Cygnus A: hot spot spectra and the condition of classical hydrodynamics

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Received January 12, accepted May 19, 1988

Summary. We have observed Cygnus A hot spots at 408 MHz with MERLIN and using previous results obtained at 151 MHz with MERLIN and at 1450 MHz with the VLA (Leahy et al., 1988) and at 5 GHz and 15 GHz with the VLA (Carilli et al., 1987, private communication) convolved with a 3 arcsecond beam, corresponding to the MERLIN beam at 151 MHz, we get an accurate determination of hot spot spectra. The spectra exhibit a low frequency turnover at \(v_\nu \approx 200 \text{ MHz} \). From the knowledge of the spectral index at high frequencies, i.e. \(\alpha \approx 1.0\) and the steepening frequency \(v_\nu \approx 2 \text{ GHz}\), we show that it is a priori possible to make a model of hot spots supplied by jets using classical hydrodynamics, i.e. \(v < c/\sqrt{3}\).

Key words: radiogalaxies – jets – hot spots

1. Introduction

The radio source Cygnus A is characterized by hot spots in the outer parts of the radio lobes. Rees (1971) and Scheuer (1974) investigated continuous flow models in which the hot spots are supplied by a beam consisting of strong electromagnetic waves or relativistic particles. Latter it was shown that hot spots are powered by very faint radio jets and are generally considered as the downstream flow of a strong shock produced by the interaction of a supersonic jet with the intergalactic medium (Blandford and Rees, 1974). Considering hot spots as the downstream flow of a strong shock, we have to involve a priori three components, the magnetic field, a relativistic gas and a thermal classical gas. The collisionless thermal plasma carries a frozen-in magnetic field and part of the flow is MHD turbulent. The relativistic gas is coupled to the thermal plasma through first order Fermi process. If we consider the phenomenology of collisionless shocks in plasma (Tidman and Krall, 1971; Biskamp, 1973), one cannot ignore the thermal plasma in the downstream flow, i.e. in the hot spots. Radio observations show that the magnetic field is generally quasi perpendicular to the jet in the case of hot spots. The phenomenology of quasi perpendicular shocks is quite well understood (Leroy et al., 1982) and indicates that the main entropy production process is the thermal proton heating through partial reflection on magnetic and electrostatic barriers.

In the following we will use the frame of reference associated with the shock wave; indices 1 and 2 will correspond to upstream and downstream quantities. Thus indices 1 and 2 are related to the jet and the hot spot respectively. We will consider here that the shock is a magnetized mixed one, i.e.

1) we have an arbitrary ratio \(\theta\) between the post-shock classical pressure \(p_{c2}\) and the post-shock relativistic pressure \(p_{r2}\), namely \(\theta \equiv p_{c2} / p_{r2}\). The pressure is related to the energy density \(W\), by the relation \(p = (\gamma - 1) W\), with \(\gamma_c = 5/3\) and \(\gamma_r = 4/3\) for the classical and the relativistic components respectively. We will call \(\theta\) the heating ratio.

2) the ratio \(\dot{\beta}_3 \equiv \dot{p}_{\text{net}2} / p_{\text{net}3}\) is greater than one, where \(\dot{p}_{\text{net}2}\) and \(p_{\text{net}3}\) are respectively the particle and the magnetic pressures.

In the next section we will recall the previous observations of Cygnus A. In Sect. 3, we will describe 408 MHz observations of Cygnus A hot spots obtained with MERLIN and we will present hot spot spectra and adopted values for hot spots. In Sect. 4, we will derive the minimum pressure magnetic field, the constraints on the physical parameters of the plasma inside hot spots, and we will show how to estimate their advance speed in the intergalactic medium. Finally, in Sect. 5 we will obtain the condition to describe Cygnus A hot spots using classical hydrodynamics.

2. Previous observations of Cygnus A

The references will concern mostly articles published after 1970.

2.1. The optical and IR observations

Optical spectra and their analysis have been obtained by Minton and Minton (1972), Osterbrock and Miller (1975), Simkin (1977), Baldwin et al. (1981), Osterbrock (1983) and Pierce and Stockton (1986). Images of the galaxy have been made by van den Bergh (1976), Krönberg et al. (1977) and Thompson (1984). The cluster around Cygnus A have been studied by Spinrad and Stauffer (1982).

IR detection of Cygnus A is reported by Rieke and Low (1972). Moreover, Cygnus A has been detected by IRAS, and is associated with the source 19577+4035 of the point source catalog (IRAS, Astronomical Satellite Catalogs and Atlases, 1985). The coordinates are \(\alpha = 19\text{h}57\text{m}44.2\text{''}\) and \(\delta = 40\deg 35\arcmin 43\arcsec\), and the IR source coincides with the optical galaxy. IR flux densities are \(S(12\mu) \approx 0.25\text{ Jy}\), \(S(25\mu) \approx 1.07\text{ Jy}\), \(S(60\mu) \approx 2.29\text{ Jy}\) and \(S(100\mu) \approx 8.11\text{ Jy}\).

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2.2. The X-ray observations

Arnaud et al. (1984) have presented detailed Einstein observations from the hot intergalactic medium surrounding the radio source. More recently, Arnaud et al. (1987) published EXOSAT observations of Cygnus A.

2.3. The radio observations

2.3.1. Intensity observations of the extended lobes

The existence of double hot spots in Cygnus A was established by Miley and Wade (1971). We will adopt Hargrave and Ryle (1974) notation of hot spots, namely, the western extended component contains a main hot spot A and a fainter hot spot B, and the eastern component contains a main hot spot D.


The radio spectrum of Cygnus A can be found in Baars et al. (1977).

2.3.2. Polarisation observations of the extended lobes

Polarisation observations of the extended lobes of or the hot spots have been made by Mitton (1971), Hargrave and Ryle (1974), Flett and Henderson (1979), De Young et al. (1979), Dreher (1979), Berlin et al. (1981), Dreher (1981), Alexander et al. (1984) and Dreher et al. (1987).

2.3.3. Observations of the radio nucleus

Radio fluxes of the very compact component associated with the nucleus of Cygnus A have been obtained by Hargrave and Ryle (1974, 1976), Bentley et al. (1975), Hachenberg et al. (1976), Hobbs et al. (1978), Kafatos et al. (1980) and Berlin et al. (1981).

High resolution maps using VLBI observations have been made by Kellerman et al. (1975), Linfield (1981), Kellerman et al. (1981) and Linfield (1985).

2.3.4. Can we describe Cygnus A by a jet model?

Observation of the jet detected by Perley et al. (1984) show a first deflection and a probable second one before entering in hot spot B (see the map obtained by Schuefer, Laing and Perley and published by Verschuur 1987). Moreover, if we admit that the direction of the counter jet is the same as the direction of the jet, it is easy to see that the counter jet does not point toward D. These observations can be explained if the direction of the jet varies with time.

From the comparison of the radio spectra of the two extended lobes, Pariskii and Sobolova (1980) found that the western component is shifted toward the blue and the eastern toward the red with a radial velocity smaller than 0.03 c. As the advance speed of the hot spots is about 0.05 c (see Sect. 4.4), we conclude from radio observations that the radio axis makes an angle greater than 45° with the line of sight. As we have seen before the direction of the radio axis is variable with time. Thus to take into account possible projection effects, for numerical applications we will suppose that the jet makes a mean angle of 70° with the line of sight.

The double hot spots can be explained if we have a strong oblique shock which produces the first hot spot and then a strong shock perpendicular to the jet which produces the second hot spot. The best example of a radio source containing such double hot spot is 3C388 (Burns and Christiansen, 1980). With such scenario, there exists a maximum angle for the deflection of the jet which is $\theta_{\text{max}} \approx 37°$ (see Sect. 92 of Landau and Lifhitz, 1987). It is easy to see that if the jet is not perpendicular to the line of sight, a deflection with $\theta \approx \theta_{\text{max}}$ can produce on the plane of the sky a deflection as great as 90°. Hot spots A and D lie symmetrically apart from the optical galaxy and 151 MHz map (Leahy et al., 1988) show in the eastern lobe a component symmetric to hot spot B. This component has probably the same origin as hot spot B, i.e. it is due to an oblique shock. The jet entering hot spot D is detected in the high resolution map of hot spot D (Perley, 1986). The origin of the fainter component E, which appears as a plateau in the high resolution map, is not clear. However, as the hot spot B will advance in the extended lobe, the jet will stop to power hot spot A. In such scenario, the characteristic time of the lifetime of one hot spot is shorter than 10^5 years and is small compared to the age of Cygnus A. Numerical simulations of a jet with a variable direction to produce double hot spots have been made by Williams and Gull (1985).

In the case of the western component, the intrinsic direction of the magnetic field projected on the plane of the sky is about 60° which is compatible with a quasi perpendicular magnetic field. In the case of the eastern component, high frequency observations Perley (1986) indicate that the magnetic field may be quasi perpendicular to the jet in hot spot D. However, projection effects allow to observe a quasi perpendicular magnetic field as a quasi parallel field.

As pointed out by Pelletier and Roland (1988), there exists an important difference between physical parameters of hot spots when the magnetic field in the jet is either quasi perpendicular or quasi parallel. In the former case with an Alfvénic Mach number $M_A \approx 6$ we have $\beta_2 \approx 2.5$ but in the latter case with the same Alfvénic Mach number, i.e. $M_A \approx 6$ we have $\beta_2 \approx 50$.

3. MERLIN observations and hot spot spectra

3.1. MERLIN 408 MHz observations

The observations were made with the six station Jodrell Bank MERLIN array on June 30th 1981. Because of the angular size and extreme brightness of Cygnus A there are several unusual features associated with the dataset. In order to avoid image smearing at the outer edges of the source structure, the observing bandwidth was reduced to 625 kHz compared with the full MERLIN 408 MHz bandwidth of 4 MHz. The set noise is dominated by Cygnus A itself; it is therefore difficult to accurately calibrate the data by conventional point source observation techniques since all such candidates are of very much lower flux density. The data were thus initially calibrated by detailed series of on/off source total power measurements. Although high values of correlation were achieved on the short spacing maxima, at no point did the correlator deviation from linearity exceed 2%, and thus no subsequent corrections have been applied.
Fig. 1. The 408 MHz map obtained with MERLIN. The beam is 3" corresponding to the 151 MHz MERLIN beam. The lack of very short baseline did not allow us to obtain the map of extended lobes in a proper way. The peak flux is 138 Jy/beam. The contour levels are $-2, 2, 5, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120, 130$ Jy/beam.

The data were analysed by closure-phase techniques starting from a model derived from the VLA 1450 MHz clean delta functions. The 408 MHz image was finally rescaled to bring it into alignment with the measured source spectral index between 151 MHz and 1450 MHz (Leahy et al., 1988) in regions of the extended lobes away from the hot spots where the index is a simple power law. These regions were restricted to lie not more than 20' from the hot spots since at distances greater than this there is evidence to suggest that flux density is missing from the 408 MHz image because of a lack of very short baseline coverage. The map given in Fig. 1 has been convolved to 3" resolution since for comparison purposes this is the limiting resolution set by the MERLIN 151 MHz image.

3.2. Hot spot spectra

An accurate knowledge of the hot spot spectra is a difficult task because flux densities are obtained with various resolutions and because of the presence of surrounding emission. To get the best determination of the spectra we have used MERLIN and VLA observations convolved with the same beam, i.e. 3" which is the resolution of MERLIN at 151 MHz. The MERLIN 151 MHz data and VLA observations at 1450 MHz are taken from Leahy et al. (1988), data obtained with MERLIN at 408 MHz are those published in this article and VLA data at 5 GHz and 15 GHz were kindly provided to us before publication by Carilli, Dreher and Perley (1987). Flux densities were obtained from the 5 frequency maps convolved with a 3" beam, using the JMFIT program of AIPS. Results are presented Table 1.

The spectra of the two main hot spots A and D are presented in Fig. 2-a and Fig. 2-b respectively. Although Cygnus A hot spots have been observed several times at various wavelengths and various resolutions, we have only used in Figs. 2a and b data given in Table 1 and we have indicated the upper limit of 100 Jy obtained by Tsien and Duffet-Smith (1982) at 81.5 MHz from the lack of scintillation of Cygnus A hot spots.

Table 1. Flux densities of hot spots at 0.15, 0.4, 1.5, 5 and 15 GHz with a 3" beam

<table>
<thead>
<tr>
<th>Frequency (GHz)</th>
<th>0.15</th>
<th>0.4</th>
<th>1.5</th>
<th>5</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hot spot A</td>
<td>(Jy)</td>
<td>175.9 (1)</td>
<td>149.2 (2)</td>
<td>68.4 (1)</td>
<td>26.8 (3)</td>
</tr>
<tr>
<td>Hot spot B</td>
<td>(Jy)</td>
<td>111.9 (1)</td>
<td>070.1 (2)</td>
<td>26.3 (1)</td>
<td>(Jy)</td>
</tr>
<tr>
<td>Hot spot D</td>
<td>(Jy)</td>
<td>158.9 (1)</td>
<td>155.7 (2)</td>
<td>82.2 (1)</td>
<td>40.6 (3)</td>
</tr>
</tbody>
</table>

The high frequency spectral index \( \alpha \approx 1.0 \), was already noted by Dreher (1981), Wright and Birkinshaw (1984) and Alexander et al. (1984).

From optical observations and the lack of detection of optical emission from Cygnus A hot spots (Kronberg et al. 1977), we deduce that the high energy cut-off of the spectrum of the synchrotron emission of the ultrarelativistic electrons is in the IR range.

3.3. Adopted values for Cygnus A hot spots

Assuming the Hubble constant to be \( H_0 \approx 50 \text{ km s}^{-1} \text{ Mpc}^{-1} \), the distance of Cygnus A is \( D \approx 320 \text{ Mpc} \) (Minton and Minton, 1972; Osterbrock and Miller, 1975; Simkin, 1977; Spinrad and Stauffer, 1982; Osterbrock, 1983 and Pierce and Stockton, 1986) and then 1° corresponds to 1.55 kpc.

The density of the hot intergalactic gas surrounding Cygnus A hot spots is found by Arnoud et al. (1984), to be \( n_{\text{hot}} \approx 6 \times 10^{-3} \text{ cm}^{-3} \) and its temperature is \( T_{\text{hot}} \approx 4.9 \times 10^7 \text{ K} \), Arnoud et al. (1987).

Using the radio spectrum of Cygnus A presented by Baars et al. (1977), we deduce the radio luminosity of the extended lobes excluding hot spots A and D is \( L_{\text{rel}} \approx 1.8 \times 10^{44} \text{ erg s}^{-1} \).

The model will be fitted using data concerning hot spot D, i.e. the brightest one. We will adopt for the characteristics of the radio spectrum of the hot spot, a flux density at 5 GHz, \( S(5 \text{ GHz}) \approx 40 \text{ Jy} \), a spectral index \( \alpha_0 \approx 0.5 \) between 200 MHz and 5 GHz, although the spectrum begins to steepen at \( v_s \approx 2 \text{ GHz} \), and a high frequency spectral index \( \alpha_1 \approx 1 \) between 5 GHz and \( 2 \times 10^{14} \text{ Hz} \), corresponding to the adopted high frequency cut-off. Then the integrated synchrotron luminosity of one hot spot is \( L_{\gamma} \approx 2.8 \times 10^{44} \text{ erg s}^{-1} \).

The size of hot spot D is obtained from high resolution observations made with the VLA at 2 cm (see Fig. 6 of Perley, 1986). The diameter of the jet which is supposed to be equal to the diameter of the hot spot is \( 2R_{\text{hot}} \approx 1.6 \), thus the radius of the jet is \( R_{\text{jet}} \approx 1.2 \text{ kpc} \) and the length of the hot spot is \( l_{\text{hot}} \approx 0.7 \approx 1.1 \text{ kpc} \). Thus the volume of the hot spot is \( V \approx 1.3 \times 10^{65} \text{ cm}^3 \).

4. Constraints on the parameters

4.1. Main results from the hydrodynamical model

The main results from the hydrodynamical model can be found in Tables 1 and 2 from Pelletier and Roland (1986). They can be summarized as follow:

i) the thermal density inside hot spots is \( n_{\gamma} \approx 10^{-3} \text{ cm}^{-3} \), value mostly independent of the parameters \( \theta \) and \( \chi \), where \( \chi \) is the ratio between the pressure of the relativistic protons and the pressure of the relativistic electrons, namely \( \chi = p_{\text{pro}}/p_{\text{elec}} \).

ii) the jet velocity \( v_j \) in the frame of reference of the shock wave is \( 0.15 \leq v_j \leq 0.58 \).

The lowest velocity, \( v_j \approx 0.15 \), which is obtained for \( \theta = 0 \) and \( \chi = 0 \) is mostly excluded because the advance speed of the hot spot, given by \( v_{\text{sep}} \approx v_j (n_j/n_{\text{hot}})^{1/2} \) then \( v_{\text{sep}} \approx 0.02 \), value too low to fit radio observations. Then the radio properties of Cygnus A hot spots can be obtained, either if the jet velocity is classical, i.e. \( v_j \approx 0.3 \), with a downstream pressure dominated by the thermal classical component or if the jet reaches a relativistic velocity, i.e. \( v_j \approx c/\sqrt{3} \), with a downstream pressure dominated by the relativistic protons.
If more generally $\chi \leq 1$, the jet velocity is always classical and it is only for $\chi \approx \chi_{\text{max}}$ and $\theta \approx 0$ that the jet becomes relativistic.

iii) Assuming the magnetic field is quasi perpendicular to the jet, in hot spots it is $B_\perp \approx 3 \times 10^{-4}$ G and the ratio $\beta$ is $\beta_2 \approx 2 - 3$, which indicates that the hydrodynamical assumption is valid only to the first approximation and it is necessary to introduce magnetic corrections for mixed shocks to describe formation of hot spots.

It is in the scope of this article to show that it is possible to make a model of Cygnus A hot spots involving classical hydrodynamics and an electron proton plasma and then we will investigate the description of Cygnus A hot spots with $r_1 \approx 0.3c$, $\theta \geq 1$ and $\beta \approx 2 - 3$.

4.2. The Alfvénic Mach number and the heating ratio

Assuming the magnetic field quasi perpendicular in Cygnus A hot spots, we know from Pelletier and Roland (1986) that the parameter $\beta$ is $\beta_2 \approx 2 - 3$. It has been shown in Pelletier and Roland (1988) that the parameter $\beta$ is related to the compression ratio $r$ and the Alfvénic Mach number $M_\text{A}$ by the relation

$$M_\text{A}^2 \approx \frac{r^3}{2(r-1)(1 + \beta_2)}.$$  \hfill (1)

If we suppose $\theta \geq 1$, we have $r \approx 4$ and from $\beta_2 \approx 2 - 3$ we deduce from (1) $M_\text{A} \approx 6 \pm 1$. Thus the Alfvénic Mach number in the jet is well determined and consequently in the following we will start adopting for the Alfvénic Mach number the value $M_\text{A} \approx 6$.

The high frequency spectral index of hot spots is $\alpha_1 \approx 1.0$ and if we take into account synchrotron losses, we deduce that the radio spectral index just after the shock is $\alpha_\text{s} \approx 0.5$. From $M_\text{A} \approx 6$ and $\alpha_\text{s} \approx 0.5$ and we obtain that the heating ratio $\theta$ is $\theta \approx 6$ (Fig. 3). Thus the dominant pressure inside hot spots is the pressure of a classical component constituted by the proton gas. This is a natural consequence of the dissipation of the kinetic energy of the jet. The compression ratio of the shock is given by

$$r = \frac{r_0}{1 + \frac{r_0}{3} \frac{k_0^2}{r^2}},$$ \hfill (2)

where $r_0 = (7 + 4\theta)/(1 + \theta)$. Then $r \approx 4$ with $M_\text{A} \approx 6$ and $\theta \approx 6$ and using relation (1) we find $\beta_2 \approx 2.4$.

4.3. The minimum pressure magnetic field and constraints on the parameters

With the first step assumption that the downstream pressure is dominated by the ultrarelativistic electrons, i.e. $\theta \approx 0$ and $\chi \approx 0$, let us estimate the magnetic field that inside hot spots minimizes the sum of the magnetic pressure and of the ultrarelativistic electron pressure. We will call it, the minimum pressure magnetic field, namely $B_{\text{mp}}$. As indicated Sect. 3.3, synchrotron spectra of hot spots have a spectral index $\alpha_\text{s} \approx 0.5$ between 200 MHz and 5 GHz and a spectral index $\alpha_1 \approx 1.0$ at high frequencies. Calling $E_0$ and $E_\text{s}$ the energies of ultrarelativistic electrons radiating at $v_0 \approx 200$ MHz and at $v_\text{s} \approx 5$ GHz, the energy density of the ultrarelativistic electrons is

$$W_\text{u} \approx K_0 \frac{E_\text{s}}{E_0} \log \frac{E_0}{E_\text{s}} + K_1 E_\text{s}^{-1},$$ \hfill (3)

where $K$ is given by the synchrotron theory (Ginzburg, 1978).

Assuming that we minimize the sum of the magnetic pressure and of the ultrarelativistic electron pressure, we have $p_\text{m,mp} \approx (3/4)p_\text{e,mp}$ and the minimum pressure magnetic field is then $B_{\text{mp}} \approx 1.9 \times 10^{-4}$ G.

The advance speed of the bow shock in front of the hot spot is related to the pressure inside hot spots by (Landau and Lifshitz, 1987)

$$v_\text{sup}^2 \approx \frac{1}{2} (\gamma + 1) \frac{\gamma}{\gamma - 1}.$$ \hfill (4)

As the pressure inside hot spots is dominated by the classical component, we have $\gamma \approx 4/3$. It follows that

$$p_\text{s} \approx \frac{3}{4} n_{\text{ext}} m_\text{p} v_\text{sup}^2.$$

Thus, inside hot spots, the sum of the four partial pressures is maximum at the stagnation point and is

$$p_\text{re} + p_\text{e} + p_\text{sup} \approx \frac{3}{4} n_{\text{ext}} m_\text{p} v_\text{sup}^2,$$

where $p_\text{re}$ and $p_\text{e}$ are the pressures of the relativistic electrons and the relativistic protons.

Secondly, we know the minimum value of $p_\text{re} + p_\text{m}$ from radio observations. This is obtained when we make the minimum pressure hypothesis. We have

$$p_\text{re} + p_\text{m,mp} + p_\text{e} + p_\text{sup} \leq p_{\text{max}},$$ \hfill (7)

With $B_{\text{mp}} \approx 1.9 \times 10^{-4}$ G, then $(p_\text{e,mp}) \approx (4/3) p_{\text{m,mp}} \approx 1.9 \times 10^{-4}$ erg cm$^{-3}$, and we deduce from (7)

$$1 + (\gamma + 1) \leq \frac{3}{4} \frac{p_{\text{sup}}}{p_{\text{m,mp}}} \frac{3}{4},$$ \hfill (8)

4.4. The separation velocity of the hot spots

From X-ray observations, the particle density of the hot intergalactic gas surrounding hot spots is $n_{\text{ext}} \approx 6 \times 10^{-3}$ cm$^{-3}$ (Arnaud et al., 1984).

Let us first remark that there exists a minimum value to the separation velocity of the hot spots. The separation velocity is also
the advance speed of the hot spots in the intracluster medium. Assuming $\theta \approx 0$ and $\chi \approx 0$ in (8), with (5) we have

$$v_{\text{sep}} \geq v_{\text{sep,mn}} \simeq \left( 3.1 \frac{p_{m,mp}}{n_{\text{ext}} m_p} \right)^{1/2} \approx 0.022 c.$$  

(9)

Now, we can make an estimate of the advance speed from the pressure inside hot spots. We have $\beta \approx 2.4$, $\theta \approx 6$ and we have seen in Pelletier and Roland (1986) that $\chi < 1$ when $\theta > 1$. So, we will assume at this step $\chi \approx 0.5$ to determine the separation velocity of the hot spots. Thus

$$p_2 \approx p_m + p_{re} + p_{re} + p_e \simeq \frac{1 + \beta}{\beta} (1 + \theta)(1 + \chi) p_{re} \simeq 15 p_{re}.$$  

(10)

With $\theta \approx 0$, $\chi \approx 0$ and the minimum pressure hypothesis, we obtain

$$p_{2,mp} = (p_m + p_{re})_{\text{mp}} \simeq 1.8 p_{re, mp}.$$  

(11)

So, when $B \geq B_{mp}$, we have $p_{re} \lesssim p_{re, mp}$, and an upper limit to the advance speed is given by

$$v_{\text{sep}} \leq \frac{B}{2p_{2, mp}} \leq 0.063$$  

(12)

This indicates that the separation speed is between 0.022 and 0.063 c, i.e. the mean separation speed is $(v_{\text{sep}}) \approx 0.037$; this within a factor less than 2.

In the following we will adopt for the advance speed of Cygnus A hot spots the value $v_{\text{sep}} \approx 0.05 c$, which is in good agreement with radio observations, see Hargrave and Ryle (1974), Winter et al. (1980), Pariskii and Soboleva (1980) and Alexander et al. (1984). Consequently the age of Cygnus A is $T \approx 6.6 \times 10^{6}$ yr, with the assumption that the radio axis makes a mean angle of 70° with the line of sight.

From relations (8) and (5), with $v_{\text{sep}} \approx 0.05 c$ we finally get

$$(1 + \theta)(1 + \chi) \leq 9.$$  

(13)

From condition (13) and $\theta \approx 6$, we must have $\chi \lesssim 0.3$, i.e. the relativistic protons in Cygnus A hot spots have an energy density smaller or equal to the energy density of the relativistic electrons. In the following, for numerical illustration, we will adopt the value $\chi \approx 0.2$.

5. The condition of classical hydrodynamics

The pressure inside hot spots is given by (5). It follows that the pressure of the classical component is

$$p_{2} + 0.75 \frac{\beta}{1 + \beta} \frac{\theta}{1 + \chi} n_{\text{ext}} m_p v_{\text{sep}}^2 \lesssim 7.1 \times 10^{-25} n_{\text{ext}} v_{\text{sep}}^2.$$  

(14)

From the definition of $\beta_2$, i.e. $\beta_2 = (p_{2} + p_{e})/\rho_{m,2}$ and $p_{m,2}$, the magnetic field in hot spots is given by

$$B_2 \simeq \frac{8 \pi \beta_2}{p_{2}} \left( 1 + \theta \right)^2 \simeq \frac{3}{4} p_{m,2} \left( 1 + \beta_2 \right)^{1/2} v_{\text{sep}} \lesssim 3.1 \times 10^{-12} n_{\text{ext}}^{1/2} v_{\text{sep}}^2.$$  

(15)

and thus, inside hot spots we have $B_2 \approx 3.6 \times 10^{-4}$ G.

The condition of classical hydrodynamics, i.e. $v_e \lesssim c/\sqrt{3}$, gives an upper limit to $v_{\text{sep}}$, which can be obtained as follows.

The radio spectrum of hot spots shows a steepening for frequencies greater than $v_e \approx 2$ GHz (Figs. 4a-b). So writing $v_2 \approx v_{\text{HS}}/\tau_{\text{sync,s}}$, where $v_{\text{HS}}$ is the length of the hot spot and $\tau_{\text{sync}}$ is the synchrotron lifetime of an electron radiating at $v_e$, the condition of classical hydrodynamics implies

$$v_2 \approx \frac{v_{\text{HS}}}{\tau_{\text{sync,s}}} < c/\sqrt{3}.$$  

(16)

From the synchrotron radiation theory (Ginzburg, 1978) and using (15), we deduce

$$\tau_{\text{sync}} \approx 1.1 \times 10^{-26} v_{\text{sep}}^{-3/2},$$  

(17)

with an hot spot length $h_{\text{HS}} \approx 1.1$ kpc, we obtain

$$v_{\text{sep}} \lesssim 0.088 c.$$  

(18)

Let us note a fundamental point. The condition of classical hydrodynamics is low in the case of Cygnus A, because the external density is very high. Suppose a radio source with the same characteristics than Cygnus A, i.e. $v_e \approx 2$ GHz and $l_{\text{HS}} \approx 1.1$ kpc, writing $B_2 \approx 3.1 \times 10^{-12} n_{\text{ext}}^{1/2} v_{\text{sep}}$, the condition corresponding to classical hydrodynamics becomes

$$v_{\text{sep}} \lesssim 2.1 \times 10^6 n_{\text{ext}}^{1/2},$$  

(19)

which gives $v_{\text{sep}} \approx 0.22 c$ with $n_{\text{ext}} \approx 10^{-3}$ e$^{-}$ cm$^{-3}$.

Thus we arrive to the conclusion: the extended components of Cygnus A can be modeled by classical hydrodynamics and moreover a radio source with the same characteristics as those of Cygnus A in a less dense external medium, say $n_{\text{ext}} \approx 10^{-3}$ e$^{-}$ cm$^{-3}$, can be also described by classical hydrodynamics if the separation velocity of its hot spots is smaller than 0.22 c.

6. Discussion and conclusion

We have observed Cygnus A hot spots at 408 MHz with MERLIN. Using our observations and using previous results obtained at 151 MHz with MERLIN and at 1450 MHz with the VLA (Leahy et al., 1988) and at 5 GHz and 15 GHz with the VLA (Carilli et al., 1987) and convolved with a 3° beam we get an accurate determination of the hot spot spectra. The main characteristics of the spectra are

i) the existence of a low frequency turnover at about $v_\gamma \approx 200$ MHz,

ii) a flat spectral index $\alpha \approx 0.5$--0.6 between 408 MHz and 1450 MHz,

iii) a steepening for frequencies greater than $v_e \approx 2$ GHz,

iv) a high frequency spectral index $\alpha \approx 1.0$.

From the knowledge of the density of the external medium, i.e. $n_{\text{ext}} \approx 6 \times 10^{-3}$ e$^{-}$ cm$^{-3}$, of the minimum pressure magnetic field $B_{mp} \approx 1.9 \times 10^{-6}$ G and of the maximum pressure inside hot spots, namely $p_{max} \approx 0.75 n_{\text{ext}} m_p v_{\text{sep}}$, it is possible to give a constraint on hot spot parameters, i.e.

$$(1 + \theta)(1 + \chi) \leq \frac{3}{4} \frac{p_{max}}{p_{m,2}} \frac{3}{4},$$  

(20)

where $\theta \equiv p_{2}/p_{e}$ is the ratio of the pressure of the classical component to the pressure of the relativistic component, $\chi \equiv p_{re}/p_{e}$ is the ratio of the pressure of the relativistic protons to the pressure of the relativistic electrons.

From (20) it is possible to obtain the minimum value of the advance speed of hot spots, when $B \geq B_{mp}$, $\chi \approx 0$ and $\theta \approx 0$, which is

$$v_{\text{sep}} \geq v_{\text{sep, min}} \simeq \left( 3.1 \frac{p_{m,mp}}{n_{\text{ext}} m_p} \right)^{1/2} \approx 0.022 c.$$  

(21)
Moreover, an upper limit to the advance speed is
\[ v_{\text{sep}} \leq 0.063 \, c, \]
which indicates that the adopted value \( v_{\text{sep}} \approx 0.05 \, c \)
is a reasonable value. Then, from (20) we get
\[ (1 + \theta)(1 + \chi) \leq 9. \quad (22) \]

It has been shown in a previous paper (Pelletier and Roland, 1986) that radio properties of Cygnus A hot spots could be understood either if the jet velocity is classical, i.e. \( v_{i} \approx 0.3 \, c \), with a downstream pressure dominated by the classical component, i.e. \( \theta > 1 \), or if the jet reaches a relativistic velocity, i.e. \( v_{i} \approx c/\sqrt{3} \), with a downstream pressure dominated by the relativistic protons, i.e. \( \theta \approx 0 \) and \( \chi \approx 9 \).

The purpose in this paper is to show that Cygnus A hot spot radio properties can be understood within classical hydrodynamics, i.e. \( v_{i} < c/\sqrt{3} \), and an electron-proton jet.

Supposing the magnetic field quasi perpendicular to the jet we have previously determined the value of the parameter \( \beta = p_{\text{par}}/p_{\text{n}} \) in hot spots, i.e. \( \beta \approx 2-3 \), where \( p_{\text{par}} \) and \( p_{\text{n}} \) are the pressures of particles and magnetic field respectively. Consequently the Alfvénic Mach number in the jet has to be \( M_{A} \approx 6-1 \). From the knowledge of the high frequency spectral index \( x_{\nu} \approx 1.0 \), we deduce the spectral index behind the shock is \( x_{\nu} \approx 0.5 \) and then the parameter \( \theta \) is \( \theta \approx 6 \) (Fig. 3), i.e. the dominant pressure inside hot spots is the pressure of a classical gas, which is the proton gas. Consequently from \( \theta \approx 6 \) and (22), we have \( \chi < 0.3 \), i.e. the relativistic protons have a pressure smaller than the relativistic electrons.

From the knowledge of the steepening frequency \( v_{\nu} \approx 2 \, \text{GHz} \) and the length of hot spots, we found that as soon as the advance speed of the hot spots is \( v_{\text{sep}} \approx 0.088 \, c \), hot spots can be described by non relativistic hydrodynamics. For radio sources with the characteristics of Cygnus A, but in a different external medium, the classical hydrodynamic condition becomes
\[ v_{\text{sep}} \leq v_{\text{sep,lim}} \approx 2.1 \times 10^{8} \, n_{\text{ext}}^{-1/2}, \quad (23) \]
which gives \( v_{\text{sep}} \leq 0.22 \, c \) with \( n_{\text{ext}} \approx 10^{-3} \, e^{-1} \, \text{cm}^{-3} \).

Acknowledgements. We thank C. Carrilli, J. Dreher and R. Perley for providing us data before publication, P. Mérat and P. Scheuer for useful comments and C. Douillet for typing out the manuscript. One of us (J.R.) thanks CNRS for financial support for this work.

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