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CHAPTER 4

Development of the lesson series on measurement

Classroom interaction in a deductive and in an inductive approach

4.1 Introduction

Since Slavin and Lake (2008) conclude that the didactics and curricula\(^4\) have less effect on student performance in mathematics than classroom interaction between students and the teacher has (see Chapter 1), we hypothesize that improving classroom interaction in mathematics classes will have a positive effect on student performance.

Classroom interaction can be guided in many ways: classroom discussions, dialogues, discussing theorems, quizzing, allowing room for questions after instruction, instruction through classroom interaction, scaffolding, asking students to develop test questions and discussing them, et cetera (Roefs, 2010). Traditional mathematics education suggests a deductive approach to this interaction, where students are taught how and why rules and procedures work: a rule is explained and a procedure is modeled, before students apply this procedure in different contexts and discuss how they used the rule or the procedure (Van de Craats, 2007). Interaction in this deductive approach is mostly vertical (between students and the teacher). The Realistic Mathematics Education movement (RME), however, suggests an inductive approach to this interaction, where students are guided in their joint reinventions of mathematical insights: a realistic context is chosen as a playground for discovering mathematical procedures and rules (Treffers, 1993). Although the teacher introduces the context and guides the reinvention process, in this inductive approach, interaction about the mathematical context is mostly horizontal (between students).

The purpose of this study is to describe the development of lesson series on measurement for prospective elementary school teachers using both approaches: one with a

\(^4\) Didactics are concerned with how to learn and teach, and with what teachers and learners should be doing in class. A curriculum states what is to be learned and taught, and in which order. Both can either be fixed, or process oriented (Gravemeijer & Terwel, 2000). In process oriented didactics, teachers may choose to change the way they teach depending on what they believe to be educational needs of a specific group or learner, whereas in fixed didactics teachers stick to the chosen didactics (teachers need to change didactics to suit learners, versus learners need to learn things the way they are taught). In a process oriented curriculum, teachers may choose to elaborate on certain topics that emerge from classroom discussions or daily news, whereas in a fixed curriculum teachers stick to the pre-defined (order of) topics.
pure deductive and another with a pure inductive approach to classroom interaction. Another aim is to learn what the consequences are of rigorously following guidelines belonging to each of the approaches when developing these lesson series with either a deductive or an inductive approach. In the next two paragraphs we will discuss background information on classroom interaction, on both the deductive approach and the inductive approach, and on measurement.

4.1.1 Background on classroom interaction

Nowadays an increasing number of teachers encourage and assist students to share their ideas and to explain them, instead of leaving it solely up to teachers to provide explanations. Pierson (2008) found strong correlations between using 7th grade students’ ideas in mathematics class and their gain scores on mathematics tests. It is however unlikely that students will always be able to come up with big ideas (Charles & Carmel, 2005) and solve problems on their own. Sometimes it is more efficient for teachers to give information, and to evaluate responses (Pierson, 2008).

One way to engage students is by asking them questions. Nelissen (2002) defined two types of questions: 1) check questions (i.e. reproduction questions like “how many cm in a dm?”), evaluative questions like “are we clear on this?”, diagnostic questions like “think out loud”); 2) stimulating questions (i.e. induce critical thinking “explain why”, induce reflection “are you sure?”). In the past, classroom interaction was limited to teachers asking questions and students answering them; in the last 50 years, cooperation and the more exhaustive collaboration methods emerged in classrooms. In a collaborative environment, students engage in each other’s mathematical ideas (Staples, 2007). For this engaging to be fruitful, teacher focus on mathematically significant aspects of students’ solution strategies is essential (Stein et al., 2008), because without that focus students might miss the essence of what they are expected to learn (Brown & Campione 1994; Deci & Chandler, 1986; Leeman, Wardekker, & Majoor, 2007). Stein et al. (2008) introduced a set of five key practices for using student responses in classroom interaction, in order to enrich the mathematical learning of the whole class: (1) anticipating likely student responses to cognitively demanding mathematical tasks, (2) monitoring students’ responses to the tasks during the explore phase, (3) selecting particular students to present their mathematical responses during the discuss-and-summarize phase, (4) purposefully sequencing the student responses that will be displayed, and (5) helping the class make mathematical connections between different
students' responses, and between students' responses and key ideas (like the idea that scale calculations as well as relationships within the metric system can be viewed as ratios, and that ratio tables can be used in both contexts). Van Eerde (2008) summed up the following skills for guiding classroom interaction: observing students, asking questions and responding to students in a mathematically deepening manner, non-verbally stimulating classroom interaction, stimulating student reflection, stating what has been learned (at the end of and during lessons), offering different types of guidance, and developing social and mathematical standards.

A typical deductive approach to classroom interaction on mathematics starts with a mathematical problem and the teacher explaining why a rule applies and how and why a procedure for solving the problem works. Next, the teacher uses modeling: he follows the procedure to solve the problem while quizzing several students (i.e.: “Lucy, which step of the procedure should I take next?”). After this, students solve a different problem using the explained procedure, and finally the teacher asks a few students to explain how they solved the problem, and gives them feedback on how they did. In a deductive approach, students engage primarily on the mathematical ideas of the teacher, and not or much less on the ideas of peers.

A typical inductive approach to classroom interaction on mathematics starts with a context (for example: a city map) and with the teacher challenging students to mathematically explore this context, and to explore, usually in small groups, a mathematical problem that follows the exploration (Lampert, 2001) (for example: discuss in your group how you can estimate the scale of this city map). Next, different approaches to solve the problem (and not just the teacher’s approach, as is the case in the deductive approach) are explained by students in a whole class discussion. Finally, students solve a different problem using their choice of one of the discussed procedures, and the teacher asks different students to explain how they solved the problem.

4.1.2 Background on measurement
Measurement education in elementary schools and in teacher training colleges in The Netherlands entails learning to estimate measures by developing personal reference measurements, understanding relationships within the metric system, learning to calculate length, area and volume, and learning to calculate with scale (Van Zanten et al., 2009). Students need to learn to structure and quantify reality, and therefore they need to be able to
estimate. The estimation skill requires students to have a sense of number and measurement, and reasoning skills (Lang, 1999). Students must have an idea of their own height and weight, of what the area of a football field or a tray is, and of what the volume of a soda can or a bathtub is. They must also be able to relate those measures (references) to other objects, so they can, for example, deduce what a good estimate for the length of a tree might be. Besides estimation, students must be able to fully grasp relationships in scale contexts (how high is the 26-meter building in the scale model, scale 1 to 500?) and those between different metric system measures (how many centimeters in 4 meters, how many liters in 5 m$^3$?). Furthermore, students must be able to calculate area and volume of specific two and three-dimensional figures, such as (compositions of) cuboids, beams, rectangles, and triangles.

If students only learn how to follow procedures without grasping the essentials of the system, things can go wrong. A student might think: “One can calculate an area by multiplying length by width. A circle has no length and width, therefore a circle has no area” (De Moor, 1999, p. 429). De Moor emphasizes the importance of adding background information on procedures and links with different contexts. The idea is for students to build their own personal network of meaning, references and relationships between measures, through classroom discussions and research activities (Gravemeijer et al., 2007, p. 55). An example of such an activity and discussion is to try to identify an error in argumentations such as: “2 m by 52 cm gives an area of 104” (Gravemeijer et al., 2007, pp. 39-40) and to try to determine the volume of a 3 mm high pool of milk on the floor with an area of five pieces of paper (Gravemeijer et al., 2007, p. 52).

**Research questions**

We expected teachers to encounter some difficulties if we asked them to strictly follow guidelines belonging to the deductive approach, because we believe student involvement decreases when a teacher always explains the procedures. That would have a deteriorating effect on the quality of classroom interaction, which would probably have a negative effect on student outcomes (Slavin & Lake, 2008). We searched for ways that would keep students productively engaged while holding on to the deductive approach, and to prevent students from memorizing procedures without understanding them, for example by focusing on critical judgement and flexibly applying procedures (Star, 2007). We also expected teachers to encounter some difficulties if we asked them to strictly follow guidelines belonging to the inductive approach, because students do not always come up with big ideas (Pierson, 2008), and because we believed that the students would ask the teacher to explain the procedures
instead of trying to discover them for themselves (Gravemeijer, 1994). We also took into account that teachers must have a deep mathematical knowledge for teaching (Gravemeijer, 1994; Silverman & Thompson, 2008), in order to appropriately respond to students’ ideas, and a deep pedagogical content knowledge (Shulman, 1986; Van Driel, 2008), in order to guide the inductive classroom interaction. We searched for ways to hold on to the inductive approach, even though students might demand a deductive approach. The research questions that will be answered in this chapter are: 1. What do experts and elementary school mathematics teacher educators believe to be a pure deductive approach to classroom interaction in a measurement lesson series for prospective elementary school teachers? 2. What do experts and elementary school mathematics teacher educators believe to be a pure inductive approach to classroom interaction in a measurement lesson series for prospective elementary school teachers? 3. What are the consequences of rigorously following guidelines for a deductive approach or an inductive approach?

4.2 Method

We used a systematic step-by-step approach to develop two series of five lessons. First, the practical framework and design requirements for the lessons series were set (Van Aken & Andriessen, 2011). The design of both the deductive and the inductive approach began with the (Rotterdam School of Education) course objectives: the student is able to estimate sizes using reference measures, convert between units and prefixes in the metric system, calculate with scale, and perform calculations of length, area and volume with triangular and rectangular 2D and 3D figures. Example exercises for each aspect are shown in Table 4.1. Five 50-minute lessons were scheduled for this course per student group, and each student group received one lesson per week. Homework assignments were the same in both approaches.
Table 4.1. Example exercises for each aspect.

<table>
<thead>
<tr>
<th>aspect</th>
<th>example exercises</th>
</tr>
</thead>
<tbody>
<tr>
<td>estimation</td>
<td>What is the length of a standard stair step?</td>
</tr>
<tr>
<td></td>
<td>What is the weight of a sandwich?</td>
</tr>
<tr>
<td>conversion</td>
<td>0.034 km = ... dm</td>
</tr>
<tr>
<td></td>
<td>450 are = ... m²</td>
</tr>
<tr>
<td>scale</td>
<td>The distance from Rotterdam to Paris is 450 km. My map has a scale of 1: 300,000. Calculate the distance from Rotterdam to Paris on my map in cm. Explain.</td>
</tr>
<tr>
<td></td>
<td>The area of the living room is 5 dm$^2$ on the map. In reality, the area of the living room is 45 m$^2$. What is the scale of the map? Explain.</td>
</tr>
<tr>
<td>calculation</td>
<td>The area of a rhombus is 16 dm$^2$. One diagonal is twice as long as the other. Calculate the length of the diagonals of the rhombus. Explain.</td>
</tr>
<tr>
<td></td>
<td>The volume of a pack of lemonade is 1.5 liters. The pack is 0.75 dm long, and 1 dm wide. Calculate the height of the pack. Explain.</td>
</tr>
</tbody>
</table>

Next, structured interviews were conducted with experts on what they believe to be a pure deductive and a pure inductive approach to classroom interaction in a measurement lesson series for prospective elementary school teachers, and what they believe teachers and students should be doing in both approaches. We interviewed an expert (PhD) in the field of mathematics education – specifically in the field of measurement – for elementary schools and for teacher training college with a preference for inductive education (expert 1), an expert (professor) in mathematics with a preference for deductive mathematical education (expert 2), and a retired elementary school mathematics teacher educator (expert 3). Next, two PowerPoint blueprints and teacher manuals were developed, and subsequently discussed with two focus groups of mathematics teacher educators (with questions like: do you think this lesson is purely deductive / inductive? What do you think of the classroom interaction? Which questions should the teacher ask? How should the teacher respond to student ideas?), then revised and the answers discussed again with the focus groups. The last step was a final revision with supervisors, to fully exclude inductive elements from the deductive approach and vice versa.

We chose to conduct focus groups (one to develop the inductive approach, and one to develop the deductive approach), because members would be more likely to challenge each other to think more deeply about their answers than they would in one on one interviews, and
also because the pursuit of consensus and unambiguous answers was important (Bryman, 2012). The same five elementary school mathematics teacher educators participated in both focus groups: three women, and two men (mean age of 40 years, range 33-55, mean teaching experience of eight years, range 3-15). The focus groups’ moderator was one of the researchers. After the lesson series had been taught, interviews were conducted with the five focus group members (they all taught one group using the deductive approach and one group using the inductive approach) to find out what they believed to be pros and cons with respect to the two approaches. The teacher’s preference for a deductive or an inductive approach was also recorded, since it could have an effect on both their recommendations and their teaching.

4.3 Results

4.3.1 Interviews

In this section we will describe the results of the interviews with experts and focus groups, and present the schematic version (the full version is available from the author) of the PowerPoint sheets for two series of five lessons with teacher manuals. The teacher manual contains guidelines for tables setting, lesson start, recap of the previous lesson, lesson goals, main topic introduction, questions to be asked, student tasks including the group size, responses to students’ remarks, and how to evaluate lesson objectives.

Results of the interviews with experts – deductive approach

All three experts advised us to use plenary modeling in the deductive approach. This means that the teacher should demonstrate, step by step, how to use a procedure to complete a mathematical task and explain how and why the procedure works. The purpose of modeling is that students are able to apply the general procedure in various contexts. The teacher must ask questions to keep students engaged. After modeling, students should proceed to independently applying the procedure to various problems of increasing difficulty. According to the three experts, the interaction in this approach must be vertical.

Results of the interviews with experts – inductive approach

For the inductive approach, the experts’ advice was to start by introducing a realistic context that is familiar to students, with a mathematical problem; students must come up with solutions to that problem that can be generalized to procedures. According to the three
experts, the interaction must be mostly horizontal in this approach. The teacher should guide discovery by using scaffolding in such a way that students are able to continue discovering by themselves. If the teacher uses an inductive approach, he should not explain his own solution; instead he should guide students in their thinking processes. The teacher should also encourage students to ask each other questions like: "How would you approach this problem? Why do you think this approach is useful?"

4.3.2 Results of the focus group meetings

Both focus groups agreed upon the following:

Start each lesson with a warm-up activity to create the focus on the subject of the lesson. After that the lesson plan and the lesson objectives must be addressed, and at the end of each lesson the teacher should check whether or not the lesson objectives were achieved. The focus groups recommended using illustrative images and materials (for the deductive approach to clarify the procedure, and for the inductive approach to clarify the context). For the inductive approach, they also recommended encouraging students to keep asking each other questions until a clear procedure was articulated. In the deductive approach, the teacher must continuously ask students questions until they have clearly articulated a solution for the problem in which they have used the explained and modeled procedure. Table 4.1 shows the teachers’ (focus group members’) own preferences for a deductive or an inductive approach for each mathematical aspect. Teacher1 and Teacher3 have a clear preference for the inductive approach, while the other teachers’ preferences vary per aspect. The teachers’ preference for the inductive approach is most pronounced for the aspects estimating and scale, while preferences for the other aspects vary per teacher. For the main study, a dichotomous variable mismatch between the teacher’s preference and the didactic approach was added for each of the three lessons. The value of the variable would be 1, only if the deductive approach was used in the lesson and the teacher’s preference was inductive, or if the inductive approach was used in the lesson and the teacher’s preference was deductive. Recommendations which specifically relate to the deductive or the inductive approach are described in the next sections.
### Table 4.2. Teacher’s didactic approach preference per mathematical aspect.

<table>
<thead>
<tr>
<th></th>
<th>Teacher1</th>
<th>Teacher2</th>
<th>Teacher3</th>
<th>Teacher4</th>
<th>Teacher5</th>
</tr>
</thead>
<tbody>
<tr>
<td>metric</td>
<td>I</td>
<td>I</td>
<td>D</td>
<td>D</td>
<td>I/D</td>
</tr>
<tr>
<td>scale</td>
<td>I</td>
<td>I</td>
<td>I/D</td>
<td>I</td>
<td>I/D</td>
</tr>
<tr>
<td>area</td>
<td>D</td>
<td>I</td>
<td>I/D</td>
<td>D</td>
<td>I</td>
</tr>
</tbody>
</table>

Note. “I/D” means: no preference for an inductive or deductive didactic approach. Lesson 2 topic was metric, lesson 3 topic was scale, and lesson 4 topic was area.

**Results of focus group meetings – deductive approach (first round).** In the deductive approach, the focus group suggested teachers should encourage students to participate in interactive instruction in which the teacher demonstrates step by step how to use a procedure to complete a mathematical task and explains how and why this procedure works. This approach follows Ausubel’s suggestion to confront students directly with procedures instead of guiding their discovery (Woolfolk, 2010). The teacher should ask questions during instruction to create engagement (following the above ideas mentioned by Nelissen (2002)), depending on the specific student’s understanding. Students should then individually apply the explained procedure to other tasks, the teacher should ask students to share and explain their answers, and the teacher should correct the explanations and answers until they are precise and clear. This approach follows 1) the Vygotski (1926) premise that students can reach the zone of proximal development in interaction with an expert, but not with peers; 2) Deci and Chandler’s (1986) ideas regarding an increasing chance of reaching teachers’ objectives by vertical interaction rather than horizontal interaction; and 3) the idea that a focus on horizontal interaction and self-discovery jeopardizes the development of appropriate knowledge and competences (Leeman et al., 2007). In the deductive approach, teachers must introduce procedures, and interaction is mostly vertical. The focus group suggests a U-shaped classroom table setting to encourage vertical interaction. Table 4.2 shows what the focus group suggests teachers and students should be doing in the deductive approach.
Table 4.3. *Focus group advice for the deductive approach.*

<table>
<thead>
<tr>
<th>aspect</th>
<th>what the teacher should do</th>
<th>what the students should do</th>
</tr>
</thead>
<tbody>
<tr>
<td>estimation</td>
<td>The teacher explains and models the following procedure for estimating sizes: choose an appropriate reference, determine how many reference objects fit in the object for which the size needs to be estimated, and finally, give the estimate in the appropriate standard measure.</td>
<td>Students participate in interactive instruction and apply the explained procedure to estimation problems.</td>
</tr>
<tr>
<td>conversion</td>
<td>The teacher explains the structure of the metric system, and models how measures can be converted by means of a step model ((2.3 m = 23 dm = 230 cm = 2300 mm). Using the Multibase Arithmetic Blocks, the teacher explains how many 1 cm² blocks fit on 1 dm², and how many 1 cm³ blocks fit in 1 dm³.</td>
<td>Students participate in interactive instruction and apply the explained procedure to conversion problems.</td>
</tr>
<tr>
<td>scale</td>
<td>The teacher explains and models how to use a ratio table to carry out scale calculations.</td>
<td>Students participate in interactive instruction and apply the explained procedure to scale problems.</td>
</tr>
<tr>
<td>calculation</td>
<td>The teacher explains and models how to carry out length, area and volume calculations with triangular and rectangular 2D and 3D figures.</td>
<td>Students participate in interactive instruction and apply the explained procedure to measurement calculation problems.</td>
</tr>
</tbody>
</table>
Results of focus group meetings – inductive approach (first round). In the inductive approach, the focus group suggests that students should participate in interactive instruction in which the teacher introduces a context with a mathematical task for which small groups of students should come up with procedures and explain them to group members. Although Vygotski’s ideas (1926) about needing an expert to learn can be an argument for a deductive approach, this inductive approach follows the Vygotski (1926) premise that students learn through discovery in interaction with the social environment, albeit guided by the teacher. These procedures should also be convenient for similar mathematical tasks. Student groups should discuss their procedures; the teacher should walk around and ask questions to foster deeper discussions (following the ideas of Nelissen (2002)) and to make sure the pros and cons of each procedure are discussed. This approach embraces the suggestion that deeper understanding of (the use of) procedures (De Moor, 1999) should be emphasized.

If students cannot come up with adequate pros and cons, the teacher should encourage students to think more deeply, for example, by providing examples where procedures might not work properly. If students cannot grasp the essence of the lesson, the teacher should use scaffolding to guide students towards discovering suitable procedures (i.e. the teacher can give hints to help students come up with ideas). Next, the teacher should ask students to share and explain their ideas, and ask other students to respond, so that students can individually select their preference for a procedure. The focus group advised that the classroom table setting should facilitate groups of four, to encourage horizontal interaction. Table 4.3 shows what the focus group suggested teachers and students should be doing in the inductive approach.
Table 4.4. *Focus group advice for the inductive approach.*

<table>
<thead>
<tr>
<th>aspect</th>
<th>what the teacher should do</th>
<th>what the students should do</th>
</tr>
</thead>
<tbody>
<tr>
<td>estimation</td>
<td>The teacher shows pictures of objects for which the size needs to be estimated, challenges students to explain their estimation procedure, walks around and asks questions to deepen horizontal interaction.</td>
<td>Student groups discover the need for reference measures and how to use them to estimate sizes.</td>
</tr>
<tr>
<td>conversion</td>
<td>The teacher asks students to explain the structure of the metric system by analyzing a ruler, and to think of a model with which measures can be converted. The teacher walks around and asks questions to deepen horizontal interaction.</td>
<td>Student groups discover the structure of the metric system (using a ruler and the Multibase Arithmetic Blocks) and a procedure to convert measures.</td>
</tr>
<tr>
<td>scale</td>
<td>The teacher shows a map of the approximate surroundings and asks students to come up with a model to carry out scale calculations. The teacher walks around and asks questions to deepen horizontal interaction.</td>
<td>Student groups discover a procedure for scale calculations.</td>
</tr>
<tr>
<td>calculation</td>
<td>The teacher shows examples of triangular and rectangular 2D and 3D figures, and asks students to come up with a procedure to carry out length, area and volume calculations with these figures. The teacher walks around and asks questions to deepen horizontal interaction.</td>
<td>Student groups discover a procedure to carry out calculations of length, area and volume of triangular and rectangular 2D and 3D figures.</td>
</tr>
</tbody>
</table>
Results of focus group meetings (second round). In the second round with the focus groups, we discussed the two revised PowerPoint blueprints and teacher manuals. After the first round, the metric system was covered in lessons two and three, and the estimating sizes aspect was covered in the first three lessons (length in the first lesson, area in the second, and volume and weight in the third). In the second round, the focus group believed that less time was needed for the metric system aspect. Furthermore, the focus group believed it was best to cover the estimating sizes aspect for all measures (length, area, volume and weight) in the first lesson, because they believed estimating sizes is the basis for grasping the other measurement aspects. We agreed upon the following: lesson series should contain primarily one aspect per lesson (i.e. estimate sizes using reference measures, convert between units and prefixes in the metric system, calculate with scale, or carry out calculations of length, area and volume on triangular and rectangular 2D and 3D figures), and in the fifth lesson time should be planned to discuss a practice test. Finally, the teachers in the focus group asked for expected student answers to the questions they were going to be asked during the lessons, as is suggested by Stein et al. (2008).

While developing the deductive approach, we wrote down the whole procedure, from A to Z, because our purpose was to make sure students followed the teacher procedure exactly. We wrote down questions for the teacher to ask during class, aimed at making students formulate, in their own words, how they followed the procedure. If students came up with their own procedures, teachers should respond with a short reflection on that procedure (is it any good?), and remind students that they should learn to apply the teacher procedure first. In the deductive approach, teachers should ask students who already understand the idea, to work on assignments that require them to apply the procedure (in increasingly difficult contexts that are more diverse than assignments for the regular group). Teachers should then ask these students to re-join the group for the evaluation phase, during which teachers should ask them to elaborate more deeply on their ideas, encouraging them with questions like: ‘are you certain?’ and ‘why is this correct?’.

4.3.3 Results of the final revision
In the final revision, we aimed at fully excluding inductive elements from the deductive approach and vice versa. We found that some of the instructions still left too much room for interpretation. In the deductive version, we added extra steps to procedures, to make sure the procedure could completely be modeled without assuming students would all take certain
steps in the same way. For example: the modeling of the procedure to convert between units and prefixes in the metric system was fine-tuned as follows:

Question: 2.3 hm = … dm?

Step 1: Determine which prefixes lie between one and the other, and list them all in a row (the smallest prefix on the left).

... dm ... m ... dam 2.3 hm

Step 2: With every step to the left, the unit is ten times as small, so the number is ten times as large. Fill in the numbers.

2,300 dm 230 m 23 dam 2.3 hm

Step 3: Answer the question.

2.3 hm = 2,300 dm.

In the inductive version, we added instruction for teachers on how to proceed if students did not come up with procedures, and on how to respond to questions like: teacher, could you please model the procedure for us? We instructed teachers to use scaffolding in case students did not come up with procedures. We did not want the teacher to explain the procedure, because that would be a deductive approach. We told teachers to use scaffolding and to avoid directing students in one direction. For example:

Student: “Please explain how to calculate the area of this triangle.”

Teacher: “What seems to be the problem?”

Student: “I do not know how to calculate the area of this triangle.”

Teacher: “Is there another triangle for which you do know how to calculate the area?”

Student: “Yes, for a triangle like this one” (student draws a right-angled triangle).

Teacher: “How do you calculate the area of that right-angled triangle?”

Student: (draws a rectangle around the triangle) “It is half of this rectangle”.

Teacher: “Is there a way to use that knowledge in this problem?”
Student: “Perhaps I can draw a rectangle around this triangle, too…”

Teacher: “Please try that with your group, and I will come to see you guys later on.”

4.4 Discussion

In this study on developing lesson series on measurement with classroom interaction guidelines for prospective elementary school teachers, we learned about possible consequences of rigorously following guidelines for either a deductive approach or an inductive approach.

For the deductive approach, the experts and focus groups in this study suggest to use the Van de Craats approach (2007): the teacher should encourage students to participate in interactive instruction (Slavin & Lake, 2008) in which he demonstrates step by step how to use a procedure to complete a mathematical task and explains why the procedure works. Students should apply the explained procedure individually in other tasks; the teacher should ask students to share and explain their answers and correct the explanations and answers until they are precise and clear (De Moor, 1999; Star, 2007; Van Eerde, 2008). For the inductive approach, the experts and the focus groups in this study suggest the teacher should encourage students to participate in interactive instruction (Slavin & Lake, 2008) in which he introduces a context with a mathematical task, for which small groups of students should find solutions (Lampert, 2001; Treffers, 1993). After discussing several solutions, student groups should come up with general procedures. The teacher should guide student groups in their interaction amongst themselves to discover general procedures for calculations. He should ask students to share their ideas and to explain them, and asks other students to respond (Pierson, 2008; Van Eerde, 2008), so that students can individually select their preference for a procedure. The nature of and time spent on horizontal interaction and vertical interaction differs between the approaches: in the inductive approach classroom interaction on discovery is mostly horizontal, while in the deductive approach classroom interaction on how students apply a procedure is mostly vertical.

In the development of the deductive approach, we assumed that students’ engagement in classroom interaction regarding how students used the teacher’s procedure (Van de Craats, 2007), would not always be optimal. We anticipated that by instructing teachers to use different kinds of questioning, depending on individual student’s level of understanding. In the development of the inductive approach, we assumed that students would not always be able to discover suitable procedures themselves (Charles & Carmel, 2005; Pierson, 2008) and
that students would ask for explanations (Gravemeijer, 1994) instead of engaging with each other’s mathematical ideas (Staples, 2007). We anticipated that by instructing teachers to use scaffolding and different kinds of questioning (Brown & Campione 1994; Deci & Chandler, 1986; Leeman, Wardekker, & Majoor, 2007; Stein et al., 2008).

Because a deductive lesson series contains step by step teacher explanations of procedures, we expected students to be able to flawlessly use those procedures in recognizable contexts. Because an inductive lesson series contains possibilities for students to discover procedures, we expected students to learn not only about these procedures, but also about unstructured problem-solving and to learn how to respond to other students’ ideas. In follow-up research, we want to find out what it is exactly that separates students who were taught deductively from those who were taught inductively. Will student outcomes differ between the two groups? Which types of problems would we expect to be solved more easily by the first or second group of students? Which types of students are more likely to profit from a deductive or an inductive approach?

In this study, we focused on a short lesson series of five lessons. If the deductive approach is applied over a longer period, teachers could choose to elaborate more on certain aspects in order to deepen students’ procedural knowledge (Star, 2007). If the inductive approach is applied over a longer period, teachers might not need to revert to scaffolding as quickly as is suggested in this study. Teachers could then make room for student elaborations on other aspects of mathematics (or even other subjects) as well, because the time spent on these subjects or aspects is not lost time.

Another limitation of this study, is that we interviewed only two experts in the field of mathematics education, and that the focus groups members were all teacher educators of one specific teacher college. We did not ask teacher educators of other teacher colleges, nor did we ask elementary school teachers to participate in the focus groups.

In this study, we developed lesson series for prospective elementary school teachers, a different population than elementary school pupils. Since the future teachers already had previous understanding of measurement, chances are that they would be more likely to be able to (re)discover procedures, and to come up with different kinds of procedures, than could be expected of elementary school pupils.

The focus groups’ reflections after teaching the courses were that students seemed to vary in approach preference, and that they seemed to be more engaged in the inductive approach. Discussions in the inductive approach were time-consuming, and focusing on certain topics without explaining procedures proved to be challenging (some students kept
asking the teacher to explain the appropriate procedure, which could result in a deductive approach). For some convenient procedures (with scale and the metric system), a deductive approach might be better, because the efficiency gains seem higher than would be the case with a guided discovery.

In practice, most teachers use their own personal mixture of the two (or other) approaches. The focus group reflections suggest that teachers should consider student and group characteristics and the specific topic when they build their own personal mixture of inductive and deductive approaches to classroom interaction, and switch approaches accordingly. When a teacher chooses an inductive approach and students keep asking for procedures and teacher explanations, it is probably wise to shift approaches, or at least to initiate a class discussion about how certain topics should be addressed (although students’ learning habits should not be a leading argument: they might profit from a shift in teaching approach). When a teacher chooses a deductive approach and students do not seem to be able to transfer an explained procedure to another context, it might be a good idea to focus more on student ideas, thinking and reasoning.

Teachers who choose a deductive approach must be able to explain how and why procedures work, to deepen students’ procedural knowledge, and to anticipate and address misconceptions. For an inductive approach, teachers need to have a solid teaching and mathematics background, because it is more complex to guide students’ reinventions and appropriately respond to their ideas than it is to explain procedures with which the teachers already are familiar. Furthermore, if students do not have a basic idea about the topic that is to be discussed in an inductive approach, they probably have an insufficient basis for self-discovery. Teachers who choose an inductive approach should also have a deep pedagogical content knowledge to guide the inductive classroom interaction into calm and clear waters instead of into chaos and insecurity. Teachers should therefore consider their personal teaching preferences and competences, and teacher educators should teach future teachers how to use both approaches in their teaching, using either a deductive or an inductive approach, or a mixture of the two.