx 20 \text{ MHz}$ (Baars et al., 1977), from the knowledge of the low energy cut-off of the ultrarelativistic electrons, i.e. $E_0 \approx 330 \text{ MeV}$, we deduce $B_t \approx 3 \times 10^{-5} \text{ G}$.

The Alfvén speed is then $V_{Ak} \approx 0.15 c$.

From the equilibrium between the internal pressure of the radio lobe and the external pressure, we have $\beta_t \approx (p_{\text{ext}}/p_{\text{m,1}}) - 1 \approx 3$.

The physical parameters of the extended lobe are summarized in Table 3.

4. Further consequences from the knowledge of the synchrotron spectrum of the hot spots

4.1. The high energy cut-off

For most of the hot spots of extragalactic radiosources the synchrotron emission of the relativistic electrons corresponding to the high energy cut-off occurs in the infrared and it has been detected in few cases in the optical. For Cygnus A it occurs before the optical range because no optical emission has been detected from the hot spots (Krönberg et al., 1977). Thus we can estimate the energy of the ultra-relativistic electrons corresponding to the synchrotron emission of the high energy cut-off. Calling $v_m \approx 2 \times 10^{14} \text{ Hz}$ the frequency corresponding to the synchrotron emission of the high energy cut-off, the energy of the ultrarelativistic electrons radiating at $v_m$ is $E_m/mc^2 \approx 6.6 \times 10^5$, which corresponds to a synchrotron lifetime $\tau_{\text{syn, m}} \approx 5.8 \times 10^9 \text{ s} \approx 190 \text{ yr}$. With a velocity $v_e \approx 0.06 c$, the length associated with this lifetime is $l_{\text{syn, m}} \approx 3.5 \text{ pc}$.

4.2. The diffusion coefficient and the level of the MHD turbulence

The diffusion coefficient $D_e$ of the relativistic electrons in a flow which has an angle $\theta$ with the magnetic field is

$$D_e = D_0 \cos^2 \theta + D_1 \sin^2 \theta.$$  (34)

For a particle of velocity $v = c$ and of gyroradius $r_g$, the diffusion coefficients are given by (Jokipii, 1966 and Melrose, 1980)

$$D_0 = (1/3) c^2/v_e,$$  (35)

$$D_1 = (1/3) r_g^2 v_e,$$  (36)

where $v_e$ is the pitch angle scattering frequency, which has been calculated for resonant interactions at the gyroradius $r_g$ with a spectrum of Alfvén waves in $\omega^{-1}$ in the framework of the quasi-linear theory

$$v_e = v_{\text{turb}} (v + 1) \frac{\pi c}{v(v + 2)} \left( \frac{r_g}{l_0} \right)^{-2} \left( \frac{\langle \delta B \rangle^2}{\langle B \rangle^2} \right),$$  (37)

where $v_{\text{turb}} = (\langle \delta B \rangle^2)/\langle B \rangle^2$ and $l_0$ is the long wavelength cut-off of the spectrum.

In the framework of the quasi-linear theory we must have $v_e \ll \omega_g$, where $\omega_g$ is the gyrofrequency of the particle.

In the case of hot spots, the MHD turbulence is not fully developed and the spectrum of Alfvén waves is given by the Kraichman spectrum, i.e. $\omega^{-3/2}$ and moreover as we will see later the level of turbulence is very low, i.e. $v_{\text{turb}} \ll 1$ and thus the diffusion coefficient is dominated by $D_0$. Then it is proportional to $E_0^{-0.5}$ and we can write

$$D_e \approx \frac{D_0}{E_0^{0.5}}.$$  (38)

To estimate $D_0$, we will suppose here that $E_0 \approx 4.5 \text{ E}_{\text{brem}} \approx 330 \text{ MeV}$, i.e. $E_0$ corresponds to the low energy cut-off of the spectrum of the ultra-relativistic electrons. To obtain the function $\alpha(\theta, M_\alpha)$, given Fig. 3 of Muxlow et al. (1988), it has been supposed that the diffusion coefficient does not depend of the energy, but it has been shown in Pelletier and Roland (1987) that as long as $\theta \geq 10^{-1}$ the spectral index is given by the linear theory and consequently it does not depend on the diffusion coefficient. Then the dependence of the diffusion coefficient with the energy does not affect the final result.

The high energy cut-off $E_m$ is obtained when the typical acceleration time $\tau_e$ is the same that the synchrotron loss time. As we have $\tau_e \approx \tau_{\text{syn, m}} v_1 v_2$, we deduce $D(E_m) \approx \tau_{\text{syn, m}} v_1 v_2 \approx 7.5 \times 10^{28}$ and then

$$D_0 \approx \tau_{\text{syn, m}} v_1 v_2 (E_0/E_m)^{0.5} \approx 2.4 \times 10^{27}.$$  (39)

Assuming that the long wave length cut-off of the Alfvén waves is a fraction of the size of hot spots, i.e. say $l_0 \approx 0.2 \text{ kpc}$ and $\theta \approx 80^\circ$, from the expression of the gyroradius $r_g = (E/mc^2)$ (mc^2/eB) and the knowledge of $D_0$, we can estimate the level of the MHD turbulence characterized by $\eta_{\text{turb}}$. Indeed, writing $D_e \approx D_0 \cos^2 \theta$ and from (35), (36) and (38), we have

$$\eta_{\text{turb}} \approx 0.9 c \left( \frac{mc^2 l_0}{eB} \right)^{0.5} \left( \frac{E_0}{mc^2} \right)^{0.5} \cos^2 \theta \frac{D_0}{D_1} \approx 4.6 \times 10^{-4},$$  (39)

which indicates a low level of the MHD turbulence in hot spots and we can verify that we have $D_0 \cos^2 \theta \gg D_1 \sin^2 \theta$.  

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Table 3. Physical parameters in extended lobes

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>$n_t \approx 2.4 \times 10^{-6} \text{ e}^{-} \text{ cm}^{-3}$</td>
</tr>
<tr>
<td>Heating ratio</td>
<td>$\theta_t \leq 1$</td>
</tr>
<tr>
<td>Low frequency turnover</td>
<td>$v_0 \approx 20 \text{ MHz}$</td>
</tr>
<tr>
<td>Magnetic field</td>
<td>$B_t \approx 3 \times 10^{-5} \text{ G}$</td>
</tr>
<tr>
<td>Alfvén speed</td>
<td>$V_{Ak} \approx 0.15 c$</td>
</tr>
<tr>
<td>Ratio $\beta_t$</td>
<td>$(p_c + p_t)/p_m$</td>
</tr>
<tr>
<td></td>
<td>$\beta_t \approx 3$</td>
</tr>
</tbody>
</table>
Let us finish, verifying the validity of the quasi-linear theory for the study of hot spots. As we have \( \omega_q \geq (eB/mc^2) \left( \frac{mc^2}{E_{\text{m}}} \right)^{1/2} \) and \( v_q \leq 3.8 \times 10^{-8} \), we verify the condition of the framework of the quasi linear theory, i.e. \( \omega_q \gg v_q \). This result is probably valid for most of the strong hot spots in the outer parts of the radio lobes.

4.3. The structure of the mixed shock

As indicated in previous articles (Pelletier and Roland, 1986 and 1988), there are three very different scale lengths for the study of mixed shocks responsible of the formation of the synchrotron sources. Namely they are the shokshock depth \( \delta_s \), the diffusion length of the relativistic particles \( \delta_r \), and the typical loss length \( l_{\text{syn,m}} \) of the particles corresponding to the high energy cut-off.

First, we can estimate the small parameter \( \delta \). It is defined by

\[
\delta = \frac{\delta_r}{\delta_s} = \frac{\kappa_{r}}{\kappa_{s}} \quad \text{where} \quad \kappa_{r} \approx N_{s} V_{s}/\omega_{cp} \quad \text{and} \quad \kappa_{s} \approx (c V_{s}/\omega_{cp}) \left( E_{\text{m}}/E_{0} \right)^{0.5}.
\]

So we have

\[
\delta \approx \frac{(v_{r}/c)(E_{0}/E_{\text{m}})^{0.5}}{8.10^{-3}}.
\]

Let us note that \( \alpha(\theta, M_{A}) \) presented Fig. 3 of Muxlow et al. (1988) has been computed assuming \( \delta \approx 10^{-4} \), but it is easy to verify that the result does not change significantly as long as \( \delta \leq 0.03 \) and \( \theta \geq 1 \).

The protons are heated in a layer of width \( \delta_c \) of order of the proton gyroadius \( r_{}\text{sp} \), thus

\[
\delta_c \approx r_{\text{sp}} \approx \frac{m_{\text{p}}c^{2}}{eB} \approx 3 \times 10^{-9} \text{ pc}.
\]

and consequently

\[
\delta_c = \delta_{c}/\delta \approx 4 \times 10^{-7} \text{ pc}.
\]

So we have \( \delta_c \approx 3 \times 10^{-9} \text{ pc} \ll \delta_r \approx 4 \times 10^{-7} \text{ pc} \ll l_{\text{syn,m}} \approx 3.5 \text{ pc} \), which exhibits the three very different scale lengths involved in such mixed shock and which justify models of adiabatic mixed shocks for the study of the formation of extragalactic synchrotron radio sources.

4.4. X-ray synchrotron emission behind mixed shocks

For most of the hot spots, synchrotron emission corresponding to the high energy cut-off is in the IR or in the visible range. But under some circumstances the synchrotron emission can reach the X-ray band. We have seen that in Cygnus A hot spots the parameter \( \eta_{\text{turb}} \) is very small compare to one. If the long wavelength cut-off of the spectrum of the Alfven waves is of order of few parsecs or less and in the same time if the turbulence in the downstream flow becomes stronger, then \( \eta_{\text{turb}} \) can be of order of unity and the parallel diffusion coefficient reaches its lowest value and the perpendicular diffusion coefficient reaches its highest value, see (35) and (36). Moreover when \( v_{r} \sim c_{q} \), the quasi-linear assumption is no longer valid and the diffusion coefficient is given by the Bohm law, i.e.

\[
D_{r} \approx D_{\text{Bohm}} \approx (1/3) c r_{\text{sp}},
\]

and is proportional to the energy.

In such case, we can estimate the high energy cut-off of the spectrum of the ultrarelativistic electrons, writting that the acceleration time is equal to the synchrotron loss time. Thus from

\[
\tau_{\text{syn,m}} \approx \tau_{r} \approx D_{r}/v_{r},
\]

we deduce

\[
E_{\text{m}}/mc^{2} \approx \left( 1.5 \times 10^{6} \frac{v_{r}^{2}}{c} \frac{e}{mc^{2}} B \right)^{1/2} \approx 10^{8},
\]

which provides synchrotron X-ray emission.

In this case, we can verify that the length corresponding to this synchrotron emission is still greater than the characteristic lengths of the mixed shock. Indeed, with \( v_{r} = 0.06c \), we obtain \( l_{\text{syn,m}} \approx 2.3 \times 10^{-3} \text{ pc} \). Thus even in the case of X-ray synchrotron emission the description of the shock wave as an adiabatic shock is still valid.

4.5. In situ reacceleration and acceleration by the bow shock

It is possible to show that either in situ acceleration in hot spots or acceleration by the bow shock occur and it is even possible to evaluate them.

If previously we have studied the acceleration of the ultrarelativistic electrons by the strong shock at the end of the jet, part of the ultrarelativistic electrons responsible of the synchrotron emission can be accelerated by the bow shock. High resolution maps of Cygnus A indicate such possibility (see Fig. 6 from Perley, 1986).

The creation rate of ultrarelativistic electrons by the strong shock is \( \pi R_{\text{Sp}}^{2} v_{r} W_{\text{Sp}} \approx 1.5 \times 10^{44} \text{ erg s}^{-1} \). This value which is half of the luminosity of the hot spot indicates that we need in situ reacceleration and/or acceleration of ultrarelativistic electrons by the bow shock. Inside hot spots we can show that in situ reacceleration by MHD turbulence does not play a fundamental role. As seen previously, hot spots are not strongly turbulent, i.e. \( \eta_{\text{turb}} \approx 10^{-4} \) and assuming a Kraichnan spectrum for the MHD turbulence, the synchrotron frequency \( v_{\text{crit}} \) after which in situ reacceleration by MHD turbulence is inefficient is (Pelletier and Zaninetti, 1984)

\[
v_{\text{crit}} \approx 10^{4} \left( B_{-4} \right)^{-1} (R_{1}^{-1} (n_{-3})^{-1} \text{ Hz} \approx 20 \text{ MHz},
\]

where \( B_{-4} \approx 3.6 \left( 10^{-4} \text{ G} \right), \ R_{1} \approx 1.2 \text{ (kpc)} \) and \( n_{-3} \approx 1.1 \left( 10^{-3} \text{ cm}^{-3} \right) \).

Thus in situ reacceleration by MHD turbulence cannot play a significant role in hot spots. Consequently acceleration by the bow shock cannot be neglected and is of the same order that the acceleration due to the strong shock at the end of the jet. However, we must take into account acceleration by the strong oblique shock which produces the first hot spot. This acceleration is of the same order as the acceleration due to the strong shock at the end of the jet. Moreover in extended lobes, it is easy to see from (45) that in situ reacceleration by MHD turbulence cannot be efficient for particles radiating at frequencies greater than about 100 MHz. The minor role of in situ reacceleration for high frequency radiating electrons in extended lobes of radio galaxies similar to Cygnus A was already noted by Alexander and Leahy (1987). In extended lobes acceleration of relativistic protons can be significant.

As the creation rate of different strong shocks producing double hot spots is about \( 4.5 \times 10^{44} \text{ erg s}^{-1} \) and is about half of the luminosity of the extended lobe, the bow shock around extended lobes accelerates also part of the ultrarelativistic electrons. Acceleration of ultrarelativistic electrons by the bow shock has been observed in the case of 3C 33 by Rudnick (1988).
5. Discussion and conclusion

We have shown from an accurate determination of the synchrotron spectrum (Muxlow et al., 1988), how it is possible to describe Cygnus A hot spots using classical hydrodynamics. A self consistent model is presented. Assuming that the advance speed of hot spots is $v_{rgb} \approx 0.05 \, c$, the pressure of the classical thermal protons which is the dominant pressure in hot spots is found to be $p_a \approx 10^{-8} \, \text{erg cm}^{-3}$ and the magnetic field is $B_a \approx 3.6 \times 10^{-4} \, \text{G}$. As the thermal density is $n_a \approx 1.1 \times 10^{-3} \, \text{e}^{-} \, \text{cm}^{-3}$, the mean energy per thermal proton is $E_{th,n} \approx 8.5 \, \text{MeV}$. The jet velocity is $v \approx 0.29 \, c$, just before entering the hot spot. Consequently the nucleus of the galaxy ejects about $5 \times 10^5 \, \text{yr}^{-1}$ to power its hot spots and the synchrotron luminosity of each of these hot spots is less than 5% of the kinetic energy of the jet. As a consequence of the observed low frequency turnover, it is shown that this low frequency turnover corresponds to the low energy cut-off of the ultrarelativistic electrons with $E_0 \approx 4.5 \times m_e V_{rgb,c} \approx 330 \, \text{MeV}$. The thermal electrons are relativistic, with a mean energy per electron $E_{th,e} \approx 1.2 \, \text{MeV}$, and thus their effective density to the internal Faraday depolarisation is $n_{fe} \approx 2 \times 10^{-4} \, \text{e}^{-} \, \text{cm}^{-3}$ which agrees with the upper limit given by internal Faraday depolarisation observations, i.e. $n_e \approx 4 \times 10^{-4} \, \text{e}^{-} \, \text{cm}^{-3}$. The magnetic energy density is about a third of the classical energy density of the protons, i.e. $W_m \approx W_e/3$. The energy densities of the thermal electrons, the ultrarelativistic electrons and the relativistic protons are of the same order and are respectively $W_{th,e} \approx W_e/7$, $W_{rel,e} \approx W_e/7$, and $W_{rel,p} \approx W_e/18$.

Inside hot spots, the ultrarelativistic electrons lose their energy via inverse Compton emission on the synchrotron photon gas. The inverse Compton emission corresponding to the low energy cut-off of the ultrarelativistic electrons is about $7 \, \text{eV}$, i.e. the inverse Compton emission begins in the UV range.

The ratio $n_e/n_2$ is of order of $10^{-3}$ in hot spots, where $n_e$ is the density of the ultrarelativistic electrons and $n_2$ the density of the thermal electrons.

Assuming an electron-proton VLBI jet, it is shown how we can deduce the physical parameters of the jet ejected by the nucleus (see Table 2). From the knowledge of the width of the extended lobe and the existence of an equilibrium between the pressure of the radio lobe and the pressure of the external medium, the velocity of the VLBI jet is found to be $v_{VLBI} \approx 0.35 \, c$. The energy densities of the magnetic field and of the ultrarelativistic electrons are of the same order and are very small compared to the kinetic power of the jet, i.e. $\rho v_r^2 \approx 5500 (B_r^2/8\pi)$. Assuming a Mach number of 10 for the VLBI jet, it is found that the dominant energy is the thermal one in the jet.

As a natural consequence of the existence of ultrarelativistic electrons in the VLBI jet, these electrons have an energy greater than the threshold energy and can be reaccelerated by oblique shocks in the large scale structure jet. Then we find that the large scale structure radio jets are those associated with the VLBI jets.

Assuming a Mach number of 10, the pressure inside the large scale structure jet is found to be about twice the pressure of extended lobes. Consequently intersections of weak oblique shock waves in the jet can focus the jet. In the extended lobes, after the expansion phase, when an equilibrium is reached with the external medium, the thermal density is $n_e \approx 2.4 \times 10^{-6} \, \text{e}^{-} \, \text{cm}^{-3}$. The low frequency turnover of the extended lobe is $v_0 \approx 20 \, \text{MHz}$. With a low energy cut-off of the ultra-relativistic electrons $E_0 \approx 330 \, \text{MeV}$, the magnetic field is $B_r \approx 3.4 \times 10^{-3} \, \text{G}$. The ratio $\theta_i$ is probably smaller than unity because of the expansion, i.e. $\theta_i \lesssim 1$. Finally the ratio $\beta_i$ is about 3.

From the knowledge of the high energy cut-off of the ultrarelativistic electrons, the value of the diffusion coefficient is derived which shows that hot spots are not strongly MHD turbulent media. We verified that the shock is an adiabatic shock and estimate the three scale lengths characteristics of this kind of mixed shock.

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References