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On the continuous spectrum of the corona, by J. Woltjer Fr.

A theory of the structure of the corona must be able to account for the observed spectral distribution of energy generally as well as regards minor details. Generally speaking the light of the corona is white; however some deviations from the intensity-distribution of the mean solar spectrum appear possibly to exist.

In the region of shorter wave-lengths ($\lambda 3820-4840 \AA$) Ludendorff’s investigation of the corona spectrum at the eclipse of 1923 Sept. gives evidence of a close resemblance between the continuous spectrum of the corona and the mean solar spectrum as regards distribution of energy. On the other hand in the red end of the spectrum there appears to be evidence of greater relative intensity of the coronal spectrum, though the observations are not quite concordant.*)

The purpose of this note is the investigation of the distribution of spectral intensity required by the hypothesis of scattering by free electrons, already advanced by several authors. It appears that scattering by free electrons gives rise to a spectral intensity curve which, compared with that of the mean solar spectrum, is practically identical in the visible part of the spectrum, but rather much raised in the infra-red.

However, at present, observations are not consistent enough to decide if this is the actual case.**)

1. Consider a volume element of the corona at $P$ scattering energy received from the spherical solar surface with centre $C$ in the direction towards $E$. The energy scattered at $P$ per unit time and volume is equal to:

$$\frac{\cos \theta}{r^2} x_\nu I_C r_0^2 \sin \psi d\psi d\xi d\nu$$

(1)

as far as the solar surface at $Q$ is concerned; $\xi$ is the angle between the planes $CPQ$ and $EPC$; $x_\nu$ is the mass-coefficient of scattering, $\rho$ the density, $I_C$ the intensity of radiation of frequency $\nu$. The coefficient $x_\nu$ depends on the angle $\varphi$; according to the classical theory of electrons it is proportional to

$$I + \cos^2 \varphi.$$  (2)

Hence the radiation scattered in the direction to $E$ is proportional to:

$$r_0^2 \int_0^{2\pi} \int_0^{\hat{\xi}} \cos \theta \sin \psi (1 + \cos^2 \varphi) d\psi d\xi;$$

(3)

$\psi_o$ is the acute angle that satisfies the equation:

$$\cos \psi = \frac{r_0}{\Delta}.$$  (4)

As $\cos \varphi$ is equal to:

$$\cos \chi \cos \alpha + \sin \chi \sin \alpha \cos \xi$$

(5)

and

$$\cos^2 \varphi = \cos^2 \chi \cos^2 \alpha + \sin^2 \chi \sin^2 \alpha \cos^2 \xi +$$

$$+ \frac{1}{2} \sin 2 \chi \sin 2 \alpha \cos \xi,$$

(6)

the integration with regard to $\xi$ reduces the expression (3) to:

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\[ 2\pi r_o^2 \int_0^\infty \frac{I_v}{r^2} \cos \theta \sin \theta (1 + \cos^2 \chi \cos^2 \alpha + \frac{1}{2} \sin^2 \chi \sin^2 \alpha) \, d\phi. \quad (3a) \]

It may easily be demonstrated that the stream of radiation of frequency \( \nu \) passing through a unit surface-element at \( P \) perpendicular to \( CP \) per unit of time is equal to:

\[ (1 + \cos^2 \alpha) F_v + 4\pi \frac{r_o^2}{\Delta^2} (1 + \cos^2 \alpha) \int_0^{\infty} \frac{\psi}{r^2} \cos \theta \sin \frac{1}{2} \chi \sin \alpha \, d\theta + 2\pi \frac{r_o^2}{\Delta^2} \left( \frac{\sin^2 \alpha}{2} - \cos^2 \alpha \right) \int_0^{\infty} \frac{\psi}{r^2} \cos \theta \sin \frac{1}{2} \chi \sin \alpha \, d\theta. \quad (3b) \]

The two integrals are equal to:

\[ \int_0^{\infty} I_v \cos \theta \sin \alpha \, d\theta \]

\[ \left( \int_0^{\infty} I_v \cos \theta \sin \alpha \, d\theta \right) \sin \chi \sin \alpha \, d\theta; \quad (8) \]

these expressions reduce to the products of the integral

\[ \int_0^{\infty} I_v \cos \theta \sin \alpha \, d\theta \]

with

\[ \frac{r_o^2}{4 \Delta^2} \quad \text{and} \quad \frac{r_o^2}{\Delta^2} \]

respectively if we substitute

\[ \sin \chi = \frac{r_o}{\Delta} \sin \theta \]

and neglect the higher powers of \( \chi \).

Putting:

\[ I_v = a_v + b_v \cos \theta \]

we get:

\[ \int_0^{\infty} I_v \cos \theta \sin \alpha \, d\theta = \frac{1}{2} a_v + \frac{1}{2} b_v; \quad (13) \]

further:

\[ \frac{\Delta^2}{2\pi r_o^2} F_v = \int_0^{\infty} I_v \cos \theta \sin \theta \, d\theta = \frac{1}{2} a_v + \frac{1}{2} b_v. \quad (14) \]

\[ F_v \, dv = 2\pi r_o^2 \int_0^v \frac{I_v}{r^2} \cos \theta \cos \chi \sin \theta \, d\psi \, dv. \quad (7) \]

As \( F_v \) measures the energy of frequency \( \nu \) in the mean solar spectrum, its introduction into \( (3a) \) is appropriate; performing this operation \( (3a) \) appears in the form:

\[ F_v \left( 1 + \cos^2 \alpha \right) + \frac{r_o^2}{\Delta^2} \sin^2 \alpha \int_0^{\infty} \frac{\psi}{r^2} \cos \theta \sin \alpha \, d\theta \]

Hence the value of \( (3) \) is found to be equal to:

\[ F_v \left( 1 + \cos^2 \alpha \right) + \frac{r_o^2}{\Delta^2} \sin^2 \alpha \left( \frac{15 a_v + 8 b_v}{3 a_v + 2 b_v} \right). \quad (3c) \]

The following table *) contains the value of the factor depending on \( \nu \).

<table>
<thead>
<tr>
<th>( \lambda ) ( (\AA, \mu) )</th>
<th>( a_v )</th>
<th>( b_v )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3230</td>
<td>3860</td>
<td>4330</td>
</tr>
<tr>
<td>4560</td>
<td>5340</td>
<td>6040</td>
</tr>
<tr>
<td>6700</td>
<td>8660</td>
<td>10310</td>
</tr>
<tr>
<td>12250</td>
<td>0.474</td>
<td>0.48</td>
</tr>
</tbody>
</table>

If \( \frac{r_o}{\Delta} \) is nearly unity, the value \( (3c) \) is rather inaccurate; however, as we are only concerned with the order of magnitude of the variation with \( \nu \), this circumstance is not serious.

The dependence of the factor of \( F_v \) in \( (3c) \) on \( \nu \) involves a distribution of the intensity in the coronal spectrum deviating from that of the mean solar spectrum and superposed on any deviation arising from other sources. However the values of the preceding table show that between wave-lengths 3230 and 12250 \( \AA \mu \) the factor of \( F_v \) in \( (3c) \) is constant with regard to \( \nu \) within \( 0.444 - 0.416 \) to \( 1 + 0.416 \) and \( 0.020 \) of its value, corresponding to \( 0.02 \) magnitude.

2. The preceding section contains an analysis of the distribution of intensity in the spectrum of the

*) ABBOT's observations from MILNE, Phil. Trans. A 233, p. 209; the last value is a theoretical limit.
corona in so far the factor \( x_\nu \) in (1) is supposed to be independent of \( \nu \). In this section I consider the variation of \( x_\nu \) with \( \nu \), the variation of the coefficient of scattering with wave-length.

In first approximation this coefficient, according to the classical theory of electrons, is independent of wave-length; hence the approximately white colour of the corona.

In a second approximation we must take account of the reaction of the motion of the electron induced by the incident radiation on the process of scattering.

According to the modern theory of the process of scattering of light by free electrons the quanta of the incident stream of radiation undergo "collisions" with the electrons by which collisions they change their direction, energy and colour. The amount of energy scattered in a given direction is determined by a frequency function, that gives the number of collisions of specified type per unit volume and unit time.

W. Pauli Jr.,*) has derived the dependence on the radiation density of this frequency function for isotropic radiation; he showed that if this function were proportional to:

\[
\rho_\nu + \frac{\alpha^2}{8\pi h^3} \rho_\nu \rho_{\nu t}
\]  
(15)

the conditions of thermodynamical equilibrium would be satisfied; \( \rho_\nu \) is the density of radiation of frequency \( \nu \); \( \nu \) is the frequency of the incident radiation; \( \nu_t \) of the scattered radiation; the other constants have their usual meaning.

In the case of the solar corona we may neglect the difference between \( \nu \) and \( \nu_t \), but as the radiation field is not isotropic the expression (15) should be modified. We might suppose the probability of scattering in a direction specified with regard to the direction of the incident radiation of intensity \( I_\nu \) to be proportional to:

\[
I_\nu + I_\nu \int \varphi I_\nu d\omega;
\]  
(16)

\( \varphi \) being a coefficient depending on the relative orientation of \( I_\nu \) and \( I_{\nu t} \), the intensity of frequency \( \nu_t \) in a different direction; \( d\omega \) is an element of direction.

However, as a determination of the order of magnitude of the second order effect in the case of the solar corona is our principal purpose, we will use the expression (15) also for non-isotropic radiation and hence suppose the coefficient of scattering to depend on \( \nu \) by a factor:

\[
1 + \frac{\alpha^2}{8\pi h^3} \rho_{\nu t}
\]  
(17)

3. The density of radiation at the point \( P \) is equal to:

\[
2 \pi \frac{\rho_\nu}{c} \int_0^{\varphi} \cos \theta \sin \varphi \; d\varphi.
\]  
(18)

This expression is equal to:

\[
\frac{\pi}{2} \left\{ F_\nu + 4\pi \frac{r_\nu^3}{\Delta^3} \int_0^\varphi \cos \theta \sin \varphi \sec \chi \sin^2 \frac{\chi}{2} d\varphi \right\}.
\]  
(18a)

Approximating as in (8) we get:

\[
\frac{F_\nu}{c} \left\{ 1 - \frac{r_\nu^2 \frac{15}{2} a_o + 8 b_o}{3 a_o + 2 b_o} \right\}.
\]  
(18b)

Substituting a mean value from the table of p. 104 we get:

\[
\frac{F_\nu}{c} \left\{ 1 + 0.22 \frac{r_\nu^2}{\Delta^2} \right\}.
\]  
(18c)

Hence the influence of the second term may be neglected.

If \( T_\nu \) is the effective temperature of the sun we have:

\[
4\pi \Delta^2 F_\nu = 4\pi \frac{r_\nu^3}{c^2} \frac{1}{e(\hbar k T_\nu) - 1}.
\]  
(19)

Thus the coefficient of scattering depends on \( \nu \) according to (17) by the factor:

\[
1 + \frac{1}{4} \frac{r_\nu^3}{\Delta^2} \frac{1}{e(\hbar k T_\nu) - 1}.
\]  
(17a)

The spectrum of an element of the corona at a given spherical distance from the solar centre depends on the spectral distribution of intensity of light scattered by volume-elements corresponding to different values of \( \Delta \). Hence this spectrum depends on the distribution of density within the corona. Neglecting this difficulty by putting \( \frac{r_\nu}{\Delta} = 1 \) in (17a) we see that the distribution of intensity in the continuous coronaspectrum near the limb of the sun is equal to that of the mean solar spectrum multiplied for each frequency by the factor:

\[
1 + \frac{1}{4} \frac{1}{e(\hbar k T_\nu) - 1}.
\]  
(17b)

The following table contains the value of this factor for different wave-lengths with \( T_\nu = 5860^\circ \).

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*) W. Pauli Jr. Über das thermische Gleichgewicht zwischen Strahlung und freien Elektronen. Z. f. Ph. 18, p. 272 etc.
tron gives a distribution of intensity in the continuous spectrum of the corona that does not deviate to an appreciable extent from the mean solar spectrum in the visible part, but presents a large preponderance of the infra-red radiation relative to the corresponding region of the mean solar spectrum.

The negative charge of the corona caused by the presence of the free electrons must be compensated by positive ions and the question arises: what is the influence of the radiation scattered by these ions?

According to the classical theory of electrons the ratio of the coefficient of scattering by free electrons and by bound electrons is equal to:

\[ \left( \frac{\nu_0^2 - \nu^2}{\nu^2} \right) \quad (20) \]

\( \nu_0 \) being the frequency corresponding to the quasi-elastic binding of the electron. As \( \nu_0 \) will be far into the ultra-violet, the influence of the bound electrons is probably very small.

4. Combining the results of the preceding sections we reach these conclusions: scattering by free electrons gives a distribution of intensity in the continuous spectrum of the corona that does not deviate to an appreciable extent from the mean solar spectrum in the visible part, but presents a large preponderance of the infra-red radiation relative to the corresponding region of the mean solar spectrum.

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