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CHAPTER 1

General introduction

This dissertation examines classroom interaction and two different didactic approaches (deductive and inductive) to the teaching of the measurement aspect of numeracy (measurement numeracy) to students of elementary school teacher training colleges. An important goal of the research is to estimate the effect of these approaches on classroom interaction and on student performance. Before going directly into the details of the research, let us first look at what measurement numeracy entails, and the challenges that teacher educators face. Measurement numeracy includes the following aspects: estimating measures, understanding relationships within the metric system, calculating with scale, and calculating length, area and volume. For instance, students learn how to convert units in the metric system, and how to calculate the size of a scale model of a living room or an airport. Do you remember how you learned to do that, and how your understanding grew? Let us look at a problem in this mathematical domain.

The volume of a pack of lemonade is 1.5 liters.  
The pack is 7.5 cm long, and 1 dm wide.  
Calculate the height of the pack. Explain.

What do you need to know, and what skills must you have, to solve these types of problem? If you were an elementary school teacher, and you wanted your pupils to be able to solve these types of problem, how would you go about it? What do teacher educators need to know, and what skills must they possess, to make sure they can effectively teach prospective teachers to solve this type of problem? If you were a teacher educator, and you wanted your students to be able to solve this type of problem, how would you go about it? If you were a teacher educator, and you wanted your students to be able to teach elementary school pupils to solve this type of problem, how would you go about it? While thinking about these questions, you could have concluded that teaching mathematics at an elementary school teacher training college is profoundly multifaceted. For those of you who still have doubts about the
complexity, a chain of competences of all teachers and learners involved is set out in the text box below.

| teacher trainers’ understanding of measurement, |
| teacher trainers’ ability to solve measurement problems, |
| teacher trainers’ ability to teach measurement and to guide prospective teachers in their learning, |
| ↓ |
| prospective teachers’ ability to learn about measurement, |
| prospective teachers’ understanding of measurement, |
| prospective teachers’ ability to solve measurement problems, |
| prospective teachers’ ability to teach measurement and to guide pupils in their learning, |
| ↓ |
| pupils’ ability to learn about measurement, |
| pupils’ understanding of measurement, |
| pupils’ ability to solve measurement problems. |

This introduction provides a framework for this research by discussing the role of different instructional practices within mathematics education and their influence on students’ numeracy. Classroom interaction and two specific didactical approaches (inductive and deductive) will be discussed in more detail. Measurement numeracy is defined, and the need for research on classroom interaction and didactic approaches in mathematics classes within teacher training colleges is argued. The final paragraph gives an overview of the contents of each chapter in this dissertation.

1.1 Measurement numeracy
Many students, even in higher education, have difficulty keeping up with elementary school mathematics (Expertgroep Doorlopende Leerlijnen, 2008; National Research Council, 2001). This is also true for students at teacher training colleges, who are expected to teach mathematics to elementary school children later on. These students are more likely to perform worse if they lack numeracy skills (Hill, Rowan, & Loewenberg Ball, 2005). In the Netherlands, the numeracy entrance level of these students is measured by means of a national test (WISCAT). Freshmen are required to achieve a higher score than the top 20% elementary school pupils. An example of an item with a p-value just below the norm is: A garden is 8.4 m long by 5.6 m wide. What is the length and width of the garden on a map with scale 1:20?
Although this norm for prospective teachers seems rather low (Van Zanten & Van den Brom-Snijders, 2007), many first-year students (25%) do not pass this test (Keijzer & Hendrikse, 2013). Especially freshmen with MBO education, who accounted for 36% of the Dutch teacher training college freshmen between 2007 and 2012, seem to lack numeracy skills: on average, their WISCAT score has been 97, which is below the norm of 103 (Eggen & Straetmans, 2013). While for most learners it might be enough to know only one strategy to solve a problem, and gain only procedural knowledge, prospective teachers must also have conceptual knowledge of mathematics (Anderson, 1983; Hiebert, 2013). For teaching to be effective, a high level of professional knowledge is required, including a firm grasp of the mathematics itself (Ball, Thames, & Phelps, 2008; Shulman, 1986). The Royal Netherlands Academy of Arts and Sciences (KNAW) suggested that immediate action is required to improve the quality of mathematics education in Dutch teacher training colleges (KNAW, 2009).

Elementary school mathematics in The Netherlands encompasses the following domains: whole numbers, ratios, decimals, percentages, fractions, data representation (diagrams, tables, graphs), geometry, and measurement (Van Zanten, Barth, Faarts, Gool, & Keijzer, 2009). Teacher training colleges cover the same domains. Mathematics curricula in the United States and high-performing Asian countries have many similarities (Chen, Reys, & Reys, 2009), and they differ only slightly from the Dutch curriculum. Some Dutch teacher training colleges choose to teach numeracy and the didactics of mathematics in the same courses; others choose to teach those subjects separately. The teacher training college of Rotterdam University of Applied Science (where this research took place) has eight separate numeracy courses in its curriculum, in addition to the didactics courses. Although it is not known how teacher training students perform on different aspects of numeracy nationwide, the experience of the School of Education of Rotterdam University of Applied Sciences is that measurement is one of the aspects that leaves the most room for improvement. Also, the Trends in International Mathematics and Science Study (TIMSS) shows that on average, Dutch elementary school children score lower on measurement in comparison with their

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1 Dutch children attend elementary school between the ages of four and twelve. After that, the elementary school advises children - depending on their potential - to continue their education at preparatory middle-level applied education (VMBO), higher general continued education (HAVO), or preparatory scholarly education (VWO). Many pupils with a VMBO-diploma go on to Secondary vocational education (MBO). MBO is oriented towards vocational training, and it takes up to four years, depending on the level of training. Holders of a level 4 MBO certificate may go on to higher professional education (HBO). HBO in the Netherlands (for example: the teacher training college) is open to graduates of MBO, HAVO, and VWO. If a student is 21 years old or older, and he does not hold a VWO, HAVO, or MBO degree, a successful admission test can also grant access to the teacher training college.
overall mathematics score (524 versus 540 in TIMSS 2011, 522 versus 535 in TIMSS 2007) (Mullis, Martin, Foy, & Arora, 2012, p. 142; Mullis, Martin, Foy, & Arora, 2008, p. 120). For that reason, this study focuses on the measurement aspect of numeracy. In this dissertation we refer to measurement as the process of determining dimensions of a physical object (we will not capture the measurement of mental states like anxiety or happiness). In measurement courses, students learn to structure and quantify reality by estimating measures through developing personal reference measurements, by understanding relationships within the metric system, by calculating with scale, and by calculating length, area, and volume.

Measurement differs from, for example, whole numbers, ratios, decimals, percentages, and fractions, in that there is an obvious relationship between measurement and daily practice, also in most text books and mathematics classes. In the past, calculations with whole numbers, fractions, and ratios were mostly exercised without any context (i.e.: \( 27 + 49 = \ldots \), \( \frac{1}{4} \times 16 = \ldots \)). Nowadays, influenced by the Realistic Mathematics Education (RME) movement (e.g. Gravemeijer, 1997; Torbeyns, Verschaffel, & Ghesquière, 2005; Treffers, 1993), most educators make an effort to use contexts as the basis for learning, even for simple calculations. However, pupils still need to learn specific procedures (the most efficient ones), and they are still required to solve problems without any context. In most measurement problems, students need to read and interpret a context, establish a plan to find the answer, select the appropriate strategies, models, and/or procedures, perform the calculations, and formulate an answer. It would appear that one needs more and other skills for measurement numeracy than for other mathematical domains. This may affect instructional practices, which will be discussed in the next section.

1.2 Instructional practices
Over the last 50 years, mathematics education has been prone to reform. After the success of behaviorism (Skinner, 1938; Watson, 1925), constructivism (Dewey, 1938; Piaget, 1957; Vygotsky, 1926) changed mathematics education in Europe and the United States. Although constructivism is a theory of learning (students must build their own knowledge, i.e. it is a theory about what happens with learners, not with teachers), teaching also changed. Some would argue that this was the start of a mathematics war between traditional mathematics education and mathematics education reform based on constructivism. In the Netherlands, this new way of teaching mathematics is known as Realistic Mathematics Education (RME). Research shows that pure discovery learning (without any guidance) is ineffective, but guided
discovery (when students need it, and which can gradually be decreased as student skills grow) can be useful (Alfieri, Brooks, Aldrich, & Tenenbaum, 2011; National Research Council, 2013). In Tobias and Duffy’s Constructivist Instruction (2009), several authors from both sides (for or against constructivism) shed light on constructivism. Even though most constructivist authors think that constructivist instruction leads to better results, many believe that more traditional approaches may be favorable for well-structured domains like mathematics (Schwartz, Lindgren, & Lewis, 2009; Spiro & DeSchryver, 2009). Mayer (2009) states that constructivism is a theory of learning (not of teaching: there is much confusion about that difference in the literature), and that students’ behavior in discovery learning does little for learning (whereas cognitive activity does). Kirschner (2009) argues that children may not have the necessary skills for discovery learning, as most adults do. Clark (2009) and Sweller (2009) argue that children need accurate and complete demonstrations, because they need to imitate an expert, and Fletcher (2009) shows empirical evidence indicating that drilling and practicing have positive effects on student performance, that students appreciate this approach, and that it costs less time than discovery learning.

Under the influence of constructivists, mathematics education shifted from teachers explaining (traditional teaching) how and why rules apply and how and why procedures work, to guiding (reform teaching) students in their individual knowledge construction, and from aiming for students’ procedural expertise to discovery learning that starts in informal contexts and allegedly allows students to acquire deeper understanding about strategies (Gravemeijer, 1997; Kilpatrick, Swafford, & Findell, 2001). Before constructivism, the aim of mathematics education was to make sure students were flawlessly able to follow procedures, and to put them into practice in increasingly difficult problems. Constructivist ideas caused mathematics education to shift towards guiding students to construct meaning from their own experience, which emerges from interaction between students, and from meaningful and increasingly complex contexts. Reflection on how tasks were performed, self-regulation, cooperation with peers and problem-solving became increasingly important aspects of mathematics education. Piaget for instance believed that students do not learn enough simply from the transfer of information. Instead, they should construct their own knowledge (Ultanir, 2012). However, when students construct their own knowledge, they may also create their own misconceptions (VanLehn, 1990).

In the Netherlands, RME was questioned by Van de Craats (2007) at a national mathematics education conference in 2007 (Panama Conference), which led to heated debates in the Dutch media and schools (KNAW, 2009). Empirical studies showed that the effect of
constructivist learning on student outcomes is not always as high as constructivists would like (Birenbaum & Dochy 1996), and recent empirical research on solving multiplication and division problems (Fagginger Auer, 2016, p. 120) suggested that “while attention to informal strategies may be fruitful in the earlier stages of the educational process, performance may benefit from a focus on standardized procedures at the end of the instructional trajectory”.

Van Putten and Hickendorff (2006) reported the strong effect of teachers on students’ use of strategy in division problems, and showed that 40% of the teachers used a traditional teaching method, even though they worked with reform-type textbooks: the enacted curriculum often differed from the intended curriculum. They also showed that when compared with 1997, in 2004, more pupils answered division problems by heart (instead of a less risky written calculation). Since there are strong performance differences between strategies (Fagginger Auer, Hickendorff, Van Putten, Béguin, & Heiser, 2016), it is likely that teachers’ didactic approach will affect student performance. At present, some Dutch elementary schools revert to mathematics textbooks that use a traditional approach. One reason for this may be the required high level of teacher skills: if teachers do not adequately put the RME principles into practice, results may suffer (Gravemeijer, Bruin-Muurling, Kraemer, & Van Stiphout, 2016).

The mathematics teaching of teacher educators not only has an effect on prospective teachers’ numeracy, it also has an effect on how prospective teachers form their ideas on how to teach mathematics in elementary schools. Although teacher educators might make other educational choices in their classes with prospective teachers than elementary school teachers would in their classes with pupils, teacher educators must be aware of the possibility that prospective teachers might see them as role models. Prospective teachers may use the teacher educator’s educational methods in the elementary school class. For example, how discovery learning (Bishop, Clopton, & Milgram, 2012; Boaler & Staples, 2008) is used in class, might depend on the previous knowledge of the group. While elementary school pupils might not possess the knowledge about what needs to be discovered, chances are that in a group of prospective teachers some will have various ideas on the subject, because they learned about it earlier. With discovery learning, perhaps possessing previous knowledge of the subject is an advantage, because without that, it will be difficult to discover anything.

Besides arguing for immediate action to improve the quality of mathematics education in Dutch teacher training colleges, the KNAW report (2009) also states that fighting a war (e.g. traditional versus reform) has led us away from the heart of the matter: estimating the effects of specific instructional practices on student performance. There are two opposing
specific didactic approaches regarding mathematics teaching: a deductive approach and an inductive approach.

1.3 The didactic factor: deductive versus inductive teaching
Deductive reasoning is viewed as the most certain way of reasoning: if the premises are true, and the rules of logic are followed, the conclusion must necessarily be true. An example of deductive reasoning is the following: 1) the pressure of an enclosed gas is directly proportional to its temperature, i.e. $P_1/T_1 = P_2/T_2$, 2) therefore, the pressure of an enclosed gas increases if its temperature increases. Inductive reasoning, however, does not hold this certainty, it is a probabilistic type of reasoning. An example of inductive reasoning is the following: 1) I got a meal every day for the last five years, 2) therefore, I will probably get a meal tomorrow. The difference between the two types of reasoning is clear, especially if you think of the inductive reasoner as a turkey on the day before Thanksgiving (e.g. Bertrand Russel’s famous example). Although inductive reasoning is uncertain, we must still rely on it, because in most practical cases (for example in educational research settings: which didactic approach works best? Or in everyday life: should I take an umbrella with me?) we lack the certain laws to reason deductively.

In a deductive mathematics class, the teacher explains a general rule, after which students apply the rule in different contexts (top-down teaching). For example: after explaining the concept of ‘area’, the teacher explains how to calculate the area of a rectangle: by multiplying length by width. After that, students apply the rule to calculate the areas of different rectangles. Inductive reasoning searches for generalizations from specific contexts, whereas deductive reasoning searches for applications from a generalization (Klauer, Willmes, & Phye, 2002). In an inductive mathematics class, the teacher introduces one or more contexts, after which students search for the general rule that connects similar contexts (bottom-up teaching). For example:

![Rectangle with squares]

How many squares of 1 cm by 1 cm are in this rectangular chocolate bar?
What is the area of this rectangle?
If the rectangle were 3 squares in length, and 5 squares in width, what would be the area?
If the rectangle were 2.5 squares in length, and 4 squares in width, what would be the area?
If a rectangle were 2 cm by 6.3 cm, what would be the area?
What would be a general rule to calculate the area of a rectangle?
The deductive approach relates to traditional mathematics teaching, and the inductive approach relates to discovery learning. Most of us would not dispute the need for students to be able to apply rules in different contexts. However, despite claims that guiding students to rediscover rules might induce deeper understanding, not everyone believes that all students need to learn how to rediscover already existing generalizations (either because these generalizations do not require rediscovery, or because these claims have not been sufficiently substantiated). Besides that, most standard exams do not test for rediscovery skills. Some argue that bottom-up teaching is more time consuming than top-down teaching. Others argue that bottom-up teaching induces deeper understanding and active learning, which shortens instructional time in the future, and gives students advantages after they leave school (Schwartz et al., 2009).

1.4 The factor classroom interaction

In our aim to improve the quality of numeracy courses, other factors that correlate positively with student performance may also play a role. The aim of identifying these factors in the Netherlands is shared with many other countries, one of which being the United States of America. The National Mathematics Advisory Panel (NMAP) in the USA studied 16,000 research reports and concluded that only a small number showed significant effects of instruction variables on student performance in mathematics. Positive effects on student performance were found in studies in which teachers keep learners productively engaged, and give them opportunities to help each other to learn (Slavin & Lake, 2008). Based on the available empirical evidence, Slavin and Lake concluded that the didactics and curricula have less effect on student performance in mathematics than classroom interaction between students and with the teacher. Although these positive effects have only been found in elementary schools, we expect to find the same effects in teacher colleges. After all, productively engaged students probably learn more than silent listeners, and sharing mathematical ideas and discussing them with peers and the teacher probably sharpens the mathematical mind of teacher college students, too. Hence, we hypothesize that improving classroom interaction in mathematics classes will have a positive effect on student performance.

To test this hypothesis, an operational definition of classroom interaction is needed: how should we measure classroom interaction? Different aspects of classroom interaction have been distinguished. Classroom interaction can be guided in many ways: classroom
discussions, dialogues, discussing theorems, quizzing, allowing room for questions after instruction, instruction through classroom interaction, scaffolding\(^2\), asking students to develop test questions and discussing them, et cetera (Roefs, 2010). Classroom interaction can be organized in several ways, for example: in discussions involving the whole class, in smaller student groups where students interact with each other and where the teacher divides his time between groups, or in unguided student groups followed by whole class evaluation. In addition to these activity structures, research on classroom interaction also focused on classroom norms, identity, student engagement, student thinking, student argumentation, encouraging argumentation, animating and positioning students, re-voicing, and high-level questioning, but little research has been done on patterns in classroom talk itself (Pearson, 2008). Pearson concluded that students benefit from “elevated responsive classrooms” (teachers who ask the right question at just the right time, who ask students to elaborate on their own or others’ ideas, instead of giving away the answers too soon).

Effective mathematics teaching includes engaging students in classroom discussions and allowing them to share and learn about mathematical ideas and arguments on how and why things work, so that they learn to see things from different perspectives (Leinwand et al., 2014). Alfieri et al. (2011) suggest that teachers should use scaffolding to guide students in their tasks, give timely feedback on student explanations, and/or provide worked examples. Conversations on content should be deep and meaningful (Alexander, 2008), in order to optimize chances that students will come up with fundamental ideas, big ideas. “A Big Idea is a statement of an idea that is central to the learning of mathematics, one that links numerous mathematical understandings into a coherent whole” (Charles & Carmel, 2005, p. 10). An example is the Big Idea of equivalence: representation of numbers and measures can be done in many different ways (Askew, 2013). 50% is equivalent to 1/2 and 0.5; 999 is equivalent to 9 times 100 plus 9 times 10 plus 9 times 1, but it is also equivalent to 1 times 1000 minus 1 times 1 (which is very helpful in multiplication by 999); 100 cm is equivalent to 10 dm and to 1 m. Although it is sometimes more prudent to explain and evaluate, as students may not always make fundamental discoveries (Charles & Carmel, 2005), using students’ mathematical ideas in classroom interaction has proven to have a positive influence on student performance (Pearson, 2008).

\(^2\) The idea of scaffolding is to guide learners to find their own answers. The teacher simplifies a complex task by reducing the learner’s actions to reach the solution, so that the learner does not get frustrated. The teacher directs the learner towards the solution without spelling it out, points out relevant elements and steers away from fruitless strategies, and rephrases the learner’s strategy to an ideal action (Wood, Bruner, & Ross, 1976).
Asking questions is one of the most obvious starting points of classroom interaction. The depth of the classroom interaction depends greatly on the type of questions. Consider the two following questions: 1) A scale model of the Statue of Liberty (scale 1:1000) is 9.3 cm high, what is the height of the Statue of Liberty? 2) Ashley says that a scale model of the Statue of Liberty (scale 1:1000) cannot be higher than a bottle of milk, what do you think about this? The second type of question is more likely to induce critical thinking and a deep discussion around mathematical ideas than the first (Nelissen, 2002). The didactic approach also affects classroom interaction. In an inductive approach, classroom interaction will be mainly about students’ ideas while they are rediscovering mathematical rules and concepts. Teachers ask questions like: “Are you sure about your idea, is it certain? What do you think of Mike’s strategy?”. In a deductive approach, classroom interaction will be mainly about students’ application of rules and concepts that have been explained by the teacher. Teachers ask questions like: “Which rule applies here, and why? How do you proceed?”. Research has been done on many different aspects of classroom interaction in mathematics classes, but analysis of classroom interaction is time consuming, so most quantitative research in this area uses small samples. Therefore, little is known about the effects of particular aspects of classroom interaction on student performance (Pierson, 2008). This is also true for students’ classroom interaction time (the part of classroom interaction that is taken up by student talk). Although Pierson coded different kinds of classroom discourse, estimation of the effects on student performance was done using ratios between different types of classroom discourse, without accounting for the time spent on the discourse, and without accounting for the time students talked. Since the time students talk might be an indicator for student engagement, which has a positive effect on student performance (Slavin & Lake 2008), measuring classroom interaction time differences across didactic approaches might shed a light on the effectiveness of these approaches.

To this day, how types of teacher questions and classroom interaction time vary across didactic approaches, remains unknown. In this dissertation, the differential effects of a deductive versus an inductive didactic approach on classroom interaction time, on teachers’ question type, and on students’ measurement numeracy were estimated. Since previous research showed that the intended and enacted curriculum often differ, teacher effects were also estimated.
1.5 Contents of this dissertation
To estimate differential effects of an inductive and a deductive didactic approach on classroom interaction and on students’ measurement numeracy, a field experiment was conducted at the teacher training college (School of Education) of the Rotterdam University of Applied Sciences. Sample characteristics are reported in Chapter 2. After evaluating the dimensionality of measurement numeracy (using confirmatory factor analyses: is measurement only one skill, or can the four separate sub-skills mentioned in the Dutch mathematics knowledge base for elementary school teacher training college students be distinguished?), an instrument was developed to measure students’ measurement numeracy (Chapter 3). In Chapter 4, the development of two lesson series is reported: one with a pure deductive didactic approach, and one with a pure inductive didactic approach. Starting from literature on constructivism and traditional mathematics education, experts in the field of mathematics education were interviewed, and the lesson series were developed with two focus groups consisting of mathematics teacher educators. In Chapter 5, measurements of classroom interaction time and teacher question types are reported. Videotaped lessons were coded, and interrater reliability and a fidelity check were reported. Repeated measures ANOVA analyses were performed to estimate the effect of the didactic approach and the teacher on classroom interaction time and on the teachers’ question type. In Chapter 6, ANCOVA analyses were performed to estimate the effect of the didactic approach and the teacher on students’ learning gains in measurement numeracy. Finally, in Chapter 7 findings and limitations are reported and discussed, and future research opportunities are suggested.