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Chapter 6

Squeezing of weak single-photon light

The unconventional photon-blockade phenomenon, described in the previous chapter, can be seen as amplitude squeezing of non-classical Gaussian states of light. In this chapter we will describe and discuss in detail how this squeezing relates to the unconventional photon blockade. Furthermore, a procedure is given how quadrature squeezing could be measured for weak nonlinearities.

6.1 Introduction

In the previous chapter, the first experimental observation of the unconventional photon blockade is shown and discussed. Its main signatures are the reduction of the $n = 2$ photon Fock state component and the amplitude squeezing of the corresponding state as shown in Fig 5.5. Here we explain in more detail an alternative way to look at the unconventional photon blockade. It turns out that the unconventional photon blockade can be seen as a particular realization of optimized Gaussian amplitude squeezing [100].

In general, one refers to a bunched photon stream if the photons are grouped together, and antibunched if there is a finite distance between the photons. Physically, this statement is quantified by the second-order correlation function $g^{(2)}(0)$, where $g^{(2)}(0) > 1$ corresponds to bunched light and $g^{(2)}(0) < 1$ corresponds to antibunched light. However, measuring only the second-order correlation function is not enough to fully characterize the quantum state. Consider for example a pure two-photon Fock state. One finds that

$$g^{(2)}(0) = \frac{\sum_{n=0}^{\infty} n(n-1)P_n}{\left(\sum_{n=0}^{\infty} nP_n\right)^2} = \frac{2P_2}{(2P_2)^2} = \frac{1}{2}. \tag{6.1}$$

This two-photon state should be called a bunched state, since two photons are lumped together, but by only measuring $g^{(2)}(0)$, one might conclude it is a single-photon state with reduced purity. It is nice to draw here an analogy to the famous poem written by John Godfrey Saxe: Blind men and the elephant. If one only sees a certain part of the elephant, one might think it is something completely different from what it actually is. For example, by only touching the trunk of the elephant one might think it is a snake. Here, the situation is similar, $g^{(2)}(0)$ alone does not tell what kind of quantum state one deals with and care should be taken in drawing conclusions. We will discuss here why unconventional photon blockade can be seen as a specific case of Gaussian amplitude squeezing, by using the measurable parameters mean photon number, second-order correlation function, the amount of quadrature and amplitude squeezing. Alternatively, one could also measure higher order correlation functions or one performs full quantum state tomography, but this requires an increase in measurement time. At the end we explain a procedure to measure the amount of quadrature squeezing in a two-level system.

6.2 Minimizing the second-order correlation $g^{(2)}(0)$.

We show that the unconventional photon-blockade effect can be viewed as a particular realization of optimized Gaussian amplitude squeezing [100]. For this, we consider a single-mode Gaussian squeezed light state written as

$$D(\alpha)S(\xi)\left|0\right\rangle = \left|\alpha, \xi\right\rangle. \tag{6.2}$$

Here, $D(\alpha)$ is the displacement operator and $S(\xi)$ is the squeezing operator. Using this, the two-photon probability becomes

$$\begin{aligned} \left|\left\langle 2\right| D(\alpha)S(\xi)\left|0\right\rangle\right|^2 &= \left|\left\langle 2\right| \exp(\alpha\hat{a}^\dagger - \alpha^*\hat{a}) \exp\left(\frac{1}{2}(\xi\hat{a}^2 + \xi\hat{a}^{\dagger 2})\right)\left|0\right\rangle\right|^2 \\ &\approx ((\bar{a})^2 - r)^2/2, \end{aligned} \tag{6.3}$$

with $\bar{a}$ the mean photon number and r the squeezing parameter. The interplay between a coherent displacement and a squeeze operation can cause the probability for a two-photon state to go to zero. Since it is experimentally hard to determine the exact photon-number distribution, it is more convenient to look at the second-order correlation $g^{(2)}(0)$, which is relatively easy to measure. Now, the full second-order correlation expression for a displaced squeezed vacuum state reads [100]

$$g^{(2)}(0) = 1 + \frac{\cosh(2r)}{\bar{a}^2 + \sinh^2(r)} - \frac{\bar{a}^2(1+\sinh(2r))}{\left(\bar{a}^2 + \sinh^2(r)\right)^2}. \tag{6.4}$$

Minimizing $g^{(2)}(0)$ as a function of r gives the value of r where the amount of antibunching is maximal. For any $\bar{\alpha}$ one finds that $g^{(2)}(0) < 1$, except for $\bar{\alpha} \to \infty$ where $g^{(2)}(0)$ approaches 1. Only at a low mean photon number $|\bar{\alpha}|^2 < 0.1$ one finds that$g^{(2)}(0) < 0.1$. This is the regime where we observe the unconventional photon blockade.

6.3 Amplitude squeezing

The state obtained by the unconventional photon blockade is a Gaussian state with a low mean photon number. Here we quantify and discuss how to measure the amount of amplitude squeezing. A state is amplitude squeezed if the photon-number variance is smaller than the variance of a coherent state with the same mean photon number. Since the photon-number variance of a coherent state is equal to the mean photon number, we define that a state is amplitude squeezed if

$$\left\langle (\Delta N)^2 \right\rangle < \langle N \rangle . \tag{6.5}$$

In order to measure amplitude squeezing, one determines the fluctuations from the actual detected count rate (σ_{SPS}) and divide it by the fluctuations of a shot-noise limited source of the same intensity (σ_{SN}). The theoretically predicted ratio is

$$\frac{\sigma_{SPS}}{\sigma_{SN}} = \sqrt{1 - \zeta\rho}, \tag{6.6}$$

where ζ denotes the overall detection efficiency and ρ is the probability that a pulse creates a single photon. In case of a single-photon source with $\rho = 1$, meaning that every pulse creates a single photon, and assuming $\zeta = 1$, we find that $\frac{\sigma_{SPS}}{\sigma_{SN}} = 0$, or in other words, 100 % squeezing. In the case of unconventional photon blockade in cavity-QED, we operate at a very low mean photon number, resulting in $\rho \approx 0$. This means that there is almost no amplitude squeezing, and reduced collection and detection efficiencies makes it hard to detect amplitude squeezing.

6.4 Quadrature squeezing

The amount of amplitude squeezing appearing in unconventional photon blockade is too small to detect with the current technologies, however it is possible to measure quadrature squeezing in the system. In case of quadrature squeezing it is crucial to first define the generic quadrature operator

$$\hat{X}(\phi) = \frac{1}{2}\left(\hat{a}e^{-i\phi} + \hat{a}^{\dagger}e^{+i\phi}\right). \tag{6.7}$$

This operator specifies the quadrature which is squeezed. For $\hat{X}(0)$ $\left(\hat{X}(\pi)\right)$ we squeeze the position (momentum) quadrature in the language of a generic harmonic oscillator. The condition for quadrature squeezing is now given as

$$\left\langle (\Delta X(\phi))^2 \right\rangle < \frac{1}{4}, \tag{6.8}$$

which is bounded by the Heisenberg uncertainty relation. A normalized value for the amount of quadrature squeezing is given by introducing the squeeze parameter [10],

$$s(\phi) = \frac{\left\langle \left(\Delta\hat{X}(\phi)\right)^2 \right\rangle - 1/4}{1/4} = 4\left\langle \left(\Delta\hat{X}(\phi)\right)^2 \right\rangle - 1 = 4\left\langle : \left(\Delta\hat{X}(\phi)\right)^2 : \right\rangle. \tag{6.9}$$

Here, squeezing exist whenever $-1 \leq s(\phi) < 0$. The notation $\langle :: \rangle$ means that the creation an annihilation operators are normal ordered. Experimentally, the amount of squeezing is often expressed as $10\log_{10}(1 + s(\phi))$. Typically, if there is a limited amount of amplitude squeezing, there exist an axis along which the system is quadrature squeezed. However, there is one particular case, a photon Fock state, where this is not true. A photon Fock state has no quadrature squeezing, however the photon-number distribution is squeezed infinitely. This explains why Fock states are so fundamentally different from other quantum states of light.

In Ref. [103], Vogel developed a method to determine the amount of quadrature squeezing of resonant fluorescence from an atom using homodyne interference. We explore the same method here, but assume that the fluorescent light comes from a QD. The fluorescent light is written as $\hat{E}_{fl}$ with $g^{(2)}(0) \ll 1$. For the homodyne intensity correlations, the light state $\hat{E}_{fl}$ is mixed with light from a local oscillator $\hat{E}_{LO}$, where $\hat{E}_{LO}$ is coherent laser light from the continuous-wave laser that excites the QD. The setup for homodyne intensity correlations is shown in Fig. 6.1. This setup is preferred over a homodyne cross correlations setup, since the sub-Poissonian statistics of the signal field directly contributes to the sub-poissonian statistics of the superimposed light. Therefore, nonclassical effects in the signal field can be interpreted as contributions to an overall nonclassical effect in the homodyne intensity scheme.

The output of the first beamsplitter consists of light from the QD and the local oscillator. This superimposed light field $\hat{E}_{SL}$ is written as

$$\hat{E}^{+}_{SL}(t) = \frac{1}{\sqrt{2}}\left(\hat{E}^{+}_{fl}(t) + e^{i\phi}\hat{E}^{+}_{LO}(t)\right), \tag{6.10}$$

where $\hat{E}^{+}_{fl}(t)$ and $\hat{E}^{+}_{LO}(t)$ represent the positive frequency at time t for the QD light and light from the local oscillator respectively. The light from the QD is described by a single mode field

$$\hat{E}_{fl} = \hat{E}^{+}_{fl}(t) + \hat{E}^{-}_{fl}(t) \sim i\hat{a}\exp(-i\omega t) - i\hat{a}^{\dagger}\exp(i\omega t), \tag{6.11}$$

which is used as a basis to determine the unnormalized second-order correlation function. The unnormalized second-order correlation function of the superimposed light field

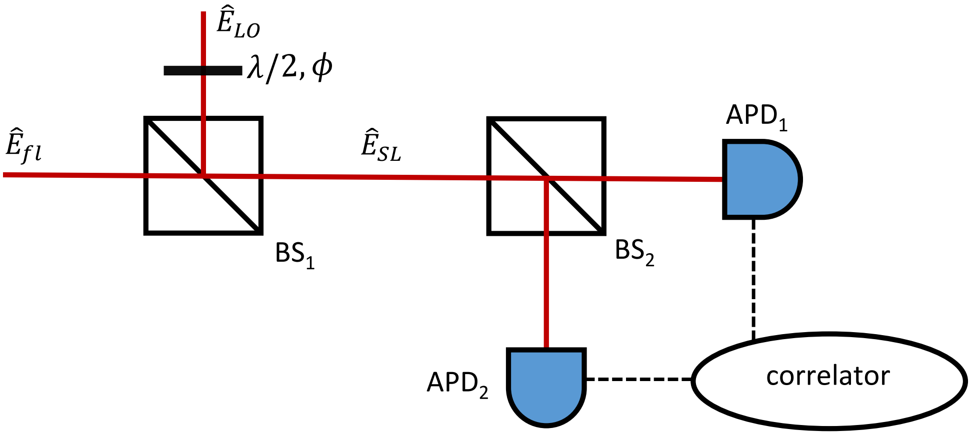

Figure 6.1: Homodyne intensity correlation scheme [79]. The light state from the QD in the micropillar $\left(\hat{E}_{fl}\right)$ is superimposed by the first beamsplitter (BS$_1$) with the local oscillator $\left(\hat{E}_{LO}\right)$, the resulting superimposed light $\left(\hat{E}_{SL}\right)$ is recorded by means of an Hanbury Brown Twiss (HBT) detection scheme.

$$G^{(2)}(t,t+\tau) = \left\langle \hat{E}^-_{SL}(t)\hat{E}^-_{SL}(t+\tau)\hat{E}^+_{SL}(t+\tau)\hat{E}^+_{SL}(t)\right\rangle \tag{6.12}$$

produces the well-known anti-bunched second-order correlation function of the resonance fluorescence $G^{(2)}_{fl}$ in the absence of a local oscillator.

Following the procedure by Vogel [79, 103], it is possible to write $G^{(2)}(t,t+\tau)$ as the sum of five local oscillator terms $|E_{LO}|^n$ with $n = 0,1,2,3,4$. Since the local oscillator behaves according to a classically theory it can be taken out of the averaging brackets and $G^{(2)}(\phi,t,t+\tau)$ can be written as an expansion of the local oscillator amplitude, Eq. 6.12 becomes

$$G^{(2)}(t,t+\tau) = \sum_{n=0}^{4} G^{(2)}_n(t,t+\tau), \tag{6.13}$$

The five terms, under the assumption that we are in the stationary regime, which allows one to drop the t dependence, become

$$G^{(2)}_0(\phi,\tau) = \frac{1}{4}\left\langle \hat{E}^-_{fl}(0)\hat{E}^-_{fl}(\tau)\hat{E}^+_{fl}(\tau)\hat{E}^+_{fl}(0)\right\rangle \tag{6.14}$$

$$\begin{aligned} G^{(2)}_1(\phi,\tau) = \tfrac{E_{LO}}{4}\Big(& e^{-i\phi}\left\langle \hat{E}^-_{fl}(\tau)\hat{E}^+_{fl}(\tau)\hat{E}^+_{fl}(0)\right\rangle + e^{-i\phi}\left\langle \hat{E}^-_{fl}(0)\hat{E}^+_{fl}(\tau)\hat{E}^+_{fl}(0)\right\rangle + \\ & e^{i\phi}\left\langle \hat{E}^-_{fl}(0)\hat{E}^-_{fl}(\tau)\hat{E}^+_{fl}(0)\right\rangle + e^{i\phi}\left\langle \hat{E}^-_{fl}(0)\hat{E}^-_{fl}(\tau)\hat{E}^+_{fl}(\tau)\right\rangle\Big) \end{aligned} \tag{6.15}$$

$$\begin{aligned} G^{(2)}_2(\phi,\tau) = \frac{E^2_{LO}}{4}\Big(& e^{2i\phi}\left\langle \hat{E}^-_{fl}(0)\hat{E}^-_{fl}(\tau)\right\rangle + e^{-2i\phi}\left\langle \hat{E}^+_{fl}(\tau)\hat{E}^+_{fl}(0)\right\rangle \\ & \left\langle \hat{E}^-_{fl}(\tau)\hat{E}^+_{fl}(0)\right\rangle + \left\langle \hat{E}^-_{fl}(0)\hat{E}^+_{fl}(\tau)\right\rangle + \\ & \left\langle \hat{E}^-_{fl}(0)\hat{E}^+_{fl}(0)\right\rangle + \left\langle \hat{E}^-_{fl}(\tau)\hat{E}^+_{fl}(\tau)\right\rangle\Big) \end{aligned} \tag{6.16}$$

$$G_3^{(2)}(\phi,\tau) = \frac{E_{LO}^3}{4}\left(e^{i\phi}\left\langle \hat{E}_{fl}^-(0)\right\rangle + e^{i\phi}\left\langle \hat{E}_{fl}^-(\tau)\right\rangle + e^{-i\phi}\left\langle \hat{E}_{fl}^+(\tau)\right\rangle + e^{-i\phi}\left\langle \hat{E}_{fl}^+(0)\right\rangle\right) \tag{6.17}$$

$$G_4^{(2)}(\phi,t,t+\tau) = \frac{E_{LO}^4}{4} \tag{6.18}$$

From these equations one notices that $G_3^{(2)}(\phi,\tau)$ and $G_4^{(2)}(\phi,\tau)$ are independent of the time delay and can be neglected. Later, a detailed explanation is given about the physical meaning of $G_0^{(2)}(\phi,\tau)$, $G_1^{(2)}(\phi,\tau)$, $G_2^{(2)}(\phi,\tau)$. Since we assume that there are no correlations for $\tau \to \infty$, it is advantageous to compare the short-time value of the correlation function with its long-time value. In this sense we may introduce the following measure for photon-pair correlations:

$$\Delta G^{(2)}(\phi,\tau) = G^{(2)}(\phi,\tau) - \lim_{\tau_1\to\infty} G^{(2)}(\phi,\tau+\tau_1). \tag{6.19}$$

Now, we derive all terms of $\Delta G^{(2)}(\phi,\tau)$ and show that $\Delta G_2^{(2)}(\phi,\tau)$ is a measure for the amount of quadrature squeezing. Using Eq. 6.13 the terms of $\Delta G^{(2)}(\phi,\tau)$ become

$$\Delta G_0^{(2)}(\phi,\tau) = \frac{1}{4}\left\langle \hat{E}_{fl}^-(0)\hat{E}_{fl}^-(\tau)\hat{E}_{fl}^+(\tau)\hat{E}_{fl}^+(0)\right\rangle - \frac{1}{4}\left\langle \hat{E}_{fl}^-\hat{E}_{fl}^+\right\rangle\left\langle \hat{E}_{fl}^-\hat{E}_{fl}^+\right\rangle = \left(G_0^{(2)}(\tau) - \frac{I_{fl}^2}{4}\right), \tag{6.20}$$

$$\Delta G_1^{(2)}(\phi,\tau) = G_1^{(2)}(\phi,\tau) - \frac{E_{LO}}{4}\left(e^{-i\phi}\left\langle \hat{E}_{fl}^-(\tau\to\infty)\hat{E}_{fl}^+(\tau\to\infty)\right\rangle\left\langle \hat{E}_{fl}^+(0)\right\rangle + e^{-i\phi}\left\langle \hat{E}_{fl}^-(0)\hat{E}_{fl}^+(0)\right\rangle\left\langle \hat{E}_{fl}^+(\tau\to\infty)\right\rangle + e^{i\phi}\left\langle \hat{E}_{fl}^-(0)\hat{E}_{fl}^+(0)\right\rangle\left\langle \hat{E}_{fl}^-(\tau\to\infty)\right\rangle + e^{i\phi}\left\langle \hat{E}_{fl}^-(\tau\to\infty)\hat{E}_{fl}^+(\tau\to\infty)\right\rangle\left\langle \hat{E}_{fl}^-(0)\right\rangle\right) \tag{6.21}$$

$$\begin{aligned}\Delta G_2^{(2)}(\phi,\tau) = {} & \tfrac{E_{LO}^2}{4}\left(e^{2i\phi}\left\langle \hat{E}_{fl}^-(0)\hat{E}_{fl}^-(\tau)\right\rangle + e^{-2i\phi}\left\langle \hat{E}_{fl}^+(\tau)\hat{E}_{fl}^+(0)\right\rangle + \right. \\ & \left. + \left\langle \hat{E}_{fl}^-(0)\hat{E}_{fl}^+(\tau)\right\rangle + \left\langle \hat{E}_{fl}^-(\tau)\hat{E}_{fl}^+(0)\right\rangle + \left\langle \hat{E}_{fl}^-(0)\hat{E}_{fl}^+(0)\right\rangle + \left\langle \hat{E}_{fl}^-(\tau)\hat{E}_{fl}^+(\tau)\right\rangle\right) - \\ & \tfrac{E_{LO}^2}{4}\left(e^{2i\phi}\left\langle \hat{E}_{fl}^-(\tau\to\infty)\right\rangle^2 + e^{-2i\phi}\left\langle \hat{E}_{fl}^+(\tau\to\infty)\right\rangle^2 + \right. \\ & \left. +2\left\langle \hat{E}_{fl}^-(\tau\to\infty)\right\rangle\left\langle \hat{E}_{fl}^+(\tau\to\infty)\right\rangle + 2\left\langle \hat{E}_{fl}^-(\tau\to\infty)\hat{E}_{fl}^+(\tau\to\infty)\right\rangle\right) \\ = {} & \tfrac{E_{LO}^2}{4}\left(e^{2i\phi}\left\langle \hat{E}_{fl}^-(0)\hat{E}_{fl}^-(\tau)\right\rangle + e^{-2i\phi}\left\langle \hat{E}_{fl}^+(\tau)\hat{E}_{fl}^+(0)\right\rangle + \right. \\ & \left. \left\langle \hat{E}_{fl}^-(0)\hat{E}_{fl}^+(\tau)\right\rangle + \left\langle \hat{E}_{fl}^-(\tau)\hat{E}_{fl}^+(0)\right\rangle\right) - \\ & \tfrac{E_{LO}^2}{4}\left(e^{2i\phi}\left\langle \hat{E}_{fl}^-(\tau\to\infty)\right\rangle^2 + e^{-2i\phi}\left\langle \hat{E}_{fl}^+(\tau\to\infty)\right\rangle^2 + \right. \\ & \left. 2\left\langle \hat{E}_{fl}^-(\tau\to\infty)\right\rangle\left\langle \hat{E}_{fl}^+(\tau\to\infty)\right\rangle\right),\end{aligned} \tag{6.22}$$

$$\Delta G_3^{(2)}(\phi,\tau) = 0, \tag{6.23}$$

$$\Delta G_4^{(2)}(\phi,\tau) = 0. \tag{6.24}$$

Here, we use that in the limit $\tau_1 \to \infty$ one finds $\left\langle \hat{E}_{fl}^{-}(0)\hat{E}_{fl}^{-}(\tau_1 \to \infty)\right\rangle \to \left\langle \hat{E}_{fl}^{-}(\tau_1 \to \infty)\right\rangle^2$. This mean that the correlations between the electric field expectation values disappear and one can separate them. Note that the expectation value of the electric field is independent of τ although we have written $\hat{E}(\tau_1 \to \infty)$ for clarity. This leads to $\left\langle \hat{E}_{fl}^{-}(0)\hat{E}_{fl}^{+}(0)\right\rangle = \left\langle \hat{E}_{fl}^{-}(\tau_1)\hat{E}_{fl}^{+}(\tau_1)\right\rangle = \left\langle \hat{E}_{fl}^{-}\hat{E}_{fl}^{+}\right\rangle = I_{fl}$. Using these relations it is easy to go from the first to the second step in Eq 6.22. Now, we relate $\Delta G_2^{(2)}(\phi,\tau)$ to the quadrature squeezing by defining a squeezing operator. For a single mode field the position (X_1) and momentum operator (X_2) of the squeezing are, using Eq. 6.9, defined as

$$\begin{aligned}
\hat{X}_1(\phi) &= \frac{1}{2}C\left(\hat{a}\exp(-i\phi) + \hat{a}^\dagger \exp(i\phi)\right) = \\
&\frac{1}{2}\left(\hat{E_s}^{+}\exp(-i\phi) + \hat{E_s}^{-}\exp(i\phi)\right) = \frac{1}{2\epsilon}\left(\hat{E}_{fl}^{+}\exp(-i\phi) + \hat{E}_{fl}^{-}\exp(i\phi)\right), \\
\hat{X}_2(\phi) &= \frac{1}{2i}C\left(\hat{a}\exp(-i\phi) - \hat{a}^\dagger \exp(i\phi)\right) = \\
&\frac{1}{2i}\left(\hat{E_s}^{+}\exp(-i\phi) - \hat{E_s}^{-}\exp(i\phi)\right) = \frac{1}{2\epsilon i}\left(\hat{E}_{fl}^{+}\exp(-i\phi) - \hat{E}_{fl}^{-}\exp(i\phi)\right).
\end{aligned} \tag{6.25}$$

Here, C is a constant which connects the electric field E_s to the annihilation and creation operators with $|C|^2 = 2I_{fl}$. The factor 2 is a normalization constant which appears because of the $\frac{1}{\sqrt{2}}$ when adding $\hat{E}_s^{+}$ and $\hat{E}_s^{-}$ to obtain the total field $\hat{E}_s$. $|\epsilon|^2$ depends on the product of the decay rate, collection and detection efficiencies. In order to relate the abstract theory to experiments we include a parameter V, the visibility of the interferometer, which includes the spatial mode overlap of the beams. Using the definition for X_1 and X_2 one finds that

$$\begin{aligned}
\Delta G_2^{(2)}(\phi,\tau) &= V^2 E_{LO}^2 |\epsilon|^2 \left(\left\langle : \hat{X}_1(\phi,0)\hat{X}_1(\phi,\tau) :\right\rangle - \left\langle \hat{X}_1(\phi)\right\rangle^2\right) \\
&= V^2 E_{LO}^2 |\epsilon|^2 \left\langle : \Delta\hat{X}_1(\phi,0)\Delta\hat{X}_1(\phi,\tau) :\right\rangle.
\end{aligned} \tag{6.26}$$

This shows that $\Delta G_2^{(2)}(\phi,\tau)$ is proportional to the amount of quadrature squeezing defined in Eq. 6.9. The question that now arises, is how to determine $\Delta G_2^{(2)}(\phi,\tau)$ experimentally. It is trivial to measure $\Delta G^{(2)}(\phi,\tau) = \Delta G_0^{(2)}(\phi,\tau) + \Delta G_1^{(2)}(\phi,\tau) + \Delta G_2^{(2)}(\phi,\tau) + \Delta G_3^{(2)}(\phi,\tau) + \Delta G_4^{(2)}(\phi,\tau)$, which is a second-order correlation measurement from which the background is subtracted at long time delays. Since $\Delta G_3^{(2)}(\phi,\tau) = 0$ and $\Delta G_4^{(2)}(\phi,\tau) = 0$, one can determine $\Delta G_2^{(2)}(\phi,\tau)$ by measuring $\Delta G_0^{(2)}(\phi,\tau)$ and $\Delta G_1^{(2)}(\phi,\tau)$. From Eq. 6.20 it follows that $\Delta G_0^{(2)}(\phi,\tau)$ can be obtained by measuring the second-order correlation function for only the QD light and thus blocking the local oscillator light. The last term, $\Delta G_1^{(2)}(\phi,\tau = 0)$, can be obtained by using the periodicity in the phase. By rewriting Eq. 6.21, one notices that

$$\Delta G_1^{(2)}(\phi) = \tfrac{E_{LO}}{4}\left(\left\langle : \left(\hat{E}_{fl}^{+}(0)e^{-i\phi} + \hat{E}_{fl}^{-}(0)e^{i\phi}\right)\hat{E}_{fl}^{-}(0)\hat{E}_{fl}^{+}(0) : \right\rangle\Big|_{\phi=0} 2\cos(\phi)\right)$$
$$-\tfrac{E_{LO}}{4} I_{fl}\left(\left\langle \hat{E}_{fl}^{+}(0)\right\rangle + \left\langle \hat{E}_{fl}^{-}(0)\right\rangle\right)\Big|_{\phi=0} 2\cos(\phi) \tag{6.27}$$
$$= V\tfrac{E_{LO}}{2}\left\langle : \Delta\hat{E}_{fl}(\phi)\Delta\hat{I}_{fl}(\phi) : \right\rangle\Big|_{\phi=0}\cos(\phi),$$

which has a 2π periodicity. Using this and the fact that $\Delta G_2^{(2)}(\phi, \tau = 0)$ is periodic in τ, with period π, Eq. 6.21 is written as

$$\Delta G_1^{(2)}(t, \phi) = \frac{\Delta G^{(2)}(t, \phi = 0) - \Delta G^{(2)}(t, \phi = \pi)}{2}\cos(\phi). \tag{6.28}$$

Combining the above derivations, the amount of quadrature squeezing can be evaluated by measuring only second-order correlation functions. In order to do this, the phase ϕ is modulated. The effectively means that fraction of local oscillator light that interacted with the single photon light is changed. Finally, the total amount of squeezing can be obtained from

$$\left\langle : \Delta\hat{X}_1(\phi, 0)\Delta\hat{X}_1(\phi, t) : \right\rangle = \frac{\Delta G^{(2)}(\phi, t) - \Delta G_0^{(2)}(t, \phi) - \Delta G_1^{(2)}(t, \phi)}{V^2 E_{LO}^2 |\epsilon|^2}. \tag{6.29}$$

The parameters V^2, E_{LO}^2, $|\epsilon|^2$ have to be determined separately. V is the interference visibility single photon light, E_{LO}^2 is the intensity of the local oscillator and $|\epsilon|^2$ can be determined from the detected RF count rate combined with the Rabi frequency and radiative decay rate of the QD.

Weak local oscillator

There are two possible regimes of the mixing of a local oscillator with QD light: the weak and strong local oscillator regime. First, we consider the regime with a weak local oscillator, where the largest amount of squeezing is generated. This is when the intensity of the fluorescence light from the QD and local oscillator is of similar magnitude. Taking only the negative part of $\Delta G_2^{(2)}(\phi, \tau)$ into account Eq. 6.22 becomes

$$\Delta G_{2max}^{(2)}(\phi, \tau) = -\frac{E_{LO}^2}{4}\left(e^{2i\phi}\left\langle \hat{E}_{fl}^{-}(\tau)\right\rangle^2 + e^{-2i\phi}\left\langle \hat{E}_{fl}^{+}(\tau)\right\rangle^2 + \right. \tag{6.30}$$
$$\left. 2\left\langle \hat{E}_{fl}^{-}(\tau)\right\rangle\left\langle \hat{E}_{fl}^{+}(\tau)\right\rangle\right)$$
$$= -\frac{E_{LO}^2}{4}\left(\left\langle \hat{E}_{fl}^{-}(\tau)\right\rangle + \left\langle \hat{E}_{fl}^{+}(\tau)\right\rangle\right)^2 2\cos(2\phi)$$
$$= -I_0 I_{fl}\frac{1}{2}\cos(2\phi).$$

The maximal total squeezing is given by

$$\left\langle : \Delta\hat{X}_1(\phi, 0)\Delta\hat{X}_1(\phi, \tau) : \right\rangle_{max} = \frac{-I_0 I_{fl}\frac{1}{2}\cos(2\phi)}{V^2 E_{LO}^2 |\epsilon|^2}. \tag{6.31}$$

Removing the detection efficiency, visibility factor, and making the squeezing dimensionless by dividing by the uncorrelated count rate of the total field, leads to a maximal theoretical squeezing of

$$\begin{aligned} s(\phi)_{max} &= 4\left\langle :\Delta\hat{X}_1(\phi,0)\Delta\hat{X}_1(\phi,\tau):\right\rangle \\ &= \frac{-4I_0 I_{fl}\frac{1}{2}\cos(2\phi)}{\left\langle \hat{E}_{SL}^{-}\hat{E}_{SL}^{+}\right\rangle^2} \\ &= \frac{-4I_0 I_{fl}\frac{1}{2}\cos(2\phi)}{\frac{1}{4}\left[I_0 + I_{fl} + 2(I_0 I_{fl})^{1/2}\cos(\phi)\right]^2} = -0.5, \end{aligned} \tag{6.32}$$

by choosing $\phi = 2\pi n$ and $I_0 = I_{fl}$. In other words, the squeezing of the fluorescence leads to an effect of 50% (or $1 - 0.5 = 0.5 \rightarrow -3.0$ dB). If $I_{fl} > I_0$, i.e., a bright fluorescence signal of single-photon nature and a weak local oscillator, $s(\phi)_{max}$ of Eq. 6.32 decreases.

Strong local oscillator

In the regime of a strong local oscillator, one is limited by the amplitude fluctuations of the local oscillator since the local oscillator noise is not balanced out. Remember that the classical fluctuations of the local oscillator can be attenuated in the same manner as its amplitude. This limits the use of a strong local oscillator to determine the amount of quadrature squeezing. Mathematically, a local oscillator with stationary Gaussian amplitude fluctuations is written as:

$$E_{LO} = E_0 + \delta E(t). \tag{6.33}$$

Here $\overline{\delta E(t)} = 0$, because averaging over the classical laser fluctuation gives zero. Now, we reconsider the quantity $\Delta G_4^{(2)}(\phi,\tau)$ and observe that

$$\begin{aligned} \Delta G_4^{(2)} &= \frac{1}{4}\left(4E_0^2\overline{(\delta E)^2} + 2\left(\overline{(\delta E)^2}\right)^2\right) \\ &\approx E_0^2\overline{(\delta E)^2}, \end{aligned} \tag{6.34}$$

where we made use of the suitable assumption that the relative amplitude noise of the local oscillator is small,

$$\frac{\overline{(\delta E)^2}}{E_0^2} = \epsilon \ll 1. \tag{6.35}$$

As a result of the amplitude fluctuations, we cannot measure $\Delta G_2^{(2)}$ independently from $\Delta G_4^{(2)}$. Since the stationary regime is considered, the effect of $\Delta G_3^{(2)}$ is neglected. To quantify the effect of $\Delta G_4^{(2)}$ on $\Delta G_2^{(2)}$ it is useful to write this in the signal-to-noise-ratio form

$$\left|\frac{\Delta G_2^{(2)}}{\Delta G_4^{(2)}}\right| = \frac{\left\langle :\Delta\hat{X}_1(\phi,0)\Delta\hat{X}_1(\phi,\tau):\right\rangle}{\epsilon E_0^2}. \tag{6.36}$$

From this it is easily seen that the usually preferred strong local oscillator may prevent the detection of the quantum noise of the signal field we are interested in. That is why strong local oscillators are not useful here.

6.5 Quadrature squeezing and unconventional photon blockade

In order to determine the amount of quadrature squeezing for the unconventional photon blockade one sends the transmitted light to a beamsplitter together with the excitation laser and measures coincidences in the superimposed signal. Here, we investigate theoretically if this experiment is archievable. Using Eq. 6.3 we observe that a vanishing two-photon probability is obtained if the squeeze parameter r is equal to $\bar{a}^2$, which is the mean photon number. By defining the amount of quadrature squeezing as $\left\langle (\Delta X_1)^2 \right\rangle = \frac{1}{4}e^{-2r}$ and considering a $\langle n_{out} \rangle \approx 0.004$ (Fig. 5.5(a)), this condition leads to $10 \log_{10}(e^{-0.008}) = -3 \times 10^{-2}$ dB squeezing. In order to confirm this rough estimation we calculate the amount of squeezing from our quantum master simulation. In Fig. 6.2, the amount of squeezing is shown as a function of the $\lambda/2$ and $\lambda/4$ wave plate orientation in the transmission path where the axes are similar to the axes in Fig 5.5 (b) and Fig 5.5 (c). The expected amount of quadrature squeezing in the region of the unconventional photon blockade is close to -3×10^{-3} dB as indicated by the black arrow. The deviation is because the theoretical analysis is only a rough approximation. This shows that the expected amount of squeezing is very small and the second-order correlation function needs to be measured extremely accurately to determine the amount of squeezing via homodyning. This is hard, since we are limited by the detector jitter due to the fast decay of the cavity, which effectively shortens the QD lifetime even further.

The use of a local oscillator combined with second-order correlation measurement is a nice tool to relatively easily get a qualitative assessment of the quantum state of light. By changing the phase ϕ and the laser power while measuring the second-order correlation function, one can determine the squeezing direction and the amount of squeezing.

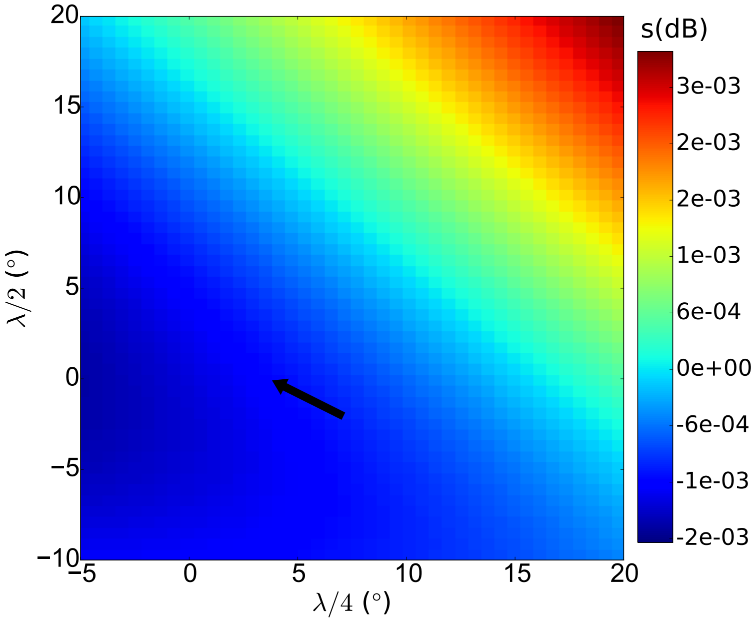

Figure 6.2: False color plot of the amount of squeezing s(dB) as a function of the orientation of the $\lambda/2$ and $\lambda/4$ wave plate in the transmission path. The black arrow indicates the small amount of squeezing in the region of unconventional photon blockade and corresponds to the black arrow in Fig. 5.5.