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Counting points on K3 surfaces and other arithmetic-geometric objects

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Nothing will come of nothing

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Lear, KING LEAR, Scene 1.1, line 82

Note about the epigraphs: the spelling of names, quoted line numbers, line breaks, interpunction, and exact wording are taken from [Sha08].

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