A model for the wind of the M supergiant VX Sagittarii

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Abstract. The velocity distribution of the stellar wind from the M supergiant VX Sgr deduced from interferometric measurements of maser lines by Chapman and Cohen (1986) has been modelled using the linearised theory of stellar winds driven by short period sound waves proposed by Pijpers and Hearn (1989) and the theory of stellar winds driven by short period shocks proposed by Pijpers and Habing (1989). The effect of the radiative forces on the dust formed in the wind is included in a simple way. Good agreement with the observations is obtained by a range of parameters in the theory. A series of observations of the maser lines at intervals of one or a few days may provide additional constraints on the interpretation.

Key words: hydrodynamics – stars: mass loss – non-radial pulsations – stars: individual (VX Sgr)

1. Introduction

To determine the mechanism driving the mass loss from cool supergiants it is necessary to know the mass loss rate and the outflow velocity at various distances from the star. Masing molecules such as SiO, OH and H2O provide an excellent probe for the outflow velocity because the intrinsic line widths are very small. A combination of a high resolution radio image of the stellar envelope with measurement of the Doppler shift of all the maser line emission features in that image can yield much useful information on the velocity profile, and therefore clues to the accelerating mechanism.

Recently the velocity distribution of the outflow from the supergiants S Per (Diamond et al., 1987) and VX Sgr (Chapman and Cohen, 1986) has been measured. The results for VX Sgr are of such an accuracy that it becomes feasible and useful to fit a model stellar wind to the data. VX Sgr is a semi-regular variable with an optical period of 732 days (Kholopov et al., 1987). Its spectral classification varies between M4eIa and M9.5. VX Sgr is one of the most luminous stars known and has an extremely high mass loss rate. The circumstellar envelope is a strong source of SiO, OH and H2O maser emission.

Figure 1, which is Fig. 5 of Chapman and Cohen (1986), shows the radial velocities of the maser features in the wind of VX Sgr relative to the central source, plotted against their angular separation from the assumed stellar position. As usual this position is taken to be the mean position of all features for each of the four types of masers present. Each dashed ellipse traces the expected position-velocity locus for a uniformly expanding spherical shell. The angular radius of each shell can be obtained from the point of intersection of the ellipses with the abscissa and the outflow velocity is given by the intersection with the ordinate. The various loci were fit to the maser features for each of the four masers, although the H2O and OH 1665/7 MHz features show considerable scatter.

The mass loss rate of VX Sgr has been determined by various methods. Hagen (1983) used Si emission lines and derived a mass loss rate of 4 × 10^{-7} M_\odot yr^{-1}. Engels (1983), using the OH maser line velocity and the assumption of a radiation pressure dominated mass loss, derived a much higher value of 2.5 × 10^{-4} M_\odot yr^{-1}. Skinner and Whitmore (1988b) also obtain a value of 2.5 × 10^{-4} M_\odot yr^{-1} using a relation between the mass loss rate and the strength of the silicate emission feature at 9.7 μm, which was calibrated using mass loss rates determined from CO line emission (Skinner and Whitmore, 1988a). They discuss the possibility that free-free radiation gives a large contribution to the IR flux, which can cause the mass loss rate to be over estimated. Taking this into account a mass loss rate of 3.5 × 10^{-5} M_\odot yr^{-1} is obtained. Independent CO measurements done by Knapp et al. (1989) suggest a somewhat lower mass loss rate of 1.3 × 10^{-5} M_\odot yr^{-1}. Most of these mass loss rate determinations depend strongly on model assumptions and therefore the later determinations can be considered consistent.

From Fig. 1 a few qualitative properties of the wind of VX Sgr can be deduced. It is a slowly accelerating wind: the difference in velocity of the features emitting 1665/7 MHz OH maser line radiation and the features emitting in the 1612 MHz line implies that matter is still being accelerated beyond 10 stellar radii. Furthermore the wind has a final velocity that is about half the escape velocity from the surface of the star. The mass loss rate of between 10^{-5} and a few times 10^{-4} M_\odot yr^{-1} is quite high, and comparable to the mass loss rate of OH/IR stars which are not supergiants but stars on the asymptotic giant branch (AGB) and therefore have a mass of only a few times that of the Sun. Classical Miras have a mass loss rate that is typically two orders of magnitude smaller.

The latter qualitative properties are common to most cool giants and supergiants. There is also perhaps some evidence for slow acceleration in the winds of AGB stars, which can be inferred from the difference between the outflow velocity deduced from the OH maser lines and the velocity deduced from the CO lines (van der Veen and Rugers, 1989). Any models for the winds of cool
give the right order of magnitude for the dissipation length, the explanation of the winds from cool giants by Alfvén waves does not seem very probable.

Radiation forces on dust grains as the driving mechanism which was first proposed by Kwok (1975) to explain the winds of cool giants, has two important shortcomings. Since dust will not condense at temperatures higher than 1500 K (Gail and Sedlmayr, 1988), the dust can only form at a distance of several stellar radii. In a hydrostatic stellar atmosphere the density at such distances is so low that no appreciable mass loss can be generated, unless another mechanism operates to bring matter up from the photosphere. The second problem is that once dust has formed, the acceleration occurs very rapidly. To obtain the slow and continuous acceleration observed in cool giants, one needs to assume a dust opacity that increases slowly with distance from the star (Chapman and Cohen, 1986). There is no physical reason for assuming such an increasing dust opacity. Attenuation of the radiation field by an optically thick dust shell, as proposed by Netzer (1989) may also cause a slower increase in outflow velocity, but it is evident that this works only in the region where the dust optical depth is larger than unity.

Stellar pulsation has been proposed by Wood (1979) and Willson and Hill (1979) as a mass loss mechanism. Recent calculations by Bowen (1988) have shown that radial pulsations alone cannot drive the observed mass loss. If dust forms sufficiently close to the star, the radial pulsations extend the star so much that a significant density of matter is present at the region of dust formation so that the observed mass loss rates can be explained. This model is very attractive for Miras because their pulsation has a very large amplitude and is very effective in increasing the atmospheric density scale height. For supergiants it is less attractive because the pulsation of supergiants is generally much less regular and of a smaller amplitude.

In this paper the model for a sound wave driven wind, developed in the paper by Pijpers and Hearn (1989) and applied to AGB stars by Pijpers and Habing (1989), will be applied to the wind of VX Sgr. The influence of radiation forces on dust, forming at some ten stellar radii, will also be considered.

2.1. The models

The outflow of gas from VX Sgr measured by Chapman and Cohen (1986) has been fitted using three different models. The first of these models was developed in a paper by Pijpers and Hearn (1989). In this model the stellar wind is driven by linear acoustic waves. The second model is an extension of this theory which describes a wind driven by periodic shock waves. This model is described in a paper by Pijpers and Habing (1989). In the third model, radiation forces on dust are included in the two previous models. The third model is discussed in Sect. 2.4. The first two models are briefly described in Sects. 2.2 and 2.3.

2.2. Winds driven by linear acoustic waves

Sound waves propagating through a stellar atmosphere exert a pressure on the gas in addition to the thermal gas pressure. This wave pressure is proportional to the energy density of the waves. For waves propagating radially through the accelerating stellar wind the wave pressure decreases with increasing radius because of the divergence of the geometry and the acceleration of the flow even if no wave energy is dissipated. This gives a negative gradient
of the wave theory sound waves dissipate with a characteristic length scale \( \lambda \) measured in stellar radii which is treated as a free parameter. This treatment here follows exactly the theory developed by Pijpers and Hearn (1989).

The models for the wind of VX Sgr are fully determined by:

1. a choice of the sound wave dissipation length \( \lambda \) in stellar radii;
2. a choice of \( \alpha \), which is a dimensionless parameter related to the wave energy flux at the base of the wind;
3. a choice of wave type, either adiabatic or isothermal.

The physical assumptions made in this model are the following:

1. The flow is stationary.
2. The wind is spherically symmetric.
3. The matter in the wind is a perfect gas of homogeneous composition.
4. Viscosity is neglected.
5. The direction of the gas flow and the acoustic wave vectors is radial.
6. Only monochromatic waves are used.
7. The wavelength of the waves is shorter than the typical density scale height in the wind, which is of the order of a stellar radius.
8. The dissipation length of the sound waves is constant.
9. Either purely adiabatic or purely isothermal waves are used.
10. Neither the formation nor the presence of dust in the flow is included. Therefore all radiative forces on the gas are neglected.
11. The atmosphere is isothermal.

Assumptions 1 to 4 do not need comment since they are common to most existing models of stellar winds. The generalisation of the model resulting from removing assumptions 5 and 6 will not lead to any essential difference in the acceleration. Departures from spherical symmetry will probably occur if much of the wave energy comes from nonradial modes such as in Be stars (Vogt and Penrod, 1983). Assumption 7 is essential to the averaging procedure used to obtain the wave pressure from the acoustic waves in a linearised approximation. Time dependent non-linear calculations are necessary to model long wavelength sound waves. Assumptions 8 and 9 are related in that the detailed energy balance between a wave and the surrounding medium determines the dissipational behaviour of the waves. For linear acoustic waves the difference between winds driven by adiabatic waves and winds driven by isothermal waves is small. For large amplitude nonlinear shocks there may be a marked difference in the results. This is discussed in the next section. The dissipation length of the sound waves depends on the amplitude of the waves which varies as a function of radius. The assumption that the wave dissipation length \( \lambda \) is constant is therefore not strictly valid. The value of \( \lambda \) has to be interpreted as an appropriate mean of the dissipation length throughout the stellar wind. Assumption 10 can be relaxed somewhat and this is discussed in Sect. 2.4. Assumption 11 is included to avoid full scale radiative transfer calculations. The gradient of the thermal pressure gas is a very minor source of momentum and the outflow will be little affected by a temperature gradient.

Pijpers and Hearn (1989) have shown that these assumptions lead to the following equation of motion for the gas in dimensionless quantities:

\[
\frac{1}{M} \frac{dM}{d\xi} = -\frac{2\Psi}{\xi^2} - \frac{2}{\xi} \left( 1 + \frac{3}{2} \frac{b_2}{b_1} \right) \frac{3b_1}{2\lambda}
\]

(1)

where \( \xi \) is a dimensionless distance, expressed as a ratio of the stellar radius, and the Mach number \( M \) is a dimensionless velocity, expressed as a ratio of the isothermal or adiabatic speed of sound, \( a \). \( \lambda \) is the dimensionless dissipation length. \( \Psi \) is the dimensionless energy density of the waves. \( \Psi \) is the distance of the critical point in stellar radii for the thermally driven Parker wind (Parker, 1958). This parameter is inversely proportional to the hydrostatic density scale height at the stellar surface, that is the density scale height the stellar atmosphere would have if the star were in hydrostatic equilibrium. \( \Psi \) is related to the stellar mass \( M_* \) and radius \( R_* \) in solar units, to the mean molecular weight of the gas \( \mu \) in atomic mass units, and to the temperature \( T \) in kelvin in the wind by:

\[
\Psi = \frac{b_0 G M_*}{2a^2 r_0} = 1.15 \times 10^{17} \frac{M_*}{R_*} \frac{T}{K}.
\]

(2)

The first term on the right hand side of Eq. (1) involves the force due to gravity, the second term contains the thermal pressure gradient and the wave pressure gradient and the third term includes the dissipation of the sound waves. The constants \( b_0 \), \( b_1 \), and \( b_2 \) are always of the order of unity. The minor difference between adiabatic waves and isothermal waves is accounted for by these constants (see Table 1). The dimensionless energy density \( \epsilon \) is given by:

\[
\epsilon = \frac{M}{(M + 1)^2} \exp \left( \frac{1 - \xi}{\lambda} \right).
\]

(3)

The dimensionless parameter \( \alpha \) defines the energy density of the sound waves at the base of the wind \( E_0 \):

\[
\alpha = \frac{2}{3} \frac{(M_0 + 1)^2}{\rho_0 \rho a^2} E_0.
\]

(4)

where \( M_0, \rho_0 \) are the Mach number and the mass density at the base of the wind and \( a \) is the speed of sound. It can be easily shown that:

\[
\epsilon = \frac{b_0}{3} \frac{\langle \delta u^2 \rangle}{a^2}
\]

(5)

where \( \langle \delta u^2 \rangle \) is the square of the velocity amplitude of the sound waves averaged over the wave period.

Table 1. Constants \( b \) for an ideal monoatomic gas

<table>
<thead>
<tr>
<th></th>
<th>( b_0 )</th>
<th>( b_1 )</th>
<th>( b_2 )</th>
</tr>
</thead>
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<tr>
<td>Isothermal</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Adiabatic</td>
<td>5/3</td>
<td>4/3</td>
<td>1/3</td>
</tr>
</tbody>
</table>

2.3. The hybrid model: winds driven by periodic shock waves

The dissipative behaviour of shock waves is very different from linear sound waves. The amplitude \( \delta \) of spherical shock waves moving outwards in a homogeneous medium (Landau and Lifshitz, 1959) varies as:

\[
\delta \propto \frac{1}{\rho_0 r^2} \log^{-1}(r/r_0)
\]

(6)
where \( r \) is the distance from the centre of the star and \( r_0 \) is the radius of the base of the wind. Two effects are important in the evolution of the amplitude of a propagating shock wave, geometrical effects and strong dissipation. In a stellar atmosphere the background mass density \( \rho_0 \) is not constant but is a function of \( r \). With the equation of continuity in spherical symmetry:

\[
\rho r^2 = \text{const.}
\]  

Eq. (6) becomes:

\[
\frac{\partial \nu}{\partial t} \sim v^{1/2} \log^{-1/2}(r/r_0).
\]  

This means that the strong damping of shock waves in a spherically symmetric, accelerating outflow does not lead to a rapid decrease in amplitude. The competing effects of increasing amplitude resulting from the decrease in background density and increasing shock dissipation with increasing shock amplitude tends to produce waves with a constant, finite amplitude, in contrast to waves that dissipate with a characteristic length scale. This means that the dimensionless energy density of shock waves propagating through the atmosphere can be expressed by Eq. (4) but the square of the velocity amplitude of the periodic shocks averaged over the shock period \( \langle \delta u^2 \rangle \) is no longer a function of radius. Pipers and Habing (1989) have combined the energy density of shock waves with the linear theory of wave pressure to obtain a hybrid model of a shock wave driven wind. The form of the wave pressure is retained but Eq. (5) with a constant value of \( \langle \delta u^2 \rangle \) is now used instead of Eq. (3). The equation of motion for the gas in this hybrid model now has the mean square of the dimensionless velocity amplitude of the shock waves, \( \langle \delta M^2 \rangle \), as a parameter instead of \( \nu \). The parameter \( a \) disappears because the wave dissipation is implicitly taken into account. Pipers and Habing (1989) have shown that the equation of motion for this hybrid model is then:

\[
\frac{1}{M} \frac{dM}{d\zeta} (C_1 - b_0 M^2) = 2\Psi - \frac{2C_2}{\zeta}
\]  

where the constants are:

\[
C_1 = 1 + \frac{1}{2} b_0 b_1 \langle \delta M^2 \rangle
\]  

\[
C_2 = 1 + \frac{1}{2} b_0 b_2 \langle \delta M^2 \rangle.
\]

Both Eqs. (9) and (1) are modified forms of the Parker wind equation (Parker, 1958) but only Eq. (9) can be solved analytically like the Parker wind equation. The mass loss resulting from this type of wind is primarily determined by the position of the critical point. For isothermal waves \( C_2 \equiv 1 \) and the critical point is at \( \zeta = \Psi \). For adiabatic waves the critical point is at \( \zeta = \Psi/C_2 \) which can be much nearer the photosphere. The mass loss is then much greater compared with the wind driven by isothermal waves of the same amplitude.

The parameter \( a \) in Eq. (4) and \( \langle \delta M^2 \rangle \) in Eq. (10) are related since they both express the energy content of the waves at the base of the wind:

\[
\langle \delta M^2 \rangle = \frac{3a}{2b_0} \frac{M_0}{(M_0 + 1)^2}.
\]  

\( M_0 \) is the Mach number of the outflow at the base of the wind. This relation only holds there because the behaviour of linear sound waves and shock waves as they propagate outwards is very different.

Like the equation for a thermal Parker wind, Eqs. (1) and (9) do not contain the mass density. Their solution only specifies the outflow velocity as a function of the distance. The density is a free parameter in the solutions which must be specified at the base of the wind in order to define the mass loss rate:

\[
\rho r^2 = \frac{\dot{M}}{4\pi}.
\]  

This means that there is one degree of freedom left which can be used to fit the observed mass loss rate of \( 2.5 \times 10^{-4} \dot{M}_o \) yr\(^{-1} \).

2.4. The inclusion of radiation forces on dust

At some point in a cooling stellar wind, dust will form. Since dust formation is a temperature dependent process it is not possible to construct an energetically consistent isothermal wind in which dust forms. In the equation of motion only the momentum of the gas is important. It is possible to include the effect of radiation forces on the wind in a wave driven wind in an elementary way if certain additional assumptions are made:

1) There is no interaction between the dust and the waves: the total momentum added to the wind is a simple sum of the individual contributions.

2) The dust forms at a distance from the star at which the flow is already supersonic, so that the velocity structure of the wind inside the radius of dust formation is not affected by the dust.

3) The region in which the dust is accelerated to its final drift velocity with respect to the gas is assumed to be small compared with all spatial scales in the wind, so that the addition of momentum to the gas by radiation forces on dust grains is not a function of gas velocity but only of radius.

4) The total optical depth of the dust is assumed to be less than unity, which reduces the radiation force term to a simple inverse square law.

The details of radiation forces on dust and the momentum coupling of dust and gas in purely radiation driven winds have been presented in a paper by Kwok (1975). With the assumptions given above, the inclusion of radiation forces in the wind driven by soundwaves or in the hybrid model reduces to a fairly straightforward rescaling of the problem. The first step in the modelling of the wind is to use Eqs. (1) or (9) to construct the subsonic wind solution. If the resulting velocity at the radius of dust formation \( R_d \) is less than the sound speed no consistent model can be constructed in this elementary fashion. If the velocity at the radius of dust formation is supersonic, radiation forces on dust grains can be easily included. The information that there is an extra force on the gas caused by the radiation forces on the dust grains cannot pass back down towards the star. The velocity structure inside the radius of dust formation is therefore the same as it was without dust.

At the radius of dust formation and further out in the wind the effective gravity, and therefore the parameter \( \Psi \), is reduced by a factor \( (1 - \Gamma) \) where \( \Gamma \) is the ratio of radiation force outwards to the force of gravity inwards. It is related to the stellar mass \( \dot{M}_* \) in units of a solar mass and to the stellar luminosity \( L_* \) in units of solar luminosity by:

\[
\Gamma = 7.8 \times 10^{-5} \frac{L_*}{\dot{M}_*}
\]  

\( \dot{M}_* \)
Table 2. The data

| Source frequency |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| SiO             | H₂O             | OH              | OH              |
| 43/86 GHz        | 22 GHz          | 1665/7 MHz      | 1612 MHz        |

<table>
<thead>
<tr>
<th>R/Rₘ</th>
<th>V (km s⁻¹)</th>
<th>Scatter (km s⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>11</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

where κ is the dust opacity in cm²g⁻¹, which is assumed to be constant throughout the dusty part of the stellar wind. The wind solution can then be continued from Rₘ using Eqs. (1) and (9) with the new value of Ψ. (1 - Γ) can become negative if the radiative forces are large, but since this is only being applied to the supersonic branch of the solutions, this is not physically inconsistent.

3. Comparison of observations and models

3.1. The data

Figure 1 shows the radial velocities of the SiO, H₂O and OH masers around VX Sgr plotted against their angular separation from the star. This figure is Fig. 5 of the paper by Chapman and Cohen (1986). Each dashed ellipse traces the expected position-velocity locus for a uniformly expanding spherical shell. The outflow velocity of the wind at various distances from the star can be deduced using the intersections of the dashed ellipses with the coordinate axes. The results of this procedure are summarized in Table 2. Knapp et al. (1989) quote an outflow velocity of ≈ 30 km s⁻¹ from their CO observations. The spectrum shown in their paper is severely contaminated by galactic CO, and this velocity determination is therefore much less certain than those by Chapman and Cohen (1986). The CO velocity determination will therefore not be considered in the analysis presented in this paper.

There is a considerable scatter in the outflow velocity, especially at intermediate distances from the star where the OH 1665/1667 MHz and the H₂O 22 GHz maser lines originate. This is to be expected in a sound wave driven wind. The passage of sound waves with a finite velocity amplitude through the masing region will shift the emission peaks back and forth in the spectrum over a range in velocity comparable to that amplitude. This can approach or even exceed the speed of sound, which is about 5 km s⁻¹ in the atmospheres of cool giants and supergiants. In a 'snapshot' of the star one will therefore always find emission over a range of velocities comparable to the sound wave amplitude.

This range in velocities hampers a simple analysis of the systematic outflow since all the measured velocities probably reflect true local gas velocities. To circumvent this problem the systematic velocity at the distance where the maser emission originates will be taken to be the mean of the data points. The scatter in the data will be interpreted as an error bound within which a good fit of the data should lie, even though the scatter may be intrinsic.

3.2. Fitting the models

Evaluation of the dimensionless parameter Ψ in Eq. (2) requires the stellar mass, the stellar radius, the gas temperature and the mean molecular weight. Chapman and Cohen (1986) quote a mass $M_\star$ of 10 $M_\odot$, and a radius $R_\star$ of 2.4 × 10¹⁴ cm which corresponds to 3.4 × 10⁵ $R_\odot$. The luminosity $L_\star$ of 4 × 10⁵ $L_\odot$ gives an effective temperature of 2500 K which would imply a mean atomic weight $\mu$ between 1 and 1.5. Assuming that the gas temperature is about equal to the stellar effective temperature these numbers give a value of Ψ of 17. Other values for the mass and luminosity have been quoted, reducing both stellar mass and stellar radius by roughly a factor of ten. This leaves the value of Ψ essentially unaltered, assuming that the temperature is accurately determined at 2500 K. A value of Ψ of 17 has therefore been adopted for all models.

The results of the linearised theory of Pijpers and Hearn (1989) for driving a wind by sound waves show that for a given star the final velocity of the wind increases with increasing $\alpha$, that is with an increasing flux of waves driving the wind. The final velocity of the wind decreases in general as the dissipation length of the waves is decreased. The final velocity of the wind calculated for both adiabatic and isothermal sound waves does not differ greatly and only the results for the adiabatic sound waves are discussed further. There are only four data points available, and two models that can be fitted to these data points. Each model has two free parameters, and possibly a third if radiation driving is included. Since the four data points cannot be expected to give good constraints on the models, it seems meaningless to do extensive $\chi^2$-testing to obtain a best fit. Instead a simple procedure was followed to construct several models consistent with the data.

Figure 2 shows the velocity of the wind plotted against the distance in stellar radii for adiabatic sound wave driven winds with a specified dissipation length $\lambda$ and where the dimensionless parameter $\alpha$ has been selected so that the solution fits the observed velocity determined from the OH 1612 MHz maser line. This is the outermost point in the velocity determination. Table 3 gives a grid of models calculated for this fit. The table also gives the distance of the critical point in stellar radii, the Mach number of the wave velocity amplitude at the base of the wind and the surface density, calculated by assuming that the mass loss rate is $2.5 \times 10^{-4} M_\odot$ yr⁻¹. The mass loss rate depends linearly on the density at the base of the wind, which is a free parameter in the theory. The final figures are the wind velocity at the base of the wind and at 100 stellar radii. The escape velocity from the surface of the star calculated from the assumed stellar parameters is 33.5 km s⁻¹. This illustrates that the observed wind velocity is about half the surface escape velocity and that the linearised theory of sound wave driven winds has no difficulty in fitting such a low final velocity.

Figure 2 shows that only the linearised sound wave solution with an infinite dissipation length gives a reasonable fit to the other three velocity points obtained from the SiO and H₂O
Fig. 2. Velocity as a function of distance in units of the stellar radius. Points are the Chapman and Cohen (1966) data. Curves are the models fitted to the outermost, velocity points, labeled with the appropriate value of the dissipation length $\lambda$. The curve labelled hybrid is wind driven by fixed amplitude shocks.

Fig. 3. As Fig. 2. Models now fitted to inner velocity points.

Fig. 4. Velocity as a function of distance for the sound wave driven wind with a dissipation length of 2 stellar radii, combined with radiation forces on dust. The curves are labelled with the appropriate values of the ratio of radiation forces to gravity $\Gamma$.

Fig. 5. As Fig. 4, now with a dissipation length of sound waves of 4 stellar radii.

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masers and the OH 1665/1667 maser line. Table 3 also shows the solutions obtained from Eq. (9) for the wind driven by short period shocks, the hybrid model. This too gives a good fit to the observed velocity distribution.

The OH 1612 MHz maser emission originates at about 87 stellar radii. Presumably dust forms closer to the star and the flow velocity will be considerably enhanced by the radiative forces on the dust. The models in Fig. 2 do not include this effect. It would be better to fit solutions to the inner two or three velocity points and then use the radiation driving by the dust described in Sect. 2.4 to fit the outermost velocity point. Figure 3 shows the results of fitting the solutions to the inner data points. Radiation driving is not yet included. The parameters used and the resulting physical quantities are given in Table 4. The hybrid solution and the solution with \( \lambda = \infty \) are the same as in Table 3. Only the solution for \( \lambda = 1.0 \) can be ruled out by the observations.

The radiation forces were included in the models with \( \lambda = 2.0 \), \( \lambda = 4.0 \) and \( \lambda = 8.0 \) by assuming that dust forms at 10 stellar radii. The solution with \( \lambda = 1.0 \) does not fit the inner data points. The hybrid model and the solution with \( \lambda = \infty \) already fit all the data. Including radiation driving with the restrictions discussed in Sect. 2.4 does not change any of the quantities in Table 4 except the outflow velocity at 100 stellar radii. Figures 4, 5 and 6 show the velocity plotted against distance for the remaining models for various values of the ratio of the radiative forces to the force due to gravity \( \Gamma \). All the curves are labelled with the value of \( \Gamma \). All models with \( \lambda \gtrsim 2.0 \) and with driving by radiation forces on dust can be made to fit the data.

4. Discussion

From Figs. 2 to 6 it is clear that a wide range of sound wave driven wind models fit the data. Conversely, at present only very poor constraints can be put on the models by the observations, but it is clear that the dissipation length of the sound waves cannot be less than about one stellar radius. Some of the models listed in Table 4, all of which fit the observed velocities at intermediate distances, have a wave amplitude and mass density at the base of the wind which are extreme or have a supersonic wind in the photosphere. This means that models with an infinite dissipation length and models with a dissipation length of less than 2 stellar radii can be ruled out. From the observed scatter in Fig. 1, an approximate upper limit to the wave amplitude may be deduced. This upper limit may not be very accurate because the column density of material at the very highest and lowest velocities in a wave profile may be too small to exhibit maser emission. The velocity scatter should however be indicative of the velocity amplitude of the waves. In Fig. 7 the velocity amplitude of the sound waves for the models listed in Table 4 is plotted against distance in stellar radii. Models with a dissipation length less than about 2 stellar radii are obviously ruled out because of the large amplitudes at the base of the wind; although in these models the assumption that the dissipation length of the sound waves is a constant throughout the wind would break down at the higher densities at the base of the wind. Models with wave dissipation lengths of between 4 and 8 stellar radii seem to fit all the physical requirements best.

Independent measurement of the surface density, or the total column density of the wind is necessary in order to constrain further the models with \( \lambda \gtrsim 2.0 \). All that can be concluded at present is that for the given mass loss rate and with the outflow
Table 3. Models fitting the outer velocity point

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$x$</th>
<th>$R_d/R_*$</th>
<th>$&lt;\delta M^2&gt;_{0}^{1/2}$</th>
<th>$\rho_0$ (cm$^{-3}$)</th>
<th>$V_0$ (km s$^{-1}$)</th>
<th>$V_{100}$ (km s$^{-1}$)</th>
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<tr>
<td>1.0</td>
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<td>1.40</td>
<td>4.99</td>
<td>$6.10 \times 10^9$</td>
<td>16.0</td>
<td>16.7</td>
</tr>
<tr>
<td>78.0</td>
<td>1.34</td>
<td>5.02</td>
<td>$5.81 \times 10^9$</td>
<td>16.8</td>
<td>18.0</td>
<td></td>
</tr>
<tr>
<td>82.0</td>
<td>1.28</td>
<td>5.07</td>
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Hybrid | 10.4 | 1.23 | $1.40 \times 10^{13}$ | $7.00 \times 10^{-3}$ | 16.7 |
| 9.46 | 1.30 | $4.02 \times 10^{12}$ | $2.44 \times 10^{-2}$ | 18.6 |
| 8.97 | 1.34 | $2.23 \times 10^{12}$ | $4.40 \times 10^{-2}$ | 18.9 |

Table 4. Models with correct intermediate velocities

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<th>$\lambda$</th>
<th>$x$</th>
<th>$R_d/R_*$</th>
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<th>$\rho_0$ (cm$^{-3}$)</th>
<th>$V_0$ (km s$^{-1}$)</th>
<th>$V_{100}$ (km s$^{-1}$)</th>
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<td>$1.52 \times 10^{19}$</td>
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<td>18.4</td>
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</table>

Hybrid | 9.96 | 1.26 | $7.75 \times 10^{12}$ | $1.26 \times 10^{-2}$ | 17.8 |

From the spatial information reported by Chapman and Cohen (1986) there is some evidence for asphericity in the mass distribution of the wind. Since the short wavelength sound waves responsible for the mass loss in this model might well be generated by non-radial pulsations of the star, such asphericities would be consistent with non-radial pulsations. The influence of such asphericities on the dynamics of the outflow has not been investigated, and a further discussion does not seem warranted at present.

A closer study of these asphericities and of velocity shifts of the maser lines in the winds of cool giants and supergiants, on time scales of a few days does seem worthwhile. These times are very short compared with the radial pulsation periods of VX Sgr and of Mira variables in general. In the models that fit the data the velocity amplitude of sound waves in the stellar photosphere is much less than the sound speed. The thermal width of atmospheric lines is of the order of the sound speed and so the presence of sound waves needed to drive the winds of red giant stars will
hardly be detectable as a broadening of atmospheric line profiles. At a few stellar radii from the star the amplitude of the linear sound waves will have increased, to become of the order of the sound speed, which means that intrinsically narrow lines such as maser lines coming from a small spatial region should exhibit Doppler shifts in the spectrum of several km s\(^{-1}\) on time scales of a few weeks or even days. Further study requires repeated long base line interferometric measurements of the velocity and position of the maser features. Presumably the SiO maser, which occurs closest to the star, would be the most sensitive to this effect.

5. Conclusions

The velocity distribution of the stellar wind from the M supergiant VX Sgr deduced by Chapman and Cohen (1986) from long baseline interferometry of the SiO, OH and H\(_2\)O masers and a mass loss rate of \(2.5 \times 10^{-4} \dot{M} \odot \text{yr}^{-1}\) can be explained by a linearized theory of a stellar wind driven by short period sound waves either with or without the inclusion of the radiation forces on dust formed in the wind. The observations are not sufficient to give a uniquely defined model. Conversely the theory has no difficulty in explaining the velocity distribution of the wind which goes up to half the escape velocity from surface of the star. Independent measurements of the density in the stellar wind or of the sound wave amplitude are necessary to constrain the interpretation further. The latter may perhaps be obtained from a series of interferometric measurements of the maser lines at intervals less than the period of the sound waves.

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References