A Multifrequency Study of the Radio Structure of 3C10, the Remnant of Tycho's Supernova

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Summary. Synthesis observations of 3C10 have been made at 6 cm and 50 cm using the Westerbork telescope. Combining these results with earlier 21 cm Westerbork maps, we have made detailed comparisons between the three wavelengths. The total intensity maps are remarkably similar: We find no evidence for changes in the spectral index of small scale features around the rim of the shell-like remnant. On the large scale, however, there appears to be a significant steepening of the spectrum near the center of 3C10. We believe this to be related to the fact that the integrated radio spectrum steepens at low frequencies, and suggest that particles presently near the outer boundary may be accelerated with a relatively flat energy spectrum. Although no linear polarization exceeding about 1% of the total intensity could be detected at 50 cm, at 6 cm the polarized emission is quite smooth, being typically 10% of the total intensity. The position angle distribution of the linear polarization is also smooth, confirming our earlier conclusion that 3C10 is surrounded by an irregular Faraday screen.

By comparing our 6 cm map with 2.8 cm and 10.4 cm results we are able to construct a rotation measure map, and confirm that the intrinsic position angles are tangentially aligned, with an average rotation measure of $-249 \text{ rad m}^{-2}$. We conclude that the ordered component of the magnetic field is radially directed, and suggest that the mechanism for producing such a configuration can be found in the Sedov solution for shock waves in a fluid medium.

Key words: supernova remnants — magnetic field structure — Faraday rotation — spectral variations

1. Introduction

The remnant of Tycho's supernova, 3C10, is an attractive object for radio study from several diverse viewpoints. The original observations (Brahe, 1573) place the explosion in early November, 1572, and enabled Baade (1945) to ascertain that the supernova was of Type I. The present radio emission is intense enough that high resolution studies of both the total and polarized radiation can be carried out at many frequencies. Furthermore, the high declination and $8'$ diameter of 3C10 make it a most suitable candidate for studies using the Westerbork Synthesis Radio Telescope. And finally, its striking circular symmetry suggests that interpretation of the radio maps will be relatively unhindered by complex projection effects.

Studies of the structure of 3C10 have been made at wavelengths ranging from 2 cm to 21 cm (Baldwin, 1968; Dickel, 1969; Weiler and Seielstad, 1971; Kundu and Velusamy, 1971; Strom and Duin, 1973, hereinafter referred to as Paper I; Hermann and Dickel, 1973). In this paper we present and discuss observations at two new wavelengths, 6 cm and 50 cm. These are compared in some detail with our own 21 cm measurements (Paper I), enabling us to detect rotation, measure and spectral index variations within the remnant.

Previous observations (Weiler and Seielstad, 1971; Kundu and Velusamy, 1971) had been used to determine rotation measures in the source. They suggested that the intrinsic position angle of the linearly polarized $E$ vector was everywhere perpendicular to radii drawn from the center of the remnant. The new 6 cm results, although requiring a very different value for the average rotation measure ($-249 \text{ rad m}^{-2}$ instead of $+37 \text{ rad m}^{-2}$), confirm the tangential alignment of the polarization vectors. This configuration for the polarized radiation means that the well ordered component of the projected magnetic field is radially directed. An explanation for this phenomenon, which appears to be common in young remnants, is put forward. Although the degree of polarization at 21 cm is generally high, there is fine scale structure in the position angle distribution. We suggested (Paper I) that this is probably the result of Faraday rotation in a screen of irregular magnetioionic material. The uniformity and strength of the 6 cm polarization confirms our hypothesis.
By intercomparing the 6 cm, 21 cm and 50 cm Westerbork maps we are able to detect changes in the energy spectrum of the radiating particles. We find that small scale features (of size \( \lesssim 1 \text{ arc}^\prime \) to 3 arc\(^\prime\)) can, bearing in mind resolution differences, be readily identified in all three maps. On the large scale, however, there is a significant radial dependence of the spectral index. Between 21 cm and 50 cm the spectrum near the edges is flatter than in the remnant center. This effect, its relationship to the spectra of other remnants and possible explanations are briefly discussed.

Some of the physical parameters derived in Section 3.2 depend on the distance to 3C10. In view of recent H\(_1\) absorption measurements (Goss et al., 1973; Williams, 1973) we shall adopt a distance of 6 kpc. The effects which changes in the distance have on derived parameters were discussed in the appendix of our earlier paper (Paper I).

2. The Measurements

The Westerbork telescope consists of twelve 25 m reflectors on an east-west baseline, ten of which are fixed at intervals of 144 m. The remaining two can be moved along a 300 m railtrack at the eastern end of the array. The hardware and electronics are described in Baars et al. (1973). Each telescope has two receiver systems, those in the fixed reflectors being sensitive to polarized radiation in the position angles 0\(^\circ\) and 90\(^\circ\), while those in the movable ones can detect radiation polarized at position angles 45\(^\circ\) and 135\(^\circ\). By correlating all twenty signals from the fixed telescopes with those from each of the four channels in the movable telescopes we can uniquely determine all four Stokes parameters along twenty ellipses in the resolution plane (Weiler, 1973).

Table 1 summarizes the relevant instrumental parameters for the observations at 6 cm and 50 cm. The spacing interval was dictated by the requirement that the radius of the first grating response should exceed the diameter of 3C10. In order to avoid saturation in the backend of the telescopes at 50 cm all channels were attenuated by a factor of 4. Although the RMS noise is the same for each Stokes parameter, gain stability and other instrumental uncertainties may impose errors in the Q, U and V maps of up to 1% of the total intensity (Weiler, 1973). At 50 cm, the linear polarization has been corrected for the variable Faraday rotation introduced by the ionosphere, using ionosonde data from the Royal Meteorological Institute of the Netherlands (KNMI) in de Bilt. The accuracy of this procedure is estimated to be better than 5\(^\circ\); however, no polarized emission was detected from 3C10 at 50 cm.

Högobom and Brouw (1974) describe the normal procedure for correcting the measurements and producing maps. During one of the four twelve hour observations at 6 cm there were linear phase drifts of up to 3\(^\circ\) per hour. By using modified nominal lengths of the baselines in the reduction of this observation, we have introduced a phase change proportional to the sine of the hourangle which approximately compensates for the observed drift. The individual telescopes had pointing errors of up to 2 arc\(^\prime\) which, although negligible at 50 cm (and 21 cm), will affect the 6 cm map of a large object like 3C10. Since the errors differ for each telescope, every interferometer will have a slightly different pointing center which will also be hour angle dependent. No attempt has been made to correct the 6 cm observations for attenuation by the primary beam, and these observations have not been used to calculate spectral indices.

A further problem with the 6 cm data is that although an 18 m spacing increment was used (Table 1), the 18 m baseline itself cannot be observed (this is because of the 25 m reflector diameter). This “missing spacing” introduces a negative bowl into the center of the synthesized antenna pattern. In a ringlike object such as 3C10 this has a serious effect because the negative contributions will add up near the remnant center and depress it below the map zero level. By “cleaning” the map—that is, deconvolving the map intensities from the theoretical antenna pattern (Högobom, 1974)—one can in principle remove such effects. However, at 6 cm this did not work very well, possibly because the pointing errors produce an effective synthesized beam somewhat different from the theoretical one.

2.1. The 50 cm Observations

At 50 cm the total intensity (Stokes parameter I) distribution of 3C10 consists of a regular circular shell, as shown in Fig. 1. The main departures from symmetry are the outward bulge in the southeast which also disrupts the rim, the existence of several peaks along the rim, and the presence of a secondary ridge which protrudes inward and north at RA = 00\(^h\)22\(^m\)15\(^s\) and Dec = 63\(^\circ\)50\(^\prime\). This latter feature is clearly visible in the higher resolution short wavelength maps, where it extends into the central region of the remnant. As at 21 cm (Paper I), emission definitely emanates from the center. In Fig. 2, the 21 cm map from our earlier
Fig. 1. Contour map of the total intensity distribution over 3C10 at 50 cm. Contour intervals are 114 m.f.u. per synthesized beam. Here and elsewhere the symbol "<" enclosed in a square indicates a local minimum and the half-power beam is shown as a shaded ellipse. The actual zero level is -60 m.f.u. per synthesized beam.

Fig. 2. Contours of the total intensity distribution over 3C10 at 21 cm convolved to a resolution of 57 x 64". The contour intervals are 61.5 m.f.u. per beam. The depression due to the lack of the zero spacing is 45 m.f.u. per beam.
The paper has been convolved to the $57'' \times 64''$ resolution of 50 cm. The similarities between Figs. 1 and 2, though striking, are not as surprising as the differences which will be discussed in Section 3.1. No polarization exceeding about 1% of the total intensity was detected at 50 cm, notwithstanding the high degree of linear polarization found at 21 cm.

2.2. The 6 cm Observations

In the 6 cm full resolution map of 3C10 (Fig. 3) an immense amount of fine scale structure can be discerned. The increase in complexity at 6 cm is partly due to the fact that nearly all isolated maxima in the 21 cm map have broken up into several individual peaks. This effect can be largely ascribed to resolution because in a convolved 6 cm map the peaks, as well as the fine scale structure generally, have nearly the same appearance as in the 21 cm map. In the central region of the remnant the bowl discussed in the first part of Section 2 has obviously depressed the zero level, making the map unsuitable for calculating the spectral index of the large scale structure, as well as the degree of polarization at 6 cm.

Close inspection of the map in Fig. 3 reveals that the outer gradient of the source remains unresolved in many places. The peaks of emission, however, are generally displaced inward, and in only a few places do they lie adjacent to the outer boundary. A related effect worth bearing in mind is that the bulk of the radio emission from 3C10 originates on average some 30 arcsec within the outermost contours. The individual peaks, which are mostly resolved, have typical flux densities of 10 m.f.u. (1 m.f.u. = $10^{-29}$ W Hz$^{-1}$ m$^{-2}$).

Although the outer boundary in Fig. 3 is more irregular than that in lower resolution maps, especially along the eastern side, its near circularity remains noteworthy. In this high resolution map it is apparent that in the southeast the entire ridge is displaced beyond the outer boundary as determined by the rest of the remnant. Just to the north of this are the two strongest peaks, which themselves lie right on the outer edge of 3C10. They are almost certainly part of the remnant.

Figures 4 and 5 present the polarization characteristics at 6 cm. The former shows the polarized intensity $P(P = (Q^2 + U^2)^{1/2})$ as a contour map, while the latter is a vector representation of $P$ and the position angle of the $E$ vectors, $\chi (\chi = \frac{1}{2} \arctan (U/Q))$. Both maps were made with the same $24'' \times 27''$ resolution as our 21 cm data (Paper 1); higher resolution maps reveal no significant additional features. Although the degree of
polarization cannot, as noted earlier, be determined with certainty, we estimate typical values of 10% in the rim, rising as high as 16% in places. The intensities in Fig. 4 show only slightly more detail than the $I$ map with the same resolution. This is surprising because the vector-like nature of polarized radiation permits cancellation of the emission, which usually results in a lumpier intensity distribution. However, from Fig. 5 it is clear that the position angle distribution is itself very smooth, and this has probably led to a smooth $P$ distribution. What is unexpected about Fig. 5 is the absence of the tangential alignment found at 2.8 cm (Kundu and Velusamy, 1971) and 10.4 cm (Weiler and Seielstad, 1971). At 6 cm, the vectors are rotated by some $-51^\circ$ on average with respect to the 2.8 cm vectors. A comparison between the 2.8 cm, 6 cm, 10.4 cm and 21 cm polarization data is made in Section 3.1.

3. Discussion

One of our principal aims in extending the observations of 3C 10 to 6 cm and 50 cm was to make high resolution comparisons at three wavelengths of the total intensity and linearly polarized radiation. In the event, our intentions could be only partially realized because of the aforementioned distortion in the 6 cm map, and because no linear polarization was detected at 50 cm. A discussion of the comparisons we were able to make follows.

3.1. Spectral Index Variations

As noted in Section 2, the spectral indices of small scale features, most of which are located in or near the
outer ridge, are very similar. Comparisons between a 21 cm map and a 6 cm map convolved to the same resolution, as well as a comparison of the small scale structure at 50 cm (Fig. 1) and that in the convolved 21 cm map (Fig. 2), make it clear that there is no variation in spectrum between individual regions of enhanced emission along the outer shell. After allowing for the shift in zero level, we calculate a spectral index between 50 cm and 21 cm of $-0.7 \pm 0.05$ for these features.

For comparison of the large scale structure we are unable to use the 6 cm map, because of the distortion resulting from lack of information in the innermost parts of the resolution plane. After correcting the 50 cm and convolved 21 cm maps for the depressed zero level, we have taken twelve radial profiles through the remnant center (RA = 00°22′31″, Dec = 63°51′6″) and averaged them as described in Paper I. The resulting curves, normalized to the same maximum intensity, are shown in Fig. 6. It is clear that the spectrum of 3C10 steepens as one proceeds radially inwards, with the spectral index at the center being $-1.0 \pm 0.1$. The simplest conclusion one can draw is that the high energy electrons are distributed in a thinner shell than the low energy ones.

This new fact could be related to earlier observations of a steepening in the spectrum of 3C10 at low frequencies (Braude et al., 1970; Roger et al., 1973): the spectral index changes from about $-0.55$ at 1400 MHz to $-0.9$ at 38 MHz. Roger et al. (1973) were able to fit the flux density data observed for 3C10 with two power law spectra having equal flux densities at 500 MHz and spectral indices of $-0.95$ and $-0.35$. The new high resolution data at 50 cm and 21 cm suggest that these two components are spatially separated, the flatter spectrum belonging to the outer regions of the remnant while the steeper one emanates from the innermost parts.

Recently, Dent et al. (1974) have confirmed that the decay rate of the flux density of Cas A is frequency-dependent, in the sense that the spectrum flattens with time. An extrapolation shows that when it reaches the present age of 3C10, Cas A will have a similar spectral index of $-0.62$. It is tempting to suppose that in analogy with 3C10, this is the result of a new population of relativistic particles with a relatively flatter energy spectrum being introduced near the outer boundary of Cas A [Rosenberg's (1970) data on Cas A appears to be inconclusive on the question of a radially varying spectral index]. This would also be consistent with the trend (Harris, 1962) that shell type remnants of greater age have flatter radio spectra.

Gull (1973) has developed a model wherein turbulent motions, produced as the result of instabilities between swept-up material and that ejected in the original explosion, accelerate particles near the shockfront to relativistic speeds. Although Gull makes no prediction about the energy spectrum expected, we tentatively suggest that the thin shell emitting a relatively flat radio spectrum is being produced near the outer boundary of 3C10, while the steep spectrum component, having developed during an earlier stage of evolution, is no longer being renewed. If all shell type remnants can be shown to have similar spectral histories, the possibility of explaining their evolution by means of a general mechanism such as Gull's would be greatly enhanced.

### 3.2. The Distribution of Linearly Polarized Emission

As noted earlier, the position angle distribution of the 6 cm linearly polarized emission is remarkably smooth. The tangential alignment found for the $E$ vectors at 2.8 cm and 10.4 cm is, however, not observed. If we assume that the 2.8 cm position angles are nearly intrinsic, and that the average rotation of $-51°$ found at 6 cm is due to the Faraday effect, then the expected rotation at 10.4 cm would be $-51° (10.4/6) = -153°$. As this is not far from $-180°$, the vectors at 10.4 cm will have a nearly tangential appearance. For various reasons, Weiler and Seielstad (1971) settled upon an average rotation measure of $+37$ rad m$^{-2}$ for 3C10, giving an almost perfect tangential alignment to the “intrinsic” position angles they derived. They were careful to point out that their derivation has a possible ambiguity of $\pm 58° n$, where $n$ is an integer. Ironically, the 6 cm results suggest $n = 3$ which yields a correction of $+174°$, leaving intact the conclusion that the intrinsic position angles are tangentially aligned.

The combined 2.8 cm, 6 cm, and 10.4 cm polarization data for 3C10 are consistent with a rotation measure of about $-250$ rad m$^{-2}$. Bearing this in mind, we have made a detailed comparison between the 10.4 cm map of Weiler and Seielstad (1971) and our 6 cm map convolved to the 10.4 cm resolution. At each point the rotation measure has been calculated, and the average value
was found to be \(-247 \pm 17 \text{ rad m}^{-2}\). It is interesting to note that sources in the same part of the sky as 3C10 have negative rotation measures (Vallée and Kronberg, 1973) so the new value fits in better with the general trend. Although no abrupt changes in rotation measure between adjacent points could be detected with the 1 arc' resolution available, it must be remembered that at 21 cm (Paper I) an irregular, cell-like structure was observed.

Ideally, we would like to use the 21 cm data to produce a detailed rotation measure map. However, the rotation measure derived above translates into an average rotation at 21 cm of \(-630^\circ \pm 45^\circ\), so any value which differs from the mean by two standard deviations will suffer from a 180° ambiguity. Therefore the individual rotation measures found in the 6 cm–10.4 cm comparison were used as reference values for the 6 cm–21 cm comparison, providing us with something of a constraint for deriving the final high resolution rotation measure map. Of course, the 21 cm map is by its very nature noncontinuous in places, but a 180° misinterpretation at 21 cm leads to only a 18° error in the intrinsic position angle derived.

The rotation measure distribution appears in Fig. 7. Using the 6 cm and 21 cm data, the average rotation measure was found to be \(-249 \text{ rad m}^{-2}\), with a standard deviation of 16 rad m\(^{-2}\). As one might expect, there is an excellent correlation between the cell structure found at 21 cm and regions of nearly constant rotation measure. It is the smoothness of the 6 cm position angle distribution, however, which confirms our earlier conclusion that the cellular structure is entirely due to the Faraday effect.

The fact that the degree of linear polarization at 6 cm and within the cells at 21 cm is nearly the same means that the irregular material producing the cells cannot be distributed throughout the synchrotron emitting shell. The sharpness of the discontinuities between adjacent cells we take as an indication that the irregularities occur in a thin screen, and it seems most probable that the screen is associated with the remnant itself. The average rotation measure of \(-249 \text{ rad m}^{-2}\) is then assumed to be due to the general Faraday rotation in the Galaxy, while the standard deviation of 16 rad m\(^{-2}\) is an indication of the integrated magnetoionic density, \(\int n_e B \cdot dl\), within the Faraday screen.

The best description of the dynamics of the swept up gas is to be found in Sedov's (1959) solution, which predicts that the bulk of swept up material will be concentrated in a very thin shell just behind the shockfront. As an approximation, we assume that most of this material lies in a constant density shell 0.5 pc thick (this is for a 7 pc radius, which depends on the distance, see Section 1). Near the outer edge where
component of the projected magnetic field perpendicular
to the line of sight is directed almost perfectly radially
everywhere. Such a configuration has also been found
in the young remnants Cas A (Mayer and Hollinger,
1968) and SN 1006 (Kundu, 1970), so it seems plausible
that this is common in young shell type supernova
remnants.

What mechanisms could give rise to this magnetic field
gyrometry? Wolfert (1972) has suggested that there may
be shearing of a frozen-in field as the radially expanding
shock front encounters irregularities in the interstellar
medium. However, the regular circular contours in
our map of 3C10 suggest no such features, at least not
on the scale of our beam size (7 arc° ≈ 4 × 10^-17 cm) or
larger, and one might expect that in the rapid heating
behind the shock such irregularities would quickly
disperse. Gull (1973) has developed a model for young
remnants wherein instabilities along a contact dis-
continuity between the ejected and swept up materials
result in radial convective motions, which stretch the
magnetic field lines. However, if the mass ratio between
swept up and ejected material in 3C10 is indeed greater
than 25 as suggested earlier, there must be some doubt
about the importance of such instabilities at its present
stage of evolution. Cas A, where the relevant masses
appear to be more nearly comparable, is more suitable
as a candidate for harboring the features of Gull's
model.

We wish to point out that implicit in the Sedov
solution itself are conditions which will tend to produce
a radially directed magnetic field. Let r_n be the initial
coordinate position of a gas element. For a ratio of
specific heats, γ = 5/3, the density of material im-
diately behind the shock will be 4ρ_0, where ρ_0 is the
ambient gas density. At a later time, the shock will have
moved to r_n, while the gas element will be at a position
r (r_n > r > r_p). Sedov's (1959) formulae enable us to
determine the gas density, ρ, as a function of the
relative position, r/r_p, and this is shown as the solid
curve in Fig. 9a. We can also relate the original
position, r_0, to the shock position, r_p, using formulae
corrected by Heiles (1964).

The key to our argument is as follows: If the expansion
retains complete spherical symmetry, then simple
geometry dictates that a mass element expands, after
shock passage, such that the ratio of its tangential
to radial scale is \( \frac{\rho}{4\rho_0} \left( \frac{r}{r_0} \right)^3 \) the dashed curve in Fig. 9a
shows that this quantity is always less than unity
behind the shock front. Thus the radial expansion
will always exceed the lateral expansion, by a greater
amount the further the gas element recedes from the
shock. If the ambient magnetic field conserves its
original ordering while passing through the shock, it
will initially be more tangential. As the gas density
decreases to ρ_0 it will return to its original configura-
tion, and thereafter a radial component will form. In

Fig. 8. Intrinsic polarization angle distribution as derived from the
6 and 21 cm data (see text). The resolution used is 24 × 27° and the
separation between points is 14°. The lengths of the vectors have
no meaning.
3C10, however, there is no evidence of a tangential field, even near the outermost boundaries. Should the magnetic field be randomized during the shock passage, an ordered radial component will appear immediately, as a result of the preferential radial expansion of the gas. By assuming that the proportion of radially ordered field is the same as the degree of stretching, we can calculate the degree of polarization as a function of distance behind the shock front (Fig. 9b). The values predicted (about 10% at a distance of 1 arc' behind the shock) do not disagree with those found at 6 cm.

In demonstrating that the conditions behind a strong shock will tend to produce a radial magnetic field, we do not mean to imply that the mechanisms investigated by Gull (1973) are not also operative. Indeed, if several different mechanisms tend to produce a radially directed field, it is no wonder that such a configuration figures so prominently in young remnants. It should be noted that Gull's model predicts that the radio shell arises some distance behind the shock front, which is precisely what we require to explain why the thin shell of dense thermal material, which produces the irregular Faraday rotation, should be completely external to the radio shell.

4. Summary and Conclusions

The present 50 cm and 6 cm observations have, in combination with the earlier Westerbork 21 cm measurements, given us considerable insight as to the nature of physical processes occurring within the remnant of Tycho's supernova. In this paper our attention has been focussed on variations in the total intensity and linear polarization distributions between the three wavelengths. We have been able to confirm our earlier conclusion (Paper I) that the radio shell is most probably surrounded by a thin shell of dense thermal material swept up from the interstellar medium. We have also conclusively demonstrated that within the radio shell, the ordered component of the magnetic field is radially directed, and have suggested that this may follow from the Sedov solution itself.

The spectral index of the radio emission between 50 cm and 21 cm is steeper near the center of 3C10 than along the rim. It seems to us of considerable importance to determine whether this is also true of other young remnants. In particular, it would be valuable to know whether our conjecture that the flattening of the spectrum of Cas A is another manifestation of the same process is correct. It is not yet clear that Gull's mechanism can account for the observed radial dependence in spectral index, but this fact certainly does not seem to contradict his model.

Finally, among the many facts we would still like to know about 3C10, one stands out: its distance. Many of the parameters derived in Section 3.2 are strongly dependent on the assumed linear size [see the Appendix of our earlier paper (Paper I) for a complete discussion of this point]. As 3C10 provides one of the few reference points for testing models of young supernova remnants, a reliable distance is immensely important.

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