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Appendix A

HNL production from hadrons

The calculation of the weak decays involving hadrons is summarized in [290]. In the absence of QED and QCD corrections the effective weak interaction Lagrangian at low energies can be written as

\[ \mathcal{L}_{\text{weak}} = \mathcal{L}_{\text{cc}} + \mathcal{L}_{\text{nc}} \]  

where the charged current terms have the form

\[ \mathcal{L}_{\text{cc}} = \frac{G_F}{\sqrt{2}} \left| \sum_{U,D} V_{UD} J_{UD}^{\mu+} + \sum_\ell J_{\ell}^{\mu+} \right|^2, \]  

where

\[ J_{UD}^{\mu+} = \bar{D} \gamma^\mu (1 - \gamma^5) U, \]  

\[ J_{\ell}^{\mu+} = \bar{\ell} \gamma^\mu (1 - \gamma^5) \nu_\ell, \]  

and \( V_{UD} \) is the CKM element which corresponds to the quark flavor transition in the hadronic current. For the neutral current, the interaction has the same form

\[ \mathcal{L}_{\text{nc}} = \frac{G_F}{\sqrt{2}} \left( \sum_f J_f^{\mu,0} \right)^2, \]  

where summation goes over all fermions,

\[ J_f^{\mu,0} = \bar{f} \gamma^\mu (v_f - a_f \gamma^5) f, \]  

\[ v_f = I_{3f} - 2Q_f \sin^2 \theta_W, \quad a_f = I_{3f} \]  

and \( I_{3f} \) is the fermion isospin projection and \( Q_f \) is its electric charge (\( Q_e = -1 \)). In the following Sections we describe different processes with HNL and hadrons.
A.1 Leptonic decay of a pseudoscalar meson

Consider a decay of pseudoscalar meson $h$ into charged lepton $\ell$ and HNL:

\[
h \rightarrow \ell + N,
\]

see left diagram in Fig. 3.2. The corresponding matrix element is given by

\[
\mathcal{M} = \frac{G_F}{\sqrt{2}} V_{UD} \langle 0 | J_{\mu}^h | h \rangle \langle \ell, N | J_{\ell,\mu} | 0 \rangle,
\]

where the corresponding quark contents of meson $h$ is $| h \rangle = | \bar{U}D \rangle$. In order to fix the notations we remind that the charged meson coupling constant, $f_h$, for a pseudoscalar meson constructed from up ($U$) and down ($D$) type quarks is defined as

\[
\langle 0 | J_{\mu}^h | h \rangle = \langle 0 | \bar{U}_\gamma \gamma_5 D \rangle \equiv i f_h p^\mu
\]

where $p_\mu$ is 4-momentum of the pseudo-scalar meson $h$. The numerical values of the decay constants for different mesons are summarized in Table C.2.

After standard calculation one finds the decay width of this reaction

\[
\Gamma(h \rightarrow \ell, \alpha N) = \frac{G_F^2 f_h^2 m_h^3}{8\pi} |V_{UD}|^2 |U_\alpha|^2 \left[ y_N^2 + y_\ell^2 - (y_N^2 - y_\ell^2)^2 \right] \sqrt{\lambda(1, y_N^2, y_\ell^2)},
\]

where $y_\ell = m_\ell/m_h$, $y_N = M_N/m_h$ and $\lambda$ is given by (3.1.12).

A.2 Semileptonic decay of a pseudoscalar meson

The process with pseudoscalar or vector meson $h'_{P/V}$ in the final state

\[
h \rightarrow h'_{P/V} + \ell + N,
\]

is mediated by the current that has $V - A$ form (see right diagram in Fig. 3.2). Properties of the hadronic matrix element $\langle h'_{P/V} | J_{\mu, \text{hadron}}^h \rangle$ depend on the type of final meson $h'$ [291]. In the case of a pseudoscalar meson only the vector part of the current plays a role:

\[
\langle h'_{P}(p') | V_\mu | h(p) \rangle = f_+(q^2)(p + p')_\mu + f_-(q^2)q_\mu = f_+(q^2) \left( p_\mu + p'_{\mu} - \frac{m_h^2 - m_{h'}^2}{q^2} q_\mu \right) + f_0(q^2) \frac{m_h^2 - m_{h'}^2}{q^2} q_\mu
\]

where $q_\mu = (p - p')_\mu$ is the transferred momentum and

\[
f_0(q^2) \equiv f_+(q^2) + \frac{q^2}{m_h^2 - m_{h'}^2} f_-(q^2).
\]
For the case of a vector meson \( h' \) in the final state both vector and axial part of the current contribute. The standard parametrization with form factors is
\[
\langle h'_V(\epsilon, p') | V^\mu | h(p) \rangle = ig(q^2)\epsilon^{\mu\nu\sigma\rho}\epsilon^*_\nu(p + p')_\sigma(p - p')_\rho, \quad (A.2.4)
\]
\[
\langle h'_V(\epsilon, p') | A_\mu | h(p) \rangle = f(q^2)\epsilon^*_\mu + a_+ (q^2)(\epsilon^* \cdot p)(p + p')_\mu + a_- (q^2)(\epsilon^* \cdot p)(p - p')_\mu, \quad (A.2.5)
\]
where \( \epsilon_\mu \) is the polarization vector of the vector meson \( h'_V \).

Using matrix elements (A.2.2–A.2.5) it is straightforward to calculate decay widths of the reactions. In the case of pseudoscalar meson \( h'_p \) we follow Ref. [219] and decompose the full decay width into 4 parts,
\[
\Gamma(h \to h'_p \ell \alpha N) = \frac{G_F^2 m_h^5 C_K^2 |V_{UD}|^2 |U_\alpha|^2 (I_{P,1} + I_{P,2} + I_{P,3} + I_{P,4})}, \quad (A.2.6)
\]
where \( I_{P,1}, I_{P,2} \) depend on \( |f_+(q^2)|^2 \), \( I_{P,3} \) on \( |f_0(q^2)|^2 \) and \( I_{P,4} \) on \( \text{Re}(f_0(q^2)f_+^*(q^2)) \).

It turns out that \( I_{P,4} = 0 \) and explicit expressions for others are
\[
I_{P,1} = \int \frac{(1-y_\mu)^2}{(y_\mu + y_N)^2} \frac{d\xi}{3\xi^3} |f_+(q^2)|^2 \Lambda^3(\xi), \quad (A.2.7)
\]
\[
I_{P,2} = \int \frac{(1-y_\mu)^2}{(y_\mu + y_N)^2} \frac{d\xi}{2\xi^2} |f_+(q^2)|^2 \Lambda(\xi) G_- (\xi) \lambda(1, y_{h'}^2, \xi), \quad (A.2.8)
\]
\[
I_{P,3} = \int \frac{(1-y_\mu)^2}{(y_\mu + y_N)^2} \frac{d\xi}{2\xi^2} |f_0(q^2)|^2 \Lambda(\xi) G_- (\xi)(1 - y_{h'}^2)^2, \quad (A.2.9)
\]
where
\[
\Lambda(\xi) = \lambda^{1/2}(1, y_{h'}^2, \xi) \lambda^{1/2}(\xi, y_N^2, y_\ell^2), \quad (A.2.10)
\]
\[
G_- (\xi) = \xi (y_N^2 + y_\ell^2) - (y_N^2 - y_\ell^2)^2, \quad (A.2.11)
\]
\[
y_\ell = \frac{m_\ell}{m_h}, \quad \xi = \frac{q^2}{m_h^2} \quad \text{and function } \lambda(a, b, c) \text{ is given by (3.1.12).} \]\n\[
C_K \text{ is the Clebsh-Gordan coefficient, see for example [292, (14)] and [293, (2.1)], } C_K = 1/\sqrt{2} \text{ for decays into } \pi^0 \text{ and } C_K = 1 \text{ for all other cases.} \]
For the decay into vector meson the expression is more bulky,

\[
\Gamma (h \to h'\ell_a N) = \frac{G_F^2 m_h^7}{64\pi^3 m_{h'}^2} |V_{UD}|^2 |U_\alpha|^2 \left( I_{V,g^2} + I_{V,f^2} + I_{V,a^+} + I_{V,a^-} + I_{V,gf} + I_{V,ga^+} + I_{V,ga^-} + I_{V,fa^+} + I_{V,fa^-} + I_{V,a^+,a^-} \right), \tag{A.2.12}
\]

where \(I_{V,FG}\) are parts of the decay width that depend on the \(FG\) form factors combination\(^1\) and \(C_K\) is the Clebsh-Gordan coefficient, \(C_K = 1/\sqrt{2}\) for decays into \(\rho^0\) and \(C_K = 1\) for all other cases in this paper. It turns out that \(I_{V,gf} = I_{V,ga^+} = I_{V,ga^-} = 0\), the other terms are given by

\[
I_{V,g^2} = \frac{m_h^2 g^2}{3} \int \frac{(1-y_{V})^2}{(y_{V}+y_N)^2} \frac{d\xi}{\xi^2} g^2(q^2)\Lambda(\xi) F(\xi) \left( 2\xi^2 - G_+(\xi) \right), \tag{A.2.13}
\]

\[
I_{V,f^2} = \frac{1}{24m_h^2} \int \frac{(1-y_{V})^2}{(y_{V}+y_N)^2} \frac{d\xi}{\xi^3} f^2(q^2)\Lambda(\xi) \times \nonumber
\times \left( 3F(\xi) \left[ \xi^2 - (y_{V}^2 - y_N^2)^2 \right] - \Lambda^2(\xi) + 12y_h^2\xi \left[ 2\xi^2 - G_+(\xi) \right] \right), \tag{A.2.14}
\]

\[
I_{V,a^+} = \frac{m_h^2}{24} \int \frac{(1-y_{V})^2}{(y_{V}+y_N)^2} \frac{d\xi}{\xi^2} a_+(q^2)\Lambda(\xi) F(\xi) \left( F(\xi) \left[ 2\xi^2 - G_+(\xi) \right] + 3G_-(\xi) \left[ 1 - y_{V}^2 \right] \right). \tag{A.2.15}
\]

\(^1\)In this computation we take all form factors as real-valued functions.
\[ I_{V,a_+} = \frac{m_h^2}{8} \int \frac{d\xi}{(y_{v+} y_N)^2} a_+ (q^2) \Lambda(\xi) F(\xi) G_-(\xi), \quad (A.2.16) \]

\[ I_{V,fa_+} = \frac{1}{12} \int \frac{d\xi}{(y_{v+} y_N)^2} f(q^2) a_+ (q^2) \Lambda(\xi) \times \]
\[ \times \left( 3 \xi F(\xi) G_-(\xi) + (1 - \xi - y_{h'}^2) \left[ 3 F(\xi) \left( \xi^2 - (y_{l}^2 - y_N^2)^2 \right) - \Lambda^2(\xi) \right] \right), \quad (A.2.17) \]

\[ I_{V,fa_-} = \frac{1}{4} \int \frac{d\xi}{(y_{v+} y_N)^2} f(q^2) a_- (q^2) \Lambda(\xi) F(\xi) G_-(\xi), \quad (A.2.18) \]

\[ I_{V,a_+a_-} = \frac{m_h^2}{4} \int \frac{d\xi}{(y_{v+} y_N)^2} a_+ (q^2) a_- (q^2) \Lambda(\xi) F(\xi) G_-(\xi) \left( 1 - y_{h'}^2 \right), \quad (A.2.19) \]

where the notation is the same as in Eqs. (A.2.7-A.2.9) and

\[ F(\xi) = (1 - \xi)^2 - 2y_{h'}^2(1 + \xi) + y_{h'}^4, \quad (A.2.20) \]

\[ G_+(\xi) = \xi \left( y_N^2 + y_{l}^2 \right) + \left( y_N^2 - y_{l}^2 \right)^2. \quad (A.2.21) \]
Appendix B

HNL decays into hadronic states

B.1 Connection between matrix elements of the unflavored mesons

B.1.1 G-symmetry

An important symmetry of the low-energy theory of strong interactions is the so-called \( G \)-symmetry which is a combination of the charge conjugation \( \hat{C} \) and rotation of 180° around the \( y \) axis in the isotopic space \( \hat{R}_y \).\(^1\) The operation of charge conjugation acts on bilinear combinations of fermions \( f_1, f_2 \) as follows:

\[
\hat{C} \bar{f}_1 f_2 = \bar{f}_2 f_1, \tag{B.1.1}
\]

\[
\hat{C} \bar{f}_1 \gamma_5 f_2 = \bar{f}_2 \gamma_5 f_1, \tag{B.1.2}
\]

\[
\hat{C} \bar{f}_1 \gamma_\mu f_2 = -\bar{f}_2 \gamma_\mu f_1, \tag{B.1.3}
\]

\[
\hat{C} \bar{f}_1 \gamma_\mu \gamma_5 f_2 = \bar{f}_2 \gamma_\mu \gamma_5 f_1. \tag{B.1.4}
\]

\( \hat{R}_y \) acts on the isospin doublet as

\[
\hat{R}_y \begin{pmatrix} u \\ d \end{pmatrix} = \begin{pmatrix} d \\ -u \end{pmatrix}. \tag{B.1.5}
\]

Acting on pion states, which are pseudoscalar isovectors, one gets

\[
\hat{G} \pi^+ = \hat{R}_y \hat{C} \bar{d} \gamma_5 u = \hat{R}_y \bar{u} \gamma_5 d = -|\bar{d} \gamma_5 u\rangle = -|\pi^+\rangle, \tag{B.1.6}
\]

\[
\hat{G} \pi^0 = \hat{R}_y \hat{C} \frac{1}{\sqrt{2}} |\bar{u} \gamma_5 u - \bar{d} \gamma_5 d\rangle = \hat{R}_y \frac{1}{\sqrt{2}} |\bar{u} \gamma_5 u - \bar{d} \gamma_5 d\rangle = -\frac{1}{\sqrt{2}} |\bar{u} \gamma_5 u - \bar{d} \gamma_5 d\rangle = -|\pi^0\rangle, \tag{B.1.7}
\]

\(^1\)The latter corresponds to the interchange of \( u \) and \( d \) quarks with an additional phase, see Eq. (B.1.5) below.
so any pion is an odd state under $G$-symmetry. As a consequence, for the system of $n$ pions

$$\hat{G} |n\pi\rangle = (-1)^n |n\pi\rangle.$$  \hfill (B.1.8)

For $\rho$ mesons, which are vector isovectors, $G$-parity is positive,

$$\hat{G} |\rho^+\rangle = \hat{R}_y \hat{C} |d\gamma_\mu u\rangle = -\hat{R}_y |\bar{u}\gamma_\mu d\rangle = |\bar{d}\gamma_\mu u\rangle = |\rho^+\rangle,$$  \hfill (B.1.9)

$$\hat{G} |\rho^0\rangle = \hat{R}_y \hat{C} \frac{1}{\sqrt{2}} |\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d\rangle = \frac{1}{\sqrt{2}} |\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d\rangle = |\rho^0\rangle,$$  \hfill (B.1.10)

while for $a_1$ mesons, which are pseudovector isovectors, $G$-parity is negative,

$$\hat{G} |a_1^+\rangle = \hat{R}_y \hat{C} |d\gamma_\mu \gamma_5 u\rangle = \hat{R}_y |\bar{u}\gamma_\mu \gamma_5 d\rangle = -|\bar{d}\gamma_\mu \gamma_5 u\rangle = -|a_1^+\rangle,$$  \hfill (B.1.11)

$$\hat{G} |a_1^0\rangle = \hat{R}_y \hat{C} \frac{1}{\sqrt{2}} |\bar{u}\gamma_\mu \gamma_5 u - \bar{d}\gamma_\mu \gamma_5 d\rangle = \frac{1}{\sqrt{2}} |\bar{u}\gamma_\mu \gamma_5 u - \bar{d}\gamma_\mu \gamma_5 d\rangle = -|a_1^0\rangle.$$  \hfill (B.1.12)

### B.1.2 Classification of currents

Unflavored quarks system interacts with electromagnetic field, $W$- and $Z$-bosons through currents

$$J_\mu^\text{EM} = \frac{2}{3} \bar{u}\gamma_\mu u - \frac{1}{3} \bar{d}\gamma_\mu d,$$  \hfill (B.1.13)

$$J_\mu^W = \bar{u}\gamma_\mu (1 - \gamma_5) d,$$  \hfill (B.1.14)

$$J_\mu^Z = \bar{u}\gamma_\mu (v_u - a_u \gamma_5) u + \bar{d}\gamma_\mu (v_d - a_d \gamma_5) d,$$  \hfill (B.1.15)

where

$$v_u = \frac{1}{2} - \frac{4}{3} \sin^2 \theta_W, \quad a_u = \frac{1}{2},$$  \hfill (B.1.16)

$$v_d = -\frac{1}{2} + \frac{2}{3} \sin^2 \theta_W, \quad a_d = -\frac{1}{2}.$$  \hfill (B.1.17)

To divide the currents (B.1.13)-(B.1.15) into $G$-odd and $G$-even parts let us
introduce isoscalar and isovector vector currents

\[ j^{V,s}_\mu = \frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d), \]  
(B.1.18)

\[ j^{V,0}_\mu = \frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d), \]  
(B.1.19)

\[ j^{V,+}_\mu = \bar{d}\gamma_\mu u, \quad j^{V,-}_\mu = \bar{u}\gamma_\mu d, \]  
(B.1.20)

and isoscalar and isovector axial currents

\[ j^{A,s}_\mu = \frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu \gamma_5 u + \bar{d}\gamma_\mu \gamma_5 d), \]  
(B.1.21)

\[ j^{A,0}_\mu = \frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu \gamma_5 u - \bar{d}\gamma_\mu \gamma_5 d), \]  
(B.1.22)

\[ j^{A,+}_\mu = \bar{d}\gamma_\mu \gamma_5 u, \quad j^{A,-}_\mu = \bar{u}\gamma_\mu \gamma_5 d. \]  
(B.1.23)

Currents (B.1.18)-(B.1.23) have a certain $G$-parity presented in Table B.1. Using these currents one can rewrite physical currents as

\[ J^{EM}_\mu = \frac{1}{\sqrt{2}} j^{V,0}_\mu + \frac{1}{3\sqrt{2}} j^{V,s}_\mu, \]  
(B.1.24)

\[ J^{W}_\mu = j^{V,-}_\mu - j^{A,-}_\mu, \]  
(B.1.25)

\[ J^{Z}_\mu = \frac{1}{\sqrt{2}}(1 - 2\sin^2 \theta_W) j^{V,0}_\mu - \frac{\sqrt{2}\sin^2 \theta_W}{3} j^{V,s}_\mu - \frac{1}{\sqrt{2}} j^{A,0}_\mu. \]  
(B.1.26)

B.1.3 Connection between the matrix elements

$G$-even part of the currents (B.1.24)-(B.1.26) belongs to one isovector family, therefore there is an approximate connection between matrix elements for the system of even number of pions or $\rho$-meson,

\[ \langle 0 | J^{EM}_\mu | 2n\pi/\rho \rangle \approx \frac{1}{\sqrt{2}} \langle 0 | J^{W}_\mu | 2n\pi/\rho \rangle \approx \frac{1}{1 - 2\sin^2 \theta_W} \langle 0 | J^{Z}_\mu | 2n\pi/\rho \rangle. \]  
(B.1.27)

The special case to mention here is the $|2\pi^0\rangle$ state. In $V\pi^0\pi^0$ vertex, where $V = \gamma/Z$, system of 2 pions should have total angular momentum $J = 1$. Pions are spinless particles, so their coordinate wavefunction has negative parity which is forbidden by the Bose-Einstein statistics. Therefore

\[ \langle 0 | J^{EM}_\mu | 2\pi^0 \rangle = \langle 0 | J^{Z}_\mu | 2\pi^0 \rangle = 0. \]  
(B.1.28)

This result is equivalent to the prohibition of the $\rho^0 \to 2\pi^0$ decay.

$G$-odd parts of the currents (B.1.24)-(B.1.26), see Table B.1, belong to one isoscalar and one isovector families, so there is only one relation between matrix

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elements for the system of odd number of pions or for $a_1$-mesons,
\[
\frac{1}{\sqrt{2}} \langle 0 | J^W_\mu | (2n+1)\pi/a_1 \rangle \approx \langle 0 | J^Z_\mu | (2n+1)\pi/a_1 \rangle + 2\sin^2 \theta_W \langle 0 | J^{EM}_\mu | (2n+1)\pi/a_1 \rangle.
\]
(B.1.29)
The last formula can be simplified in the case of the one-pion or $a_1$ state. The direct interaction between photon and $\pi^0$ is forbidden because of the $C$ symmetry, while photon-to-$a_1$ interaction violates both $P$ and $C$ symmetry. Therefore, the matrix element $\langle 0 | J^{EM}_\mu | \pi/a_1 \rangle = 0$ and
\[
\frac{1}{\sqrt{2}} \langle 0 | J^W_\mu | \pi/a_1 \rangle \approx \langle 0 | J^Z_\mu | \pi/a_1 \rangle.
\]
(B.1.30)

All the approximate relations discussed above hold up to isospin violating terms of order $(m_\pi - m_\pi^0)/m_\pi \sim 3.4\%$.

B.2 HNL decays to a meson and a lepton

There are 4 types of these decays: $N \to \ell_\alpha + h_{P/V}$ and $N \to \nu_\alpha + h_{P/V}$, where $h_P$ and $h_V$ are pseudoscalar and vector mesons respectively. Reaction $N \to \ell_\alpha + h_P$ is closely related to the process calculated in Section A.1. It utilizes the same matrix element and differs only by kinematics. Using the same notation, the decay width is
\[
\Gamma(N \to \ell_\alpha h_P) = \frac{G_F^2 f_h^2 M_N^3}{16\pi} |V_{UD}|^2 |U_{\alpha}|^2 \left[ (1 - x_\ell^2)^2 - x_h^2 (1 + x_\ell^2) \right] \sqrt{\lambda(1, x_h^2, x_\ell^2)},
\]
(B.2.1)

where $x_h = m_h/M_N$, $x_\ell = m_\ell/M_N$ and function $\lambda$ is given by eq. (3.1.12).

In the case of the neutral current-mediated decay $N \to \nu_\alpha + h_P$ the hadronic matrix element reads (see Section C.1.1 for details)
\[
\langle 0 | J^Z_\mu | h_P^0 \rangle \equiv -i \frac{f_h}{\sqrt{2}} p_\mu,
\]
(B.2.2)

where $p_\mu$ is the 4-momentum of the pseudo-scalar meson $h$, $J^Z_\mu$ current is given by Eq. (B.1.15). The decay width is
\[
\Gamma(N \to \nu_\alpha h_P) = \frac{G_F^2 f_h^2 M_N^3}{32\pi} |U_{\alpha}|^2 (1 - x_h^2)^2,
\]
(B.2.3)

where $x_h = m_h/M_N$ and $f_h$ are neutral meson decay constants presented in the right part of Table C.2.

Consider the process $N \to \ell_\alpha + h_V$. For the vector meson the hadronic matrix element of the charged current is defined as
\[
\langle 0 | J^{\mu}_{UD} | h_V \rangle \equiv ig_h \varepsilon^{\mu}(p),
\]
(B.2.4)
where $\varepsilon^\mu(p)$ is the polarization vector of the meson and $g_h$ is the vector meson decay constant. The values of the $g_h$ are given in Table C.3. Using previous notations, the decay width of this process is

$$
\Gamma(N \to \ell^- h^+_V) = \frac{G_F^2 g_h^2 |V_{UD}|^2 U_{1a}^2 M_N^3}{16\pi m_h^2} \left((1 - x_h^2)^2 + x_h^2 (1 + x_h^2) - 2x_h^4\right) \sqrt{\lambda(1, x_h^2, x_h^2)}.
$$

(B.2.5)

Finally, to calculate the HNL decay width into neutral vector meson $N \to \nu_\alpha + h_V$ we define the hadronic matrix element as

$$
\langle 0 | J_Z^\mu | h_0^V \rangle \equiv i \kappa_h g_h \sqrt{2} \varepsilon^\mu(p),
$$

(B.2.6)

where $g_h$ is the vector meson decay constant and $\kappa_h$ is the dimensionless correction factor, their values are given in Table C.3. For the decay width one obtains

$$
\Gamma(N \to \nu_\alpha h_V) = \frac{G_F^2 g_h^2 |V_{UD}|^2 U_{1a}^2 M_N^3}{32\pi m_h^2} \left(1 + 2x_h^2\right) \left(1 - x_h^2\right)^2.
$$

(B.2.7)

### B.3 HNL decays to a lepton and two pions

For the case of 2 pions the matrix element of the axial current is equal to zero, so the general expression for matrix element is (c.f. (A.2.2))

$$
\langle \pi(p') | J_\mu | \pi(p) \rangle = f_+(q^2)(p + p')_\mu + f_-(q^2)q_\mu,
$$

(B.3.1)

where $J_\mu$ is one of the currents (B.1.13)-(B.1.15) and $q_\mu = (p - p')_\mu$. Because of the isospin symmetry (B.1.27), the form factors are related as

$$
f_\pm^{\text{EM}} \approx \frac{1}{\sqrt{2}} f_\pm^{W} \approx \frac{1}{1 - 2\sin^2\theta_W} f_\pm^{Z}.
$$

(B.3.2)

Electromagnetic current conservation $q_\mu J_\mu = 0$ implies $f_\pm^{\text{EM}}(q^2) = 0$. Therefore all the matrix elements could be expressed via a single form factor, called pion electromagnetic form factor,

$$
\langle \pi(p') | J_\mu^{\text{EM}} | \pi(p) \rangle = F_\pi(q^2)(p + p')_\mu.
$$

(B.3.3)

Pion electromagnetic form factor is related to the cross section of reaction $e^+e^- \to 2\pi$ as

$$
\sigma(e^+e^- \to 2\pi) = \frac{\pi\alpha_\text{EM}^2}{3s} \beta_\pi^2(s) |F_\pi(s)|^2,
$$

(B.3.4)

where $\beta_\pi(s) = \sqrt{1 - 4m_\pi^2/s}$, so it is well-measured experimentally. There is a lot of data on electromagnetic form factor [294–299], which agree with each other.
Using matrix elements described above it is easy to find the decay widths of \( N \to \ell \pi^0 \pi^+ \) and \( N \to \nu_\ell \pi^+ \pi^- \) (see Feynman diagrams in Fig. B.2),

\[
\Gamma(N \to \ell_\alpha \pi^0 \pi^+) = \frac{G_F^2 M_N^3}{384 \pi^3} \left| V_{ud} \right|^2 \left| U_{\alpha} \right|^2 \int \frac{(m_\ell - M_N - m_\ell)^2}{4m_\ell^2} \left( 1 - x_\ell^2 \right)^2 + \frac{q^2}{M_N^2} \left( 1 + x_\ell^2 \right) - 2 \frac{q^4}{M_N^4} \right) \times \\
\times \lambda^{1/2} \left( 1, \frac{q^2}{M_N^2}, x_\ell^2 \right) \beta_\pi^2(q^2) |F_\pi(q^2)|^2 dq^2, \tag{B.3.5}
\]

\[
\Gamma(N \to \nu_\alpha \pi^+ \pi^-) = \frac{G_F^2 M_N^3}{768 \pi^3} \left| U_{\alpha} \right|^2 \left( 1 - 2 \sin^2 \theta_W \right)^2 \times \\
\times \int \frac{M_N^2}{4m_\ell^2} \left( 1 - \frac{q^2}{M_N^2} \right)^2 \left( 1 + 2 \frac{q^2}{M_N^2} \right) \beta_\pi^2(q^2) |F_\pi(q^2)|^2 dq^2, \tag{B.3.6}
\]

where \( x_\ell = \frac{m_\ell}{M_N} \) and the function \( \lambda \) is given by (3.1.12). The decay width \( \Gamma(N \to \)
\( \nu_\alpha \pi^0\pi^0 = 0 \) because of Eq. (B.1.28).

Using the VDM model, formula (B.3.5) and lifetime of the \( \tau \)-lepton we have calculated the branching ratio \( \text{BR}(\tau \to \nu_\tau \pi^-\pi^0) = 25.2\% \) which is close to the experimental value 25.5\%.

The decay into 2 pions is significantly enhanced by the \( \rho \)-resonance. It turns out, that this is the dominant channel, see Fig. 3.15 comparing the decay width of HNL into 2 pions and into \( \rho \)-meson. Therefore, one can replace the decay into 2 pions with 2-body decay into \( \rho \)-meson.
Appendix C

Phenomenological parameters

In this Section we summarize parameters used in this work. Values of the CKM matrix elements are given in Table C.1.

<table>
<thead>
<tr>
<th>$V_{ud}$</th>
<th>$V_{us}$</th>
<th>$V_{ub}$</th>
<th>$V_{cd}$</th>
<th>$V_{cs}$</th>
<th>$V_{cb}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.974</td>
<td>0.225</td>
<td>0.00409</td>
<td>0.220</td>
<td>0.995</td>
<td>0.0405</td>
</tr>
</tbody>
</table>

Table C.1: CKM matrix elements [222] adopted in this work.

C.1 Meson decay constants

The decay constants for charged pseudoscalar mesons are defined by Eq. (A.1.3), the values of $f_h$ (Table C.2) are measured experimentally and/or obtained by lattice calculations [300].

Meson decay constants for the mesons with the same-flavor quarks are defined by Eq. (B.2.2). There is a discrepancy regarding their values in the literature, therefore we have computed them directly (see Appendix C.1.1). The results of these computations are given in the right column of Table C.2. The meson decay constants for neutral mesons consisted of quarks of different flavors (such as $K^0, D^0, B^0, B_s$) are not needed in computing HNL production or decay, we do not provide them here.

For vector charged mesons the decay constants $g_h$ are defined by Eq. (B.2.4). In the literature they often appear as $f_h$, connected to our prescription by mass of the meson $g_h = f_h m_h$. Their values are presented in Table C.3. For vector neutral mesons the decay constants $g_h$ and dimensionless factors $\kappa_h$ are defined by Eq. (B.2.6). Their values are presented in Table C.3 as well.
\[ f_{\pi^+} = 130.2 \text{ MeV} \quad f_{\pi^0} = 130.2 \text{ MeV}^1 \\
\[ f_{K^+} = 155.6 \text{ MeV} \quad f_\eta = 81.7 \text{ MeV}^2 \\
\[ f_{D^+} = 212 \text{ MeV} \quad f_{\eta'} = -94.7 \text{ MeV}^2 \\
\[ f_{D_s} = 249 \text{ MeV} \quad f_{\eta_c} = 237 \text{ MeV}^3 \\
\[ f_{B^+} = 187 \text{ MeV} \quad \\
\[ f_{B_c} = 434 \text{ MeV} \]

Table C.2: Decay constants of pseudoscalar charged mesons (left table) and pseudoscalar neutral mesons (right table).

\[
g_{\rho^+} = 0.162 \text{ GeV}^2 \quad g_{D^{++}} = 0.535 \text{ GeV}^2 \quad g_{D^{*+}} = 0.650 \text{ GeV}^2 \\
\hline
h & g_h \text{ [GeV}^2] & \kappa_h \\
\hline
\rho^0 & 0.162 & 1 - 2 \sin^2 \theta_W^5 \\
\omega & 0.153 \ [238] & \frac{4}{3} \sin^2 \theta_W \\
\phi & 0.234 \ [302] & \frac{4}{3} \sin^2 \theta_W - 1 \\
J/\psi & 1.29 \ [304] & 1 - \frac{4}{3} \sin^2 \theta_W
\]

Table C.3: Decay constants of vector charged mesons (left table) and vector neutral mesons (right table). Decay constants for \(D^{*+}_{(s)}\) mesons in [303] show large theoretical uncertainty, we quote only the average value here.

C.1.1 Decay constants of \(\eta\) and \(\eta'\) mesons

To describe HNL decays into \(\eta\) and \(\eta'\) mesons we need to know the corresponding neutral current decay constants, that we define as (B.2.2)

\[
\langle 0 | J^Z_\mu | h^0 \rangle \equiv -i \frac{f_h}{\sqrt{2}} \gamma_\mu,
\]

where \(p_\mu\) is the 4-momentum of the pseudo-scalar meson \(h\), \(J^Z_\mu\) current is given by Eq. (B.1.15). The choice of the additional factor \((-1/\sqrt{2})\) is introduced in order to obtain \(f_{\eta^0} = f_{\pi^\pm}\) and \(f_{\eta^0} > 0\), see discussion below. Taking into account that for pseudoscalar mesons only axial part of the current contributes to this matrix element we can write the matrix element as

\[
\langle 0 | J^Z_\mu | h^0_F \rangle = \langle 0 | \bar{q} \gamma_\mu \gamma^5 \lambda Z q | h^0_F \rangle,
\]

where

\[
q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}, \quad \lambda^Z = \frac{1}{2} \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.
\]

\footnote{It should be equal to \(f_{\pi^+}\), according to Eq. (B.1.30).}

\footnote{See discussion in Section C.1.1.}

\footnote{See discussion in Section C.1.2.}

\footnote{See discussion in the section C.1.3.}

\footnote{See Eq. (B.1.27).}
The relevant decay constants are $f^0$ and $f^8$, they come from the set of decay constants extracted from experiments defined as [305]

$$
\langle 0 \mid J_\mu^a \mid h \rangle = i f_h^a p_\mu, \tag{C.1.3}
$$

where $J_\mu^a = \bar{q} \gamma_\mu \gamma^5 \frac{\lambda^a}{\sqrt{2}} q$ with $\lambda^a$ being the Gell-Mann matrices for $a = 1 \ldots 8$ and

$$
\lambda^0 = \sqrt{\frac{2}{3}} \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix} \tag{C.1.4}
$$

The overall factor in $\lambda^0$ is chosen to obey normalization condition $\text{Tr}(\lambda^a \lambda^b) = 2 \delta^{ab}$.

Within the chiral perturbation theory ($\chi$PT) (see [306] and references therein), the lightest mesons correspond to pseudo-Goldstone bosons $\phi^a$, that appear after the spontaneous breaking of $U_L(3) \times U_R(3)$ symmetry to group $U_V(3)$. States $\phi^a$ are orthogonal in the sense

$$
\langle 0 \mid J_\mu^a \mid \phi^b \rangle = i f^a_{\phi^b} p_\mu, \quad f^a_{\phi^b} = f^a_{\phi^b} \delta^{ab} \tag{C.1.5}
$$

where and $f^a_{\phi^b}$ are corresponding decay constants. Using

$$
\lambda^3 = \begin{pmatrix}
1 & 0 & 0 \\
0 & -1 & 0 \\
0 & 0 & 0
\end{pmatrix}, \quad \lambda^8 = \sqrt{\frac{1}{3}} \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & -2
\end{pmatrix} \tag{C.1.6}
$$

we can rewrite the axial part of the weak neutral current (B.2.2) as a linear combination of the $J_\mu^0, J_\mu^3$ and $J_\mu^8$

$$
\bar{q} \gamma_\mu \gamma^5 Z q = \frac{1}{\sqrt{2}} \left( \frac{J_\mu^0}{\sqrt{6}} - \frac{J_\mu^8}{\sqrt{3}} - J_\mu^3 \right) \tag{C.1.7}
$$

and $f_h$ is given by

$$
f_h = f^3_h + \frac{f^8_h}{\sqrt{3}} - \frac{f^0_h}{\sqrt{6}}. \tag{C.1.8}
$$

For example, $\pi^0$ meson corresponds to $\phi^3$ state in $\chi$PT, so $f^0_{\pi^0} = f^8_{\pi^0} = 0$ and Eq. (C.1.8) gives $f_{\pi^0} = f^3_{\pi^0} = f_{\pi^+}$ because of the isospin symmetry, in full agreement with Eq. (B.1.30).

For $\eta$ and $\eta'$ the application of Eq. (C.1.8) is not so straightforward. These mesons are neutral unflavored mesons with zero isospin and they can oscillate between each other. So $\eta$ and $\eta'$ do not coincide with any single $\phi^a$ state. Rather they are mixtures of $\phi^0$ and $\phi^8$ states. In real world isospin is not a conserved quantum number, so $\phi^3$ state also should be taken into account, but its contribution is negli-
gible [307], so we use \( f_\eta^3 = f_{\eta'}^3 = 0 \). Another complication is \( U(1) \) QCD anomaly for \( J_{\mu}^0 \) current that not only shifts masses of corresponding mesons but also contributes to the \( f_0^0 \) meson constant. To phenomenologically take into account the effect of anomaly it was proposed to use two mixing angles scheme [308],

\[
\begin{pmatrix}
  f_\eta^8 & f_0^0 \\
  f_\eta^3 & f_{\eta'}^0
\end{pmatrix} = \begin{pmatrix}
  f_8 \cos \theta_8 & -f_0 \sin \theta_0 \\
  f_8 \sin \theta_8 & f_0 \cos \theta_0
\end{pmatrix}.
\] (C.1.9)

Taking parameter values from the recent phenomenological analysis [305],

\[
f_8 = 1.27(2)f_\pi, \quad f_0 = 1.14(5)f_\pi, \quad \theta_8 = -21.2(1.9)^\circ, \quad \theta_0 = -6.9(2.4)^\circ,
\] (C.1.10)

we find

\[
\begin{align*}
f_\eta &= 0.63(2)f_\pi \approx 81.7(3.1) \text{ MeV}, \\
f_{\eta'} &= -0.73(3)f_\pi \approx -94.7(4.0) \text{ MeV}.
\end{align*}
\] (C.1.11) (C.1.12)

These numbers should be confronted with the values quoted in [218] and [238].

### C.1.2 Decay constant of \( \eta_c \) meson

The decay constant of \( \eta_c \) meson is defined as [309]

\[
\langle 0 | \bar{c} \gamma^\mu \gamma^5 c | \eta_c \rangle \equiv i f_{\eta_c}^{\exp} p^\mu, \tag{C.1.13}
\]

where \( f_{\eta_c}^{\exp} = 335 \text{ MeV} \), as measured by CLEO collaboration [310]. Our definition (B.2.2) differs by a factor \( \sqrt{2} \), so \( f_{\eta_c} = f_{\eta_c}^{\exp} / \sqrt{2} \approx 237 \text{ MeV} \).

### C.1.3 Decay constant of \( \rho \) meson

There are 2 parametrizations of the \( \rho \) charged current matrix element using \( g_\rho \), defined by (B.2.4), or \( f_\rho \), which is related to \( g_\rho \) are \( f_\rho = g_\rho / m_\rho \). The value of the decay constant can be obtained by 2 methods: from \( \rho \to e^+e^- \) using the approximate symmetry (B.1.27) or from the \( \tau \)-lepton decay. Results obtained in Ref. [302] by these two methods differ by about 5\%, \( f_{\rho,ee} = 220(2) \text{ MeV} \) and \( f_{\rho,\tau} = 209(4) \text{ MeV} \). We calculate

\[
\Gamma(\tau \to \nu \rho) = \frac{G_F^2 g_\rho^2 m_\tau^3 |V_{ud}|^2}{16\pi m_\rho^2} \left( 1 + 2 \frac{m_\rho^2}{m_\tau^2} \right) \left( 1 - \frac{m_\rho^2}{m_\tau^2} \right)^2,
\] (C.1.14)

\[
\Gamma(\rho \to e^+e^-) = \frac{e^4 g_\rho^2}{24\pi m_\rho^3},
\] (C.1.15)

and get \( g_{\rho,\tau} = 0.162 \text{ GeV}^2 \) and \( g_{\rho,ee} = 0.171 \text{ GeV}^2 \), which corresponds to \( f_{\rho,\tau} = 209 \text{ MeV} \) and \( f_{\rho,ee} = 221 \text{ MeV} \) in full agreement with [302]. The difference between
these results can be explained by the relation B.1.27 being approximate. So we use $g_{\rho,\tau}$ value as more directly measured one. The results of our analysis agrees with $f_{\rho}$ value in [238] (within about 10%), but differ from the value adopted in [218] by $\sim 25\%$.

C.2 Meson form factors of decay into pseudoscalar meson

To describe the semileptonic decays of the pseudoscalar meson into another pseudoscalar meson one should know the form factors $f_+(q^2), f_0(q^2), f_-(q^2)$ defined by Eq. (A.2.2), only two of which are independent. We use the $f_+(q^2), f_0(q^2)$ pair for the decay parametrization.

In turn, there are many different parametrizations of meson form factors. One popular parametrization is the Bourrely-Caprini-Lellouch (BCL) parametrization [311] that takes into account the analytic properties of form factors (see e.g. [312, 313]),

$$f(q^2) = \frac{1}{1 - q^2/M_{\text{pole}}^2} \sum_{n=0}^{N-1} a_n \left[ (z(q^2))^n - (-1)^{n-N} \frac{n}{N} (z(q^2))^N \right]$$

(C.2.1)

where the function $z(q^2)$ is defined via

$$z(q^2) \equiv \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

(C.2.2)

with

$$t_+ = (m_h + m_{h'})^2.$$  

(C.2.3)

The choice of $t_0$ and of the pole mass $M_{\text{pole}}$ varies from group to group that performs the analysis. In this work we follow the FLAG collaboration [313] and take

$$t_0 = (m_h + m_{h'}) \left( \sqrt{m_h} - \sqrt{m_{h'}} \right)^2.$$  

(C.2.4)

The coefficients $a_n^+$ and $a_n^0$ are then fitted to the experimental data or lattice results.

C.2.1 K meson form factors

Form factors of $K \rightarrow \pi$ transition are well described by the linear approximation [314, 315]

$$f_{K\pi}^{+,0}(q^2) = f_{K\pi}^{+,0}(0) \left( 1 + \lambda_{+,0} \frac{q^2}{m_{\pi^+}^2} \right).$$

(C.2.5)

The best fit parameters are given in Table C.4.
\[ f(q^2) = f(0) - c(z(q^2) - z_0) \left(1 + \frac{z(q^2) + z_0}{2}\right) \frac{1}{1 - Pq^2}, \] (C.2.6)

where \( z_0 = z(0) \). The best fit parameter values are given in Table C.5.

<table>
<thead>
<tr>
<th>( f )</th>
<th>( f(0) )</th>
<th>( c )</th>
<th>( P ) (GeV(^{-2}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f^{DK}_+ )</td>
<td>0.7647</td>
<td>0.066</td>
<td>0.224</td>
</tr>
<tr>
<td>( f^{DK}_0 )</td>
<td>0.7647</td>
<td>2.084</td>
<td>0</td>
</tr>
<tr>
<td>( f^{D\pi}_+ )</td>
<td>0.6117</td>
<td>1.985</td>
<td>0.1314</td>
</tr>
<tr>
<td>( f^{D\pi}_0 )</td>
<td>0.6117</td>
<td>1.188</td>
<td>0.0342</td>
</tr>
</tbody>
</table>

Table C.5: Best fit parameters for the form factors (C.2.6) of \( D \to \pi \) and \( D \to K \) transitions \[316\].

Form factors of \( D_s \to \eta \) transition read \[317\]

\[ f^{D_s\eta}_+(q^2) = \frac{f^{D_s\eta}_+(0)}{1 - q^2/m^{2}_{D_s}}, \] (C.2.7)

\[ f^{D_s\eta}_0(q^2) = \frac{f^{D_s\eta}_0(0)}{1 - \alpha^{D_s\eta}q^2/m^{2}_{D_s}}, \] (C.2.8)

where \( f^{D_s\eta}_+(0) = 0.495 \), \( \alpha^{D_s\eta} = 0.198 \) \[317\], \( m_{D_s} = 2.112 \text{GeV} \) \[222\]. Scalar form factor \( f^{D_s\eta}_0(q^2) \) is not well constrained by experimental data, so we take \( f^{D_s\eta}_0(q^2) = f^{D_s\eta}_+(q^2) \) by Eq. (A.2.3) and \( \alpha^{D_s\eta}_0 = 0 \).

### C.2.3 B meson form factors

Most of \( B \) meson form factors are available in literature in the form (C.2.1), their best fit parameter values are given in Table C.6. The form factors for \( B_s \to D_s \) are almost the same as for \( B \to D \) transition \[318\], so we use the same expressions for both cases.
One of the relevant HNL production channels is the pseudoscalar meson decay $h_P \rightarrow h'_V \ell_N$. To compute the decay width one needs to know the form factors $g(q^2)$, $f(q^2)$, $a_\pm(q^2)$, defined by Eqs. (A.2.4, A.2.5). The dimensionless linear combinations are introduced as

$$V^{hh'}(q^2) = (m_h + m_{h'}) g^{hh'}(q^2),$$

$$A_0^{hh'}(q^2) = \frac{1}{2m_{h'}} \left( f^{hh'}(q^2) + q^2 a_{-}^{hh'}(q^2) + \left( m_h^2 - m_{h'}^2 \right) a_{+}^{hh'}(q^2) \right),$$

$$A_1^{hh'}(q^2) = \frac{f^{hh'}(q^2)}{m_h + m_{h'}},$$

$$A_2^{hh'}(q^2) = - (m_h + m_{h'}) a_{+}^{hh'}(q^2).$$

For these linear combinations the following ansatz is used

$$V^{hh'}(q^2) = \frac{f^{hh'}}{1 - q^2/(M_V^2)} \left[ 1 - \sigma_V^{hh'} q^2/(M_V^2)^2 - \xi_V^{hh'} q^4/(M_V^4)^4 \right],$$

$$A_0^{hh'}(q^2) = \frac{f^{hh'}}{1 - q^2/(M_P^2)} \left[ 1 - \sigma_{A_0}^{hh'} q^2/(M_P^2)^2 - \xi_{A_0}^{hh'} q^4/(M_P^4)^4 \right],$$

$$A_{1/2}^{hh'}(q^2) = \frac{f^{hh'}}{1 - \sigma_{A_{1/2}}^{hh'} q^2/(M_P^2)^2 - \xi_{A_{1/2}}^{hh'} q^4/(M_P^4)^4}. $$

Best fit values of parameters are adopted from papers [319–321]. $f$, $\sigma$ parameters are given in Table C.7, while $\xi$ and the pole masses $M_V$ and $M_P$ are given in Table C.8.

### Table C.6: Best fit parameters for the form factors (C.2.1) of $B \rightarrow \pi$, $B_{(s)} \rightarrow D_{(s)}$ and $B_s \rightarrow K$ transitions [313].

<table>
<thead>
<tr>
<th>$f$</th>
<th>$M_{\text{pole}}$ (GeV)</th>
<th>$a_0$</th>
<th>$a_1$</th>
<th>$a_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f^B_{(s)} D_{(s)}$</td>
<td>$\infty$</td>
<td>0.909</td>
<td>-7.11</td>
<td>66</td>
</tr>
<tr>
<td>$f^B_{(s)} D_{(s)}$</td>
<td>$\infty$</td>
<td>0.794</td>
<td>-2.45</td>
<td>33</td>
</tr>
<tr>
<td>$f^B_{(s)} K$</td>
<td>$m_{B^*} = 5.325$</td>
<td>0.360</td>
<td>-0.828</td>
<td>1.1</td>
</tr>
<tr>
<td>$f^B_{(s)} K$</td>
<td>$m_{B^*(0^+)} = 5.65$</td>
<td>0.233</td>
<td>0.197</td>
<td>0.18</td>
</tr>
<tr>
<td>$f^{B_{(s)} K}$</td>
<td>$m_{B^*(0^+)} = 5.325$</td>
<td>0.404</td>
<td>-0.68</td>
<td>-0.86</td>
</tr>
<tr>
<td>$f^{B_{(s)} K}$</td>
<td>$m_{B^*(0^+)} = 5.65$</td>
<td>0.490</td>
<td>-1.61</td>
<td>0.93</td>
</tr>
</tbody>
</table>
Table C.7: First part of the table with parameters of meson form factors (C.3.5-C.3.7) of decay into vector meson [319–321].

<table>
<thead>
<tr>
<th>$h, h'$</th>
<th>$f_{V}^{h'}$</th>
<th>$f_{A_0}^{h'}$</th>
<th>$f_{A_1}^{h'}$</th>
<th>$f_{A_2}^{h'}$</th>
<th>$\sigma_{V}^{h'}$</th>
<th>$\sigma_{A_0}^{h'}$</th>
<th>$\sigma_{A_1}^{h'}$</th>
<th>$\sigma_{A_2}^{h'}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D, K^*$</td>
<td>1.03</td>
<td>0.76</td>
<td>0.66</td>
<td>0.49</td>
<td>0.27</td>
<td>0.17</td>
<td>0.30</td>
<td>0.67</td>
</tr>
<tr>
<td>$B, D^*$</td>
<td>0.76</td>
<td>0.69</td>
<td>0.66</td>
<td>0.62</td>
<td>0.57</td>
<td>0.59</td>
<td>0.78</td>
<td>1.40</td>
</tr>
<tr>
<td>$B, \rho$</td>
<td>0.295</td>
<td>0.231</td>
<td>0.269</td>
<td>0.282</td>
<td>0.875</td>
<td>0.796</td>
<td>0.54</td>
<td>1.34</td>
</tr>
<tr>
<td>$B_s, D_{s}^*$</td>
<td>0.95</td>
<td>0.67</td>
<td>0.70</td>
<td>0.75</td>
<td>0.372</td>
<td>0.350</td>
<td>0.463</td>
<td>1.04</td>
</tr>
<tr>
<td>$B_s, K^*$</td>
<td>0.291</td>
<td>0.289</td>
<td>0.287</td>
<td>0.286</td>
<td>−0.516</td>
<td>−0.383</td>
<td>0</td>
<td>1.05</td>
</tr>
</tbody>
</table>

Table C.8: Second part of the table with parameters of meson form factors (C.3.5-C.3.7) of decay into vector meson [319–321]. Masses of $B_c$, $D_s$ and $D_{s}^*$ are taken from [222], while for $B_c^*$ theoretical prediction [322] is used.

<table>
<thead>
<tr>
<th>$h, h'$</th>
<th>$\xi_{V}^{h'}$</th>
<th>$\xi_{A_0}^{h'}$</th>
<th>$\xi_{A_1}^{h'}$</th>
<th>$\xi_{A_2}^{h'}$</th>
<th>$M_P^h$ (GeV)</th>
<th>$M_V^h$ (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D, K^*$</td>
<td>0</td>
<td>0</td>
<td>0.20</td>
<td>0.16</td>
<td>$m_{D_s} = 1.969$</td>
<td>$m_{D_s^*} = 2.112$</td>
</tr>
<tr>
<td>$B, D^*$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.41</td>
<td>$m_{B_c} = 6.275$</td>
<td>$m_{B_c^*} = 6.331$</td>
</tr>
<tr>
<td>$B, \rho$</td>
<td>0</td>
<td>0.055</td>
<td>0</td>
<td>−0.21</td>
<td>$m_B = 5.279$</td>
<td>$m_{B^*} = 5.325$</td>
</tr>
<tr>
<td>$B_s, D_{s}^*$</td>
<td>0.561</td>
<td>0.600</td>
<td>0.510</td>
<td>0.070</td>
<td>$m_{B_s} = 6.275$</td>
<td>$m_{B_{s}^*} = 6.331$</td>
</tr>
<tr>
<td>$B_s, K^*$</td>
<td>2.10</td>
<td>1.58</td>
<td>1.06</td>
<td>−0.074</td>
<td>$m_{B_s} = 5.367$</td>
<td>$m_{B_{s}^*} = 5.415$</td>
</tr>
</tbody>
</table>
Appendix D

Production from $J/\psi$ and $\Upsilon$ mesons

D.1 Production from $J/\psi$

The process $J/\psi \rightarrow N\bar{\nu}$ allows to create HNLs with masses up to $M_{J/\psi} \simeq 3.1$GeV and therefore contribute to the production above the $D$-meson threshold.

To estimate $\text{BR}(J/\psi \rightarrow N\bar{\nu})$ let us first compare the processes $J/\psi \rightarrow e^+e^-$ and $J/\psi \rightarrow \nu_e\bar{\nu}_e$. The ratio of their width is given by [323]

$$\frac{\text{BR}(J/\psi \rightarrow e^+e^-)}{\text{BR}(J/\psi \rightarrow \nu_e\bar{\nu}_e)} = \frac{27G_F^2M_{J/\psi}^4}{256\pi^2\alpha^2} \left(1 - \frac{8}{3}\sin^2\theta_W\right)^2 \sim 4.5 \times 10^{-7}$$  \hspace{1cm} (D.1.1)

with the precision of the order of few per cent [323]. Using the measured branching ratio $\text{BR}(J/\psi \rightarrow e^+e^-) \simeq 0.06$ [222], one can estimate decay into one flavor of neutrinos, $\text{BR}(J/\psi \rightarrow \nu_e\bar{\nu}_e) \simeq 2.7 \times 10^{-8}$. The corresponding branching of $J/\psi$ to HNL is additionally suppressed by $U^2$ and by the phase-space factor $f_{PS}$:

$$\sum_\alpha \text{BR}(J/\psi \rightarrow N\bar{\nu}_\alpha) = U^2 f_{PS}(M_N/M_{J/\psi}) \text{BR}(J/\psi \rightarrow \nu_e\bar{\nu}_e)$$ \hspace{1cm} (D.1.2)

We estimate this fraction at $M_N = M_D$ (just above the $D$-meson threshold) taking for simplicity $f_{PS} = 1$. Clearly, at masses below $M_D$ the production from $D$-mesons dominates (as the $J/\psi$ production fraction $f(J/\psi) \simeq 0.01$, see [110, Appendix A], reproduced for completeness in Appendix 4.1.1). Above $D$-meson mass but below $M_{J/\psi}$ we should compare with the production from $B$ mesons. We compare the probability to produce HNL from $B$-meson and from $J/\psi$:

$$\frac{\text{HNLs from } J/\psi}{\text{HNLs from } B} = \frac{X_{ee} \times f(J/\psi) \times \text{BR}_{J/\psi \rightarrow N\bar{\nu}}}{X_{bb} \times f(B) \times \text{BR}_{B \rightarrow NX}} =$$

$$= 3 \times 10^{-4} \left(\frac{X_{ee}}{10^{-3}}\right) \left(\frac{10^{-7}}{X_{bb}}\right)$$ \hspace{1cm} (D.1.3)
where we have adopted \( f(B) \times BR(B \rightarrow N + X) \sim 10^{-2} \) (c.f. Fig. 3.4, right panel) and used \( f(J/\psi) \sim 10^{-2} \). The numbers in (3.1.6) are normalized to SHiP. We see therefore that \( J/\psi \) decays contribute subdominantly while \( X_{b\bar{b}}/X_{c\bar{c}} \gtrsim 10^{-8} \).

### D.2 Production from \( \Upsilon \)

The heavy mass of \( \Upsilon \) opens up a possibility to produce HNLs up to \( M_N \approx 10 \text{GeV} \). Similarly to Eq. (D.1.1) we can find the branching ratio

\[
BR(\Upsilon \rightarrow \nu \bar{\nu}) = 4 \times 10^{-4} BR(\Upsilon \rightarrow e^+e^-) \]  

Therefore

\[
BR(\Upsilon \rightarrow N \bar{\nu}_a) = U^2_a f_{PS}(M_N/M_\Upsilon) \frac{27G_F^2M_\Upsilon^4}{64\pi^2\alpha^2} \left( -1 + \frac{4}{3} \sin^2 \theta_W \right)^2 BR(\Upsilon \rightarrow e^+e^-) \]  

(D.2.1)

Using the latest measurement \( BR(\Upsilon \rightarrow e^+e^-) \approx 2.4 \times 10^{-2} \) [222] one finds that \( BR(\Upsilon \rightarrow \nu \bar{\nu}) \approx 10^{-5} \). We do not know the fraction \( f(\Upsilon) \) out of all \( b\bar{b} \) pairs, but one can roughly estimate it being equal to the fraction \( f(J/\psi) \sim 1\% \) (see Appendix 4.1.1 in [110]), so

\[
N_{\Upsilon \rightarrow N\nu} \approx 10^{-10} N_\Upsilon \times \left( \frac{U^2}{10^{-5}} \right) \]  

(D.2.2)

where we have normalized \( U^2 \) to the current experimental limit for \( M_N > 5 \text{ GeV} \) (c.f. Fig. 3.1).


Appendix E

Vector-dominance model

Here we provide the $F_\pi(s)$ formula, given by the vector-dominance model \[294\]

$$F_\pi(s) = \frac{BW^\text{GS}_\rho(s)^{1+c_1 BW^\text{KS}_\rho(s) + c_\rho BW^\text{GS}_\rho(s) + c_\rho^\prime BW^\text{GS}_\rho(s) + c_\rho^\prime\prime BW^\text{GS}_\rho(s)}}{1 + c_\rho + c_\rho^\prime + c_\rho^\prime\prime},$$  

(E.0.1)

where $c_i = |c_i|e^{i\phi}$ are the complex amplitudes of the Breit–Wigner (BW) functions. They are different for $\omega$ and $\rho$ mesons. For $\omega$ it is the usual BW function

$$BW^\text{KS}_\omega(s) = \frac{m_\omega^2}{m_\omega^2 - s - im_\omega \Gamma},$$  

(E.0.2)

while for $\rho$ mesons the Gounaris-Sakurai (GS) model \[324\] is taken,

$$BW^\text{GS}_\rho(s) = \frac{m_\rho^2(1 + d(m_\rho_\rho) \Gamma_\rho_\rho/m_\rho_\rho)}{m_\rho^2 - s + f(s, m_\rho_\rho, \Gamma_\rho_\rho) - im_\rho_\rho \Gamma(s, m_\rho_\rho, \Gamma_\rho_\rho)},$$  

(E.0.3)

where

$$\Gamma(s, m, \Gamma) = \Gamma \frac{s}{m^2} \left( \frac{\beta_\pi(s)}{\beta_\pi(m^2)} \right)^3 ;$$  

(E.0.4)

$$f(s, m, \Gamma) = \frac{\Gamma m^2}{k^3(m^2)} \left[ k^2(s) \left( h(s) - h(m^2) \right) + (m^2 - s) k^2(m^2) h'(m^2) \right]$$  

(E.0.5)

$$\beta_\pi(s) = \sqrt{1 - \frac{4m_\pi^2}{s}},$$  

(E.0.6)

$$d(m) = \frac{3}{\pi k^2(m^2)} \ln \left( \frac{m^2 + 2k(m^2)}{2m_\pi} \right) + \frac{m}{2\pi k(m^2)} - \frac{m_\pi^2 m}{\pi k^3(m^2)},$$  

(E.0.7)

$$k(s) = \frac{1}{2} \sqrt{s} \beta_\pi(s),$$  

(E.0.8)

$$h(s) = \frac{2}{\pi} \frac{k(s)}{\sqrt{s}} \ln \left( \frac{\sqrt{s} + 2k(s)}{2m_\pi} \right)$$  

(E.0.9)

and $h'(s)$ is a derivative of $h(s)$. 


Appendix F

Estimation of the upper bound width

In this Section we estimate the ratio between $U_{\text{max}}$ and $U_{\text{top}}$ as defined in Section 5.1.3.1.

The decay probability (5.1.9) can be rewritten in the form

$$P_{\text{decay}} \approx \int dE_X f_{E_X} e^{-l_{\text{target-det}} \Gamma X / E_X} = \int dE_X e^{-g(E_X)},$$  \hspace{1cm} (F.0.1)

where for clarity we have assumed $f_{E_X}$ to be dimensionless, and $g(E_X) = l_{\text{target-det}} \Gamma X M_X / E_X - \ln(f_{E_X})$. Using the steepest descent method, one arrives at

$$P_{\text{decay}} \approx \sqrt{\frac{2\pi}{-g''(E_{\text{peak}})}} e^{-g(E_{\text{peak}})},$$ \hspace{1cm} (F.0.2)

$E_{\text{peak}}$ is determined by the extremum criterion $g'(E_{\text{peak}}) = 0$. For the exponential spectrum of the form $f_{E_X} = f_0 e^{-E_X \delta}$ the integral and the peak energy are

$$E_{\text{peak}}^X = \sqrt{\frac{l_{\text{target-det}} \Gamma X M_X}{\delta}}, \quad P_{\text{decay}} \approx \sqrt{\pi} f_0 e^{-2 E_{\text{peak}}^X \delta} \sqrt{\frac{E_{\text{peak}}^X}{\delta}},$$  \hspace{1cm} (F.0.3)

while for the power law spectrum $f_{E_X} = f_0 E_X^{-\alpha}$ they are

$$E_{\text{peak}}^X = \frac{l_{\text{target-det}} \Gamma X M_X}{\alpha}, \quad P_{\text{decay}} \approx \sqrt{\frac{2\pi}{\alpha}} f_0 e^{-\alpha(E_{\text{peak}}^X)^{-\alpha}},$$ \hspace{1cm} (F.0.4)

Expressing then $\Gamma_X \propto U^2$ and $U^2 = U_{\text{max}}^2 (M_X) \times R$, one immediately arrives
at (F.0.5).

\[ U_{\text{top}}^{2,\text{SHIP}}(M_N) \approx U_{\text{max}}^{2,\text{SHIP}}(M_N) \frac{\ln^2 \left( f_0 \sqrt{\pi} (\delta^{-3} \langle p_B \rangle)^{\frac{1}{2}} N_{\text{prod}}(M_N, U_{\text{max}}^{\text{SHIP}}(M_N)) \right)}{4 \langle p_B \rangle \delta}, \quad (F.0.5) \]

\[ U_{\text{top}}^{2,\text{MAT}}(M_N) \approx U_{\text{max}}^{2,\text{MAT}}(M_N) \frac{\alpha}{\langle p_B \rangle} \left( \sqrt{\frac{2\pi}{\alpha^3}} N_{\text{prod}}(M_N, U_{\text{max}}^{\text{MAT}}(M_N)) \right)^{\frac{1}{\pi - 2}}. \quad (F.0.6) \]
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