Structure in the Universe from One Massive Neutrino?

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Summary. We discuss some severe problems confronting theories of galaxy formation that rely on the growth of small density perturbations and examine a scenario incorporating massive neutrinos. For one neutrino mass \(\sim 90 \text{eV} \text{K}\) we naturally find: 1 small initial density fluctuations can give both condensed systems at \(z \sim 10\) and small perturbations in the \(3K\) background \(\Delta T/T \sim 10^{-5} - 10^{-4}\), 2. non-luminous self-gravitating neutrino systems with a mass range characteristically \(10^{12} - 10^{16}\) \(\text{M}_{\odot}\), in which the baryonic material can fragment into galaxies and where the large and irregular systems with density contrast of order 1 are identified with superclusters; 3, a density parameter of the Universe \(\Omega_0 \gtrsim 1\), which gives a low age of the Universe. We also discuss the sensitivity of such a model to lower neutrino mass.

Key words: cosmology - galaxies: formation - neutrinos - superclusters

1. Introduction

It is a truth universally acknowledged, that cosmology is in want of an explanation for the origin and nature of the small density fluctuations which grow through self-gravity into the observed galaxies and larger structures. Most importantly while "making" the perturbations the overall Universe should remain Robertson Walker-Friedmann like (Weinberg, 1972). A recent additional success of these relatively simple models was the explanation of the magnitude of the baryon-antibaryon asymmetry (\(\Delta B\) creation; Nanopoulos and Weinberg, 1979), which is generated at a very early epoch (\(T \sim 10^{18} \text{Gev}, t \sim 10^{-36} \text{s}\)). This new result and the present homogeneity and isotropy on large scales (e.g. galaxy counts and the cosmic background radiation, henceforth abbreviated CBR) imply that a Friedmann model of the Universe is an excellent approximation all the way from the Planck time, before which quantum corrections to the classical Einstein gravity are expected to dominate (\(t_{\text{pl}} = (Gh/2\pi c^3)^{1/2} \sim 10^{-43} \text{s}\)), to the present \((t \sim 10^{10} \text{h}^{-1} \text{yr}, \text{with } H_0 = 100 \text{h} \text{km s}^{-1} \text{Mpc}^{-1})\). This hints to density perturbations with a Zel'dovich spectrum, i.e. all scales (\(\lambda\)) have the same density contrast \((\delta = \rho - \bar{\rho})/\bar{\rho}\) when they enter their respective horizons (Zel'dovich, 1972; Gott 1981).

This spectrum has the same (small) metric fluctuations on all scales (the transverse metric perturbation is \(h_{11} \sim (\lambda/c)t\delta\), which is constant in time, Peebles 1980).

Other possible spectra have large metric perturbations either very early or late in the history of the Universe. More specifically, the largest black hole that evaporates by now \((T_{\text{ev}} \sim 10^{15} \text{yr} (M/10^{14} \text{g}))\) has a mass very much smaller than that of a galaxy, say, which needs \(\delta_{\text{ev}} \sim 10^{-3}\); thus with no primordial black holes hanging around and the homogeneity on the presently observed largest scales only very small deviations from the \(x = 0\) Zel'dovich slope in \(\delta_{\text{nor}} \propto \lambda_{\text{nor}}^2\) are allowed. Henceforth we will assume a Zel'dovich spectrum (but we should keep in mind the possibility of a non-continuous spectrum).

Before recombination density perturbations can be decomposed in two modes (Peebles, 1980): 1. isothermal, with baryon density enhancements on a smooth radiation background, and 2. adiabatic, with equal amplitudes of baryon and radiation perturbations. Large scale density perturbations from before the epoch of \(\Delta B\) creation (e.g. from the quantum gravity period and neglecting complications as shear) only give rise to adiabatic fluctuations afterwards: locally the same \(n_{\text{b}}/n_{\text{r}}\) is created at \(T \sim 10^{15} \text{Gev},\) which in different regions in the perturbations differ only in proper time (expansion rate \(d/a \propto (G\rho)^{1/3}\)).

Let us now consider the magnitude of the Zel'dovich density contrast \((\delta \ll 1\) in order that the perturbations of the background metric are small, cf. Bardeen, 1980), for which the degree of isotropy of the CBR provides valuable information. At the epoch of recombination an adiabatic density amplitude \(\delta_{\text{a}}\) will lead to a CBR perturbation of at least \(\Delta T/T = \delta_{\text{a}}/4\) on scales of \(\sim 3H_0^{-1/2}\) degrees corresponding to the horizon at recombination (for numerical calculations see Wilson and Silk, 1981). On smaller angles \(\Delta T/T\) vanishes (Davis and Boynton, 1980) and larger scales will have \(\Delta T/T\) of the order of the density amplitude upon entering its horizon, hence fixed for a Zel'dovich spectrum (Gott, 1981). A recent experiment (Fabris et al., 1981) has found fluctuations \(\Delta T/T(6^o) \sim 310^{-5}\) and a small higher harmonic added to the dipole moment of our peculiar velocity, (see also Boughn et al., 1981) which together indicate a Zel'dovich spectrum with \(\delta_{\text{a}} \sim 10^{-4}\). But even if these were only upper limits (cf. Partridge, 1980) serious problems arise, for adiabatic fluctuations at least. In a roughly flat Universe the density contrast after recombination grows as \(\delta \sim a^{-1/2}\), with \(a(t)\) the expansion factor, the baryons no longer being locked to the radiation, hence we expect growth by a factor \(\sim 1500\). If the present density of the Universe is less than for a flat Universe \((\Omega_0 \approx 8\pi G \rho / 3H_0^2 < 1)\), the period of growth is even reduced: no growth when the Universe expands freely \((1 + z \leq \Omega_0^{-1} )\), nor between the epochs of recombination \(z_R \sim 1500\) and equal baryon radiation density.
$\left( z_{\nu} \sim 4.10^4 \Omega_\nu h^2 \left( \frac{T_0}{2.7} \right)^{-4} \right)$, when the expansion rate of the Universe is greater than the baryon perturbation growth rate (Peebles, 1980). Although a small numerical factor helps these linear estimates somewhat it appears that galaxies (δ ≳ 1) cannot be accounted for. A way out might be that the CBR has been reheated after recombination, but this seems not very plausible in view of the large energy input required and the fact that this reheating occurs at later epochs, when there are even stronger irregularities. Another problem is the origin of superclusters with masses $\sim 10^{13} - 10^{15} M_\odot$ and sizes of tens of Mpc, if they indeed have a density contrast of order 1 (cf. Einasto et al., 1980). It seems difficult to make both these large scale perturbations and galaxies directly, but hierarchical clustering also appears to be ineffective (large travel times 50 Mpc/500 km s$^{-1}$ $\sim 10^{11}$ yr; cf. Efstathiou and Eastwood, 1981). For completeness we remark that in the standard theory galaxies cannot be made directly from adiabatic perturbations, because these are damped by photon diffusion during recombination on mass scales less than $\sim 10^{13} - 10^{15} M_\odot$ for $\Omega_h \sim 1 - 0.01$. Typically galaxies are then thought to arise from fragmentation of larger structures, but the amplitude problem remains (Press and Vishniac, 1980). Should isothermal fluctuations be present somehow, then clustering of masses somewhat larger than the Jeans mass ($\sim 10^9 M_\odot$) might very well explain the correlation function up to clusters of galaxies through the mechanism of hierarchical clustering (for nor too small $\Omega_h$, superclusters remaining troublesome $\Delta T/\Delta T$ limits would be less stringent, because for a fixed Zel'dovich amplitude $\delta$ of the total density perturbation the baryon amplitudes at recombination on galaxy scales are larger: $\delta_{b,rec} \approx \delta(M/M_{b,rec})^{-1/3}$ (Gott, 1981).

In an earlier paper (Klinkhamer and Norman, 1981, henceforth KN) it was shown that neutrinos with non-zero restmass might alleviate the amplitude problem of adiabatic perturbations, because density perturbations of the collisionless neutrinos can start to grow before the epoch of recombination. Also it was shown that there might be some interesting implications for the origin of galaxy halos and larger systems of non-luminous matter. Estimates of the decay of a heavy neutrino to a lighter one indicate a negligible contribution to the UV background (de Rijula and Glashow, 1980). In this paper we present a scenario of the formation of large scale structure in the Universe, which in our opinion has a remarkable degree of consistency, but depends on one neutrino mass being quite large. Problems that arise for a smaller neutrino mass will be discussed.

We conclude this introduction with some remarks relevant to the further discussion, first on neutrino masses and then on the nature and structure of the dark material. The question of neutrino mass has witnessed a renaissance in the physics community. Some years ago it was argued that the neutrino should be massless, because with only the observed left-handed field a mass term in the Lagrangian is forbidden by lepton number conservation. But theories unifying strong, weak and electromagnetic forces at high energies ($M_\nu \sim 10^{15}$ GeV) and with quarks and leptons together in representations automatically violate baryon and lepton number, and thus predict non-negligible effects at observable energies (e.g. proton lifetime of order $10^{33}$ yr) and perhaps small neutrino masses (two reviews are Ellis, 1980 and Nanopoulos, 1980). Theories with neutrino mass typically involve superheavy fermions $F$, which might explain quark and lepton masses as radiative corrections (Barbieri and Nanopoulos, 1980) and through some sort of mixing neutrino masses $m_\nu \sim M_{\text{neutrino}} / M_F$, with $M_F$ of the order of or below the unification energy $M_\nu$ and hence $m_\nu$ in the eV range (for an easy introduction Klinkhamer et al., 1981; in the super-unification scheme Lazarides and Shafi, 1981). Presently there are three types of experiments to determine neutrino mass:

1. standard $\beta$ decay ($n \rightarrow p + e^- + \nu_e$), which is a three body decay with in the high energy tail of the Curieplot (reaction rate versus electron energy) the $m_e$ dependence. Lyubimov et al. (1980) claim $14 \leq m_e \leq 46$ eV, but molecular excitations cause uncertainties of $\sim 10^2$ eV at least.

2. if there is a mismatch between interaction $(v_{x,\alpha})$ and mass $(v_{x,\nu_i})$ eigenstates, a created $\nu_e$ is a superposition of $v_x$'s and for different $m_i$ the different phase factors $(e^{i\theta_i} E_i^2 = p_i^2 + m_i^2)$ make one find, depending on the travel time, either $v_x$ or $v_{x\nu}$ or $v_x$, i.e. neutrino oscillations. This explains approximately the factor 3 less detected solar neutrinos (created as $v_x$ in the Sun, than spread over 3 states). Preliminary reactor experiments perhaps indicate some spatial variation in the $v_x$ flux (Reines et al., 1980; Silverman and Soni, 1981) but implied mass differences are still uncertain.

3. a very recent idea of A. de Rujula (1981) is to look at internal bremsstrahlung in electron capture of a neutron deficient nucleus. This also is a three body decay $(Z \rightarrow (Z - 1) + \gamma + v)$ and for some isotopes the experiment seems promising.

Hopefully we made it plausible that small neutrino masses arise naturally in modern particle physics, although masses may be in a large range $\sim 10^{-3}$ eV.

Non-luminous matter appears to surpass the amount of visible matter (e.g. Faber and Gallagher, 1979). Flat rotation curves imply galaxy halos with a mass distribution $M(<r) \propto r$, but their extent is not known. Using statistical velocity dispersions Peebles (1981) derived a typical galaxy mass of order $5 \times 10^{12}$ $M_\odot$. Thus it is clear that the popular statement (cf. Davis et al., 1980; but for different opinions e.g. Ostriker and Turner, 1979, Spinrad et al., 1978) that mass to light ratios augment towards larger scales might be due to underestimating galaxy or binary masses, the halos extending further out. Two further conclusions of Peebles' review (1981) are 1. that halos of spirals appear to be quite standard with masses of order $10^{12}$ $M_\odot$ [there have been speculations that ellipticals have halos too (see discussion in Dekel and Hoffman) and observations of NGC 4278 by Raimond et al. (1980) and Einstein observations of M87 (Fabbricant et al., 1980) are indicative], and 2 that perhaps clusters also have a halo with $M(<r)\propto r$ for 0.5–3 Mpc. These (speculative) conclusions might have some foundation in the following scheme.

2. Scenario

In KN (see also for notation) the slight modifications induced by non-zero neutrino mass in the history of the Universe were discussed (Fig. 1). Massive neutrinos participating in adiabatic density perturbations with a Zel'dovich spectrum must have initial amplitude

$$\delta_i \sim \frac{1 + z_i}{1 + z_{\nu_i}} \sim 2 \times 10^{-4} \left( \frac{90 \text{ eV}}{\Sigma} \right) \frac{1 + z_i}{10}$$

(1)

to condense ($\delta \sim 1$) at $z_i$. Here $z_{\nu_i}$ is the epoch of equal radiation and neutrino mass density, which is determined by the total mass of the 3 left-handed neutrinos $\Sigma = \sum_{i=1}^{3} m_i$. Small scale neutrino perturbations are damped if they enter their horizon rotationally. The minimum mass is roughly determined by the mass within $3ct$ at the transition to the non-relativistic regime at $T \sim m_3 / 3 (\sim 1)$, or perhaps smaller, depends on the phase mixing process leading to the damping; cf. Gilbert 1965, Wasserman 1981):
\[ M_n \sim 7 \times 10^{13} N^3 \chi^2 \left( \frac{1}{90 \text{eV}} \right)^2 M_\odot. \tag{2} \]

If the total mass \( \sum \) is not spread equally over all the three neutrinos \((N = 3)\) but with one neutrino dominant \((N = 1)\), the perturbation mass scale is reduced, being \( \propto n(\chi)^2 \) and at earlier transition to the non-relativistic regime the \( \chi^2 \text{ term} \) wins, so the parameterisation \( N^3 \) in Eq. (2).

Let us consider one dominant neutrino mass \( \sim 90 \text{ eV} (\chi \sim 1) \), keeping in mind the large uncertainties \((\chi^2 \text{ in Eq. (2)})\). KN showed that galaxies with flat rotation curves of \( V_{\text{rot}} \sim 400 \text{km s}^{-1} \) arise naturally. But looking at the formation but only at the dynamics Peebles (1981) concluded that for a neutrino distribution function \( f = 1/(1 + r^2/\alpha^2)^{-1} \exp(-3r^2/2\alpha^2) \), a neutrino mass of order 75 eV fits the data best. It must be admitted that after collapse of the KN protohalo the neutrino occupation number at \( r = 0(10 \text{ kpc}) \) is perhaps much less than 1, which might be a problem.

We now proceed with a discussion of the mass scales to be expected. From KN (erratum) we have the following neutrino systems of characteristic mass: a) \( 1 - 3 \times M_\odot \) formed at \( z_0 \), and b) a small interval below \( M_\odot \) \( 1 - (1 + z)^{3/2} \times 3 \times M_\odot \) formed between \( z_1 \) and \( z = 0 \). The mass range given by a) occurs particularly in a scenario with one dominant neutrino mass \( M_\nu \) is still determined at \( T \sim m_\nu/3 \), but perturbations only grown when \( z < z_{c_1} \), giving masses up to \( 3 \times M_\odot \); range b) is from perturbations a little damped while relativistically entering their horizon and Wasserman (1981, Eqs. 22, 23) has shown that the damping is strong for masses below \( M_\odot \), hence only a small range of masses below \( M_\odot \) will condense at \( z < z_1 \); and range c) finally is from perturbations entering the horizon at \( z < z_{c_2} \). Figure 2 summarizes and gives approximate numerical values for \( 1 + z_1 \sim 10 \). The ranges b) and c) only apply with \( Q_\odot \sim 1.2 \left( \frac{90 \text{eV}}{V_{\text{rot}}} \right)^{-2} \geq 1 \), hence conden-
sations can form recently. Clearly we now have non-luminous systems ranging up to cluster masses. Superclusters perhaps form through hierarchical clustering leading to masses of order \( (1 + z)^{3/2} M_\odot \sim 10^{16} M_\odot \) where \( \beta \sim 1.5 - 2 \) (Press and Schechter, 1974; Efstathiou et al., 1979). These systems are expected to have low density contrast (recently formed), irregular form (cf. Doroshkevich et al., 1980, but 2-dimensional and note the spectrum cut-off used) and roughly filling space because \( \Omega_\odot \sim 1 \) (Press and Schechter, 1974). Preliminary observations confirm these characteristics (Oort, 1981). To make visible this most interesting Universe so far formed here we need to consider the dynamically negligible baryons. The baryon density perturbation is smoothed on scales less than \( M_\odot \sim 10^{14} M_\odot \left( \frac{n_{\odot}}{0.01} \right)^{-1/2} \). The dependence on the density parameters can be derived as follows (cf. Barrow, 1980): a photon with free path from Thomson scattering \( L \sim (n_{\odot} \chi)^{-1} \), random walks out of a perturbation \( l = NL \) in \( N^2 \) steps, this must take less than an expansion time at recombination \( t_z \), hence \( l < l = (c t_z L)^{1/2} \) and \( M_\odot \sim n_{\odot} L^0 \), from which we see the \( \Omega_\odot \).
dependence and the $k^{3/2}$ with neutrino mass dominated expansion gives the $\Omega_{m}^{-3/4}$ term. After recombination the baryon density quickly follows the already grown neutrino perturbations (Doroshkevich et al., 1980b; Appendix) and form through dissipation luminous cores, i.e. galaxies. For the smallest non-luminous systems $\sim M_{*}$ the baryon mass is $\sim (\Omega_{m}/\Omega_{b})M_{*} \sim 10^{10-11} M_{\odot}$, which cools to a single (spiral) galaxy; in larger non-luminous systems the baryons will fragment into galaxies (Rees and Ostriker, 1977; White and Rees, 1978).

Some implications of our scenario are:

1. the all over $M/L$ is roughly the same for $10^{12} - 10^{13} M_{\odot}$ systems,
2. In systems of range $a)$ we will have no separate galaxy halos, because the $10^{14} M_{\odot}$ perturbations and smaller irregularities $\sim M_{*}$ collapse at the same time ($z_{c}$) and during the violent relaxation the small scales lose their identity (perhaps some accretion on the galaxies has occurred, see Sect. 3).
3. Large systems ($10^{13} M_{\odot}$) may initially contain separate galaxy halos, which became bound before the collapse of the larger system, of course tidal stripping will have been effective.
4. For galaxy halos in range $b)$ with masses of order $M_{*}$ condensing at $z_{*} < z_{c}$ the turnaround radii $\propto (M_{*}/M)^{1/3}$ scale as $(1 + z_{*})^{-1}$ and their virial velocities $\propto (M_{*}/M)^{1/2}$ scale as $(1 + z_{*}^{-1})^{1/2}$ compared to KN (Eq. 8), which gives for the flat rotation curve velocities (km s$^{-1}$)

$$V_{\text{rot}} \sim 520 N_{2} \left( \frac{\Sigma}{90 \text{ eV}} \right)^{1/2} \left( \frac{1 + z_{*}}{10} \right)^{1/2} (z_{c} < z_{*}).$$

The more recently condensed halos give lower dispersion/rotation velocities and perhaps the higher $V_{\text{rot}}$ in spirals are not observed because these cores have already merged into elliptical galaxies. We note that ellipticals have quite high central velocity dispersions ($V \sim 400$ km s$^{-1}$, Davies 1981), whereas the typical flat rotation curve velocity is of order $200$ km s$^{-1}$, but the good agreement with Eq. (3) probably is a quirk of fate.
5. Systems very much larger than $10^{16} M_{\odot}$ are not to be expected as separate entities (unless $m_{*}$ smaller, see below).
6. Depending on the value of $z_{*}$ (what might be inferred from quasar densities?), the theme of late formation of the largest systems may result in some interesting observations (cf. Oort et al., 1981).

Our Local Group might perhaps be an example of point 2: from velocities of the small galaxies there appear to be two separate halos $\sim 10^{12} M_{\odot}$ but not a more massive common envelope (Lynden-Bell, private communication).

Due to the discreteness of the point masses ($z < z_{c}$) there will be a number density power spectrum $|\delta_{s}|^{2} \propto k^{n}$ with $n \sim 1$. Correlation functions for galaxies in different non-luminous systems will then be of the form $\bar{\xi} \propto x^{-\gamma}$, $\gamma \sim (9 + 3n)/(5 + n)$ $\sim 2(\xi > 1)$ Peebles, 1981). N-body simulations have shown that the correlation function at small separation is determined by close binaries in clusters (Gott et al., 1979), this also applies to the galaxies in our large non-luminous systems. Clustering between different non-luminous systems may give a power law $\bar{\xi}(x)$ with $\gamma \sim 2$ on large enough separations $x$. Also clustering of separate galaxy halos with their visible cores might give contributions to the correlation function (measured on the cores) at distances less than twice the halo radius, the neutrinos freely penetrating. At first $\xi^{*}$'s flat slope and an eventual continuity to intragalactic densities seem difficult to explain, certainly the latter is not at all evident if galaxies form dissipatively and then cluster (Peebles, 1980, § 64). But simulations indicate that a power law, at least at small separations, is quickly created, quite independent of $\Omega_{m}$, $n$ and initial features (Gott et al., 1979), also transient effects are unimportant (Fry and Peebles, 1980). At larger separations the value of $\xi$ is not yet certain (cf. Krishner et al., 1980) and simple clustering of galaxies seems to be not effective enough (Efstathiou and Eastwood, 1981) so our scenario perhaps provides an answer! Finally we note that in extracting $\xi$ magnitude-position independence is assumed, which need not be trivial in our scenario.

3. Conclusion

What then are the problems that would arise/remain if the neutrino obstinately refuses to be heavy (cf. Bond et al., 1980; Doroshkevich et al., 1980b):
1. If $\Sigma < 0.1$ eV, baryons would be the dominant form of matter, with $\Omega_{b} \ll 1$ (unless there are pregalactic stars, black-holes etc., but how to form these in a not too contrived way?) and all the problems of Sect. 1 remain.
2. If $\Sigma = 0.1$ eV the non-luminous systems would have scales

$$a) \sim M_{*} \sim 10^{16} M_{\odot}(N = 3)$$

$$b) \sim 3^{-3} M_{\odot} \text{ or } \sim 10^{-15} - 10^{-18} M_{\odot}(N = 1)$$

where $N = 3$ or 1 means the distribution of the total mass $\Sigma$ over 3 or 1 states, being the extreme cases for the neutrino mass spectrum. Note that clustering will not be effective if $z_{c} \sim 10$, the Universe freely expanding afterwards.

Probably these large systems do not occur in the Universe, and even if so, they cannot fragment in the non-luminous systems associated with the observed groups and clusters of galaxies ($10^{14} - 10^{15} M_{\odot}$). Also the amplitude problem retains its full glory: Eq. (1) gives $\delta_{2} \sim 10^{-1}$, which is augmented by a factor $\sim 10$, because $\Omega_{b} \sim 0.1$.

3. If $\Sigma = 0.1$ eV, $N = 3$, there would be non-luminous systems of $\sim 10^{14} - 10^{15} M_{\odot}$. One could explain clusters of galaxies and superclusters, as discussed in Sect. 3, but the neutrinos cannot fragment into galaxy halos, these should then originate from accretion into existing potential wells of baryon (Gunn, 1977). Small baryon perturbations present at recombination do not grow in the neutrino dominated Universe (Appendix), so accretion on baryon cores starts only after the collapse of the large non-luminous systems in which these baryon cores fragment ($z \ll z_{c}$). On the other hand halos of galaxies outside these large dark systems should result from accretion on baryon cores which were already condensed at $z \gg z_{c}$. A general problem of accretion might be the small scatter of apparent halo masses.

Clearly cosmology prefers either a non-significant neutrino mass or one ($N = 1$) quite large mass $0.1$ eV (or perhaps $N = 3$). In the latter case a number of problems may be solved, which we now summarize.

In a hot Big Bang with a Zel'dovich spectrum of adiabatic density fluctuations galaxy halos of order $10^{12} M_{\odot}$ and flat rotation curve velocities of several $100$ km s$^{-1}$ are formed. The necessary neutrino mass $0.1$ eV implies a flat or closed Universe, with a quite small age of the Universe (if $h \sim 0.75, \Omega_{b} = 2, \tau_{0} = 8$ yr). We again emphasize the uncertainties in the numerical values given (for example if $a = 1/2$). Also it is now possible to make galaxies at $z_{c} \sim 10$, while avoiding limits on the perturbation amplitude of the CBR on angular scales of a degree: $\Delta T / T \sim 10^{-4} - 10^{-5}$; but these fluctuations should be found in the future. A range of condensed non-luminous systems of $10^{12} - 10^{16} M_{\odot}$ arises naturally: galaxy halos up to irregular, low contrast superclusters. A prediction is that in non-luminous systems of mass $\sim 10^{14} M_{\odot}$ visible galaxies will not have separate halos, except for some possible accretion. The observed power law 2-point correlation function might reflect on not too large separations ($\lesssim 1$ Mpc) relaxation effects and
hierarchical clustering, but our scenario might also take account of clustering at larger scales, which would be difficult to obtain by hierarchical clustering of galaxies only. If the neutrinos have one dominant large mass this might be of paramount importance for a consistent explanation of galaxy formation and the structure in the Universe. Remains to measure neutrino mass in the laboratory.

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Appendix

Here we consider two problems of baryon perturbations in a neutrino mass dominated Universe. $P(x)$ denotes equation (x) in Peebles (1980). First let us estimate how fast a neutrino density perturbation induces a baryon perturbation of equal amplitude. For collisionless neutrinos linearising $P(9.10 + 7.9)$ gives in moving coordinates:

$$\frac{\partial^2}{\partial t^2} \delta_i + \frac{4\dot{\vartheta}}{a \dot{\vartheta}} \delta_i = 4\pi G \rho_i \delta_i, \quad \delta_i = (\rho_i - \bar{\rho}_i)/\bar{\rho}_i \quad (A1)$$

and thus $\delta_i \propto t^{2/3}$ (see below). Baryons falling in the potential wells of the neutrino perturbations obey the following equation of motion in comoving coordinates, linearised and for zero pressure

$$\frac{\partial^2}{\partial t^2} \delta_b + \frac{4\dot{\vartheta}}{a \dot{\vartheta}} \delta_b = 4\pi G \rho_i \delta_i \quad (A2)$$

where we used

$$(1 + \delta) \dot{\rho}_i = \rho_i \nabla^2 \delta = 4\pi G \rho_i (\rho_i - \bar{\rho}_i).$$

With the background equations $P(11.4 - 5)$

$$a \propto t^{2/3}, \quad \rho_i \propto a^{-3} \quad (A3)$$

$6\pi G \rho_i \dot{a}^2 = 1$

we find for (A2)

$$\frac{\partial^2}{\partial t^2} \delta_b + \frac{4\dot{\vartheta}}{3\dot{a}} \delta_b + \frac{2}{3t^2} \delta_b$$

which gives a rise of the order of the expansion time scale $t$ for the $\delta_b = \delta_b e^{rT}$ to attain the fixed $\delta_b$ amplitude. Hence neutrino density perturbations quickly induce baryon perturbations of equal amplitude. Secondly we show that separate baryon perturbations do not grow in a neutrino mass dominated Universe. The baryon perturbation equations are for no pressure and no baryon-neutrino coupling (cf. $P(10.2))$

$$\frac{\partial^2}{\partial t^2} \delta_b + \frac{4\dot{\vartheta}}{3\dot{a}} \delta_b = \frac{2}{3t^2} \delta_b$$

which with (A3) for the smooth neutrino background becomes

$$\frac{\partial^2}{\partial t^2} \delta_b + \frac{4\dot{\vartheta}}{3\dot{a}} \delta_b = \frac{2}{3\Omega_\nu \dot{\Omega}} \delta_b, \quad r = \Omega_\nu \Omega_\phi \quad (A6)$$

which gives for the growing solution $\delta_b \propto t^a$

$$a = \left(1 - \frac{1}{3} \left(\frac{2}{3} + 2r\right)^{1/2}\right) \sim 2r \quad (r \ll 1).$$

(Independently Bond et al., 1981, found the same result). Hence practically now growth. Also we check from (A6) that with $r \approx 1$ we have $a \sim 2/3$, which is the solution for (A1) too.

References

Nanopoulos, D. V.: 1980, lecture XVieme Recontre de Moriond (CERN-TH 2986)
Peebles, P. J. E.: 1980, The Large-Scale Structure of the Universe, Princeton UP
Weinberg, S.: 1972, Gravitation and Cosmology, Wiley