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Propositions
appended to the dissertation

Models of curves

The Birch and Swinnerton-Dyer conjecture & ordinary reduction

by Raymond van Bommel

1. The Birch and Swinnerton-Dyer has been numerically verified, up to a precision of 10 digits, provably only up to squares, excluding the computation of III, for the Jacobian of the hyperelliptic curve

$$y^2 + (x^6 + x^4 + 1)y = x^4 + x^2$$

of genus 5 over $\mathbb{Q}$.  

(Section 1.2, p. 4–6)

2. Let $E$ over $\mathbb{Q}(\sqrt[5]{5})$ be the elliptic curve given by

$$y^2 = x^3 + \sqrt[5]{5} \cdot x^2 - (5 + 3\sqrt[5]{5}) \cdot x + \sqrt[5]{5}(5 + \sqrt[5]{5}) .$$

Let $H$ and $H'$ over $\mathbb{Q}$ be the hyperelliptic curves given by $y^2 = x^5 - x^3 + \frac{1}{5} \cdot x$, and $y^2 = x^5 - 5 \cdot x^3 + 5 \cdot x$, respectively. Then the Birch and Swinnerton-Dyer conjecture holds for $E$ over $\mathbb{Q}(\sqrt[5]{5})$ if and only if it holds for the Jacobians $\text{Jac} H$ and $\text{Jac} H'$ over $\mathbb{Q}$.  

(Theorem 2.1.2, p. 28)

3. The Galois cover of $\mathbb{P}^1$ over $\mathbb{F}_p$ with group $D_n$ obtained by gluing $n$ copies of $\mathbb{P}^1$ in the shape of an $n$-gon, can be deformed into a smooth $D_n$-cover $C \to \mathbb{P}^1$ over $\mathbb{F}_p((t))$, such that $C$ is ordinary. Consequently, there exists a $D_n$-cover $C \to \mathbb{P}^1$ over $\mathbb{F}_p$, such that $C$ is ordinary.

(Proposition 3.5.5, p. 48)

4. For all positive integers $M$ and $g$, and all number fields $K$ except for the field $\mathbb{Q}$, 100% of hyperelliptic curves of genus $g$ over $K$ have at least $M$ primes of ordinary good reduction.

(Theorem 4.6.1, p. 56)

5. Let $C_{13}$ be the curve over $\mathbb{Q}$ given by the affine equation

$$x^3 y + x^3 - 2x^2 y^2 - x^2 y + xy^3 - xy^2 + 2xy - x - 2y^2 + 3y = 0 .$$

Then $C_{13}$ is non-hyperelliptic of genus 3 and its Jacobian is of algebraic rank 3. For this Jacobian, the Birch and Swinnerton-Dyer conjecture has been numerically verified, up to a precision of 30 digits, provably only up to squares, excluding the computation of III.

(joint with Holmes & Müller)
6. Let $q$ be a prime number and $d \geq 2$ be an integer. Then $\frac{q-1}{q}$ of monic polynomials of degree $d$ in $\mathbb{F}_q[x]$ are squarefree, or in other words, $\frac{1}{q}$ of these polynomials have discriminant 0.

7. Consider the following two ways to count (ordinary) hyperelliptic curves of fixed genus $g \geq 2$ over $\mathbb{F}_q$:
   - count each isomorphism class of $H/\mathbb{F}_q$ with weight $\frac{1}{|\text{Aut}(H)|}$;
   - count each squarefree homogeneous polynomial $f(x, z)$ of degree $2g + 2$, corresponding to the hyperelliptic curve $y^2 = f(x, z)$, with weight 1.

Then the second count is $(q^2 - 1)(q - 1)$ times the first count.

8. On the projective closure of the arithmetic surface given by
   \[ y^2 = p(x^3 + 1) \]
   over $\mathbb{Z}_p$, for $p > 2$ prime, the differential $\frac{dy}{y}$ generates the dualising sheaf, even though the function $\frac{1}{y}$ has a pole on the special fibre. \textit{(example by Maistret)}

9. Let $K$ be a field of characteristic unequal to 2 with absolute Galois group $G$. Let $g > 1$ be an integer, and let $f \in K[x]$ be a squarefree polynomial of degree $2g + 2$. Let $J$ be the Jacobian of the hyperelliptic curve given by $y^2 = f$. Then, as $\mathbb{F}_2[G]$-modules,
   \[ J[2](K) = \mathbb{F}_2\langle \alpha_i + \alpha_j : i, j \in \{1, \ldots, 2g + 2\} \rangle / \mathbb{F}_2\langle \alpha_1 + \ldots + \alpha_{2g+2} \rangle, \]
   where $\alpha_i$, for $i = 1, \ldots, 2g + 2$, are formal symbols corresponding to the roots of $f$ in $K$, and $G$ acts on them in the natural way.

10. Let $k$ be a positive integer. We consider all possible football matches in which $2k$ goals are scored in total. The number of such matches in which the end result is a draw equals the number of such matches in which the home team is never behind. (By a “football match” we mean the set of all intermediate pairs of scores that occur during the match.) \textit{(MOAWOA 2016, joint with Lyczak)}

11. For positive integers $S \leq L$, a subset of $M \subset \mathbb{Z}/L\mathbb{Z}$ is called safe, if for every $i \in \mathbb{Z}$, there is a $j \in \{i + 1, i + 2, \ldots, i + S\}$ such that $j \in M$. There exists an algorithm that given $S$ and $L$, computes the number of safe subsets of $\mathbb{Z}/L\mathbb{Z}$, modulo 123456789, in runtime $O(L)$. \textit{(BAPC 2016)}

12. An average day on any place on earth is 12 hours long. If this place is on the equator, then each day is 12 hours long. \textit{(joint with Koymans & Orecchia)}