COMMUNICATIONS FROM THE OBSERVATORY AT LEIDEN.

On the theory of the chromosphere and the corona, by F. Woltjer Jr.

1. The theory of the chromosphere has been the subject of a series of papers by E. A. Milne published during the last years. Milne has seen the possibility of a remarkable solution of the equations of mechanical equilibrium in a stellar atmosphere: a solution characterized by the balance of gravitation and radiation pressure at large distances from the stellar surface. The fact of this balancing changes the character of the density distribution thoroughly; the otherwise exponential decrease of density with increasing height above the stellar surface becomes far less rapid.

This circumstance is very important as it suits well the large height of the Ca+ chromosphere as derived from observation.

I think some grave objections may be brought forward, not against the theoretical possibility of the structure investigated by E. A. Milne, but against its actual occurrence.

Firstly I consider the intensity of the Ca+ light emitted by the lower regions of the chromosphere if constituted according to Milne's theory. As on this theory the optical depth of the Ca+ chromosphere measured along the solar radius is large, the same will be the case with the optical depth measured tangentially to the solar surface in the lower parts of the chromosphere. Hence if $E_\nu$ is the emission per unit solid angle and per unit volume of radiation of frequency $\nu$, and $x_\nu$ is the mass absorption coefficient and $\rho$ the material density, the intensity of the Ca+ light emitted by the lower parts of the chromosphere is $E_\nu/x_\nu \rho$. But $E_\nu$ is about equal to $2\pi x_\nu \rho I_\nu$ if $I_\nu$ is the intensity of the photospheric light. Hence the intensity of the Ca+ light is about $\frac{1}{3}$ $I_\nu$. If we take $I_\nu$ equal to $1/10$ of the intensity of the continuous solar spectrum we have the intensity of the Ca+ light emitted by the lower parts of the chromosphere about equal to $1/20$ of the intensity of the continuous solar spectrum of corresponding wave-length. Now this amount seems rather large if compared with experimental results of the same ratio in prominences. *)

Secondly the stability of the Ca+ chromosphere must be considered. Now Milne has shown **) how small velocities will be reduced by changes in the radiation force caused by the existence of these velocities and in this way he has demonstrated the stability of his chromospheric structure. However the time needed to reduce the velocities is long. If $v$ is the radial velocity of a group of particles the equation of motion is:

$$\frac{dv}{dt} = - \lambda v.$$

The quantity $\lambda$ is about $\frac{2 g}{3 c^2} \frac{h \nu}{kT}$ ***) the symbols have their usual meanings. Hence $\lambda = 4g/e$ and the time needed to reduce the original velocity in the ratio $1/e$ is $1/\lambda$, that is about 4 days. I think the dimensions of the chromosphere are too small to allow this reduction process to become effective.

2. The expulsion of atoms and electrons from the solar atmosphere has been investigated by many writers, who have considered the forces acting on the atoms and have connected the supposed phenomenon with terrestrial phenomena as aurorae and magnetic storms.

I think it worth while to consider the hypothesis that the chromosphere and the corona consist of these expelled atoms and electrons, which however need not all leave the sun once for all, but at least partly may fall back in the solar atmosphere. I proceed to discuss some points more in detail.

a. Optical depth. It is necessary to suppose the optical depth measured tangentially to the solar surface


**) M.N. 86, p. 578 etc.

***) Milne, l.c., p. 586.
to be very small in order to prevent a displacement of the chromospheric lines due to the motion of the gases. If we draw a line from the earth to a point of the chromosphere, along this line we first meet atoms that have a small velocity of approach to the earth, then atoms that have no velocity component in the direction of the earth, and at last atoms that have a small velocity of recession. If the optical depth is small, we see right through the chromosphere, hence no line shift is to be expected.

The small optical depth is also necessary to prevent the atoms from deepening the corresponding absorption line in the solar spectrum, as they are not allowed to produce a line shift in the absorption spectrum.

Hence contrary to the role the chromosphere plays in Milne’s theory, I suppose it to be not responsible for the blackness of the absorption lines in the solar spectrum.

b. Height of the Ca⁺ chromosphere. If Ca⁺ atoms are projected radially from the solar surface with large velocities, they will have travelled a considerable distance before they become ionised by the ultraviolet solar radiation. It is difficult to estimate exactly the time needed to ionise a Ca⁺ atom. Milne *) computes a value of about a hundred seconds, but considers it to be an underestimate. Hence accepting this number for the sake of numerical definiteness, only a mean velocity of 140 km/sec. would be necessary to have a Ca⁺ chromosphere with a height of 14,000 km. Thus the factor that determines the height of the chromosphere according to this point of view is the product of mean velocity and time needed for ionisation.

c. Connection with the corona. It is tempting to assume that the Ca⁺ atoms, once ionised, go on travelling outward on account of their large velocities and now give rise to the line spectrum of Ca⁺⁺, i.e. if Pannekoek’s **) suggestion shall appear tenable, to the well known bright line spectrum of the corona.

d. Density of the corona. Some years ago I have investigated the intensity distribution of the continuous coronal spectrum supposing it to be due to scattering by free electrons. At the same time I have computed the number of free electrons per unit volume necessary to produce the total intensity observed **). As regards this last point however it is to be remarked that the observations used determine the integrated intensity of a considerable range of wavelengths, hence include the line emission. If this line emission is preponderant the determination of the density of free electrons is spurious.

A simple computation allows some insight into this question. Suppose $N$ atoms per cm$^3$. A rather rough approximation gives the number of these atoms that are excited as $N e^{-\frac{kT}{k'}}$. Introducing the transition probability $A_{\lambda\rightarrow\lambda'}$, the total emission of unit volume is:

$$A_{\lambda\rightarrow\lambda'} h\nu N e^{-\frac{kT}{k'}} = A_{\lambda\rightarrow\lambda'} \frac{h\nu}{kT} e^{-\frac{kT}{k'}} N e^{-\frac{kT}{k'}}.$$

Take $A_{\lambda\rightarrow\lambda'} = 10^5$, $T = 5000^\circ$, then this number is equal to:

$$7 \times 10^{-5} N \frac{h\nu}{kT} e^{-\frac{kT}{k'}}.$$

As the total radiation of the corona is $10^{-5}$ in terms of the solar radiation we have the relation:

$$7 \times 10^{-5} N \frac{h\nu}{kT} e^{-\frac{kT}{k'}} \times \text{volume of corona} = 4 \times 10^{-6} \times 10^{33}$$

hence:

$$N \frac{h\nu}{kT} e^{-\frac{kT}{k'}} \times \text{volume of the corona} = 0.6 \times 10^{28}.$$