COMMUNICATION FROM THE OBSERVATORY AT LEIDEN.

On the energy required for the ionisation of the interior of a star,
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The energy content of a star in the absence of rotation consists of contributions from various sources, that may be enumerated as follows:

- a. potential gravitational energy;
- b. heat energy;
- c. radiation energy;
- d. dissociation energy;
- e. energy from subatomic origin and unknown sources.

The contributions from the first three sources may easily be calculated for a star in radiative equilibrium; the values are respectively: *)

\[ a = \frac{-\frac{3}{2} f M^2}{r_o}; \quad b = \frac{\beta f M^2}{r_o}; \quad c = \frac{1}{2} (1-\beta f M^2) \]

\( M \) is the mass, \( r_o \) the radius of the star, \( f \) the constant of gravitation; \( \beta \) is EDDINGTON's constant: the proportion of gas-pressure to total pressure. The computation of the energy from subatomic origin is impossible; the supply from unknown sources, though certainly to be kept in mind, is unaccessible even for a rough estimation.

The purpose of this note is to consider the dissociation energy more in detail. The dissociation that comes into play in the interior of a star is not the ordinary chemical dissociation, but the breaking up of the atoms: the ionisation. F. H. SEARES **) considered the energy required to reach a degree of ionisation that seemed probable for the atoms inside a star. However, I think the more important point is the computation of the energy required for the change in ionisation connected with the transition of a star from one state to another.

1. I start from the well known dissociation formula,

\[ \frac{x_{r+1}}{x_r} \left( \frac{T_s}{T_i} \right)^{\frac{3}{2}} \approx \frac{C}{k T_s} \frac{1}{k T_i} \]

also used by E. A. MILNE *), to determine the state of ionisation for the interior of a star. This formula connects \( x_r \), the relative number of atoms, that have lost \( r-1 \) electrons, for two succeeding values of \( r \), with the partial electron pressure \( P_r \), the ionisation potential \( \chi_r \), the absolute temperature \( T \) and some constants by the equation:

\[ \frac{x_{r+1}}{x_r} P_r = T^{\frac{3}{2}} e^{-\frac{\chi_r}{kT}} \left( \frac{2\pi m_e}{h^2} \right)^{\frac{3}{2}} k^{\frac{5}{2}} \sigma_r \]

\( m_e \) is the mass of an electron, \( k \) and \( h \) the usual constants relating respectively to the mean kinetic energy of a molecule and the energy-quanta; \( \sigma_r \) is a number relating to the particular kind of atom in question, its symmetry number; \( q_r \) is the "statistical weight" of the condition of this atom. The formula supposes each neutral or ionised atom to have only one stationary state, neglecting the variety of states possible for the bound electrons.

I consider a star in two stages of its evolution, distinguished by the indices 1 and 2. Neglecting the difference between electron-pressure and gas-pressure (that certainly will be small) \( P \) varies for stars of the same mass as the fourth power of \( T \). So we get the formula:

\[ \frac{x_{r+1}}{x_r} \left( \frac{T_s}{T_i} \right)^{\frac{3}{2}} e^{\frac{\chi_r}{k T_s}} \left( \frac{1}{k T_i} \right) \]

Suppose the stages of evolution 1 and 2 to be so chosen that the corresponding effective temperatures have the ratio 1:2; then the corresponding radii will have the ratio 4:1 and the mean temperatures the ratio 1:4. So we may take in (2):

\[ \frac{T_s}{T_i} = 4 \]


***) Compare e.g. EDDINGTON, Zeitschrift für Physik, 7.
thus reducing (2) to:

$$\left(\frac{x_{r+1}}{x_r}\right) = \left(\frac{x_{r+1}}{x_r}\right) = 8 \cdot e^{\frac{1}{2} \frac{f}{kT_1}}.$$  

2. Before proceeding to a discussion of equation (4) we must specify the conditions in the interior of the star we consider. Suppose the total mass $M$ equal to 5 times the mass of the sun. The total luminosity $L$ may be calculated from Eddington's equations, that furnish the absolute magnitude $-0.33$ equivalent to:

$$L = 4.64 \times 10^{35} \text{ erg sec}^{-1}.$$  

The supposed value of the effective temperature $T_2 = 3000^\circ$ furnishes the value of the radius $r_0$ in state 1:

$$r_0 = 2.82 \times 10^{14} \text{ cm}.$$  

According to Eddington, we have:

$$1 - \beta = 0.32.$$  

The mean temperature follows from the formula:

$$T_m = \frac{m \beta f M}{R} r_0.$$  

$m$ is the molecular weight, $f$ the gravitation constant and $R$ the absolute gas-constant. Taking $m = 2.11$ we get:

$$T_m = 2.02 \times 10^6 \text{ degrees}.$$  

The density corresponding to $T_m$ follows from the formula:

$$\frac{\rho}{T_0} = \frac{am}{3R} \frac{\beta}{1 - \beta}.$$  

$a$ is the energy-density of black-body radiation for $T = 1^\circ$. Substituting numerical values we get:

$$\rho = 1.13 \times 10^{-3} \text{ gram cm}^{-3}.$$  

Hence the value of the gas-pressure:

$$\rho = 0.90 \times 10^{11} \text{ dynes cm}^{-2}.$$  

Substitution in (1) shows the relative concentration of succeeding stages of ionisation:

$$\frac{x_{r+1}}{x_r} = 2.2 \times 10^{13} \frac{\sigma_r e^{-0.057 \chi_r}}{q_r};$$  

here $\chi_r$ is measured by the ionisation-potential in volts. In this way Milne (l. c.) estimates the relative abundance of different ionisation-stages within the star. Take e.g. Fe; the potential required for the ionisa-

4. It might seem that the consideration of iron as a representative element is far too special. To meet this objection we must remember that although for the lighter elements the foregoing considerations do not apply because most of them already in state 1 consist of bare nuclei of the heavier elements will behave analogous to iron, only with a difference in the level of the electrons that are being detached by the transition from state 1 to state 2.