A modification of the method of Lehmann-Filhés for determining the orbit of a spectroscopic double star, by J. Woltjer Jr.

The problem of orbit determination for spectroscopic binaries essentially consists in the derivation of $e$ and $\omega$, eccentricity and angular distance from ascending node to periastron. If $u$ is the argument of the latitude, $T$ the period, $a$ the semi-axis major, $z$ the distance from the star to the tangential plane of the celestial sphere through the centre of motion and $i$ the inclination relative to this plane, the fundamental equation for the radial velocity relative to this centre of motion is:

$\frac{dz}{dt} = \frac{2\pi}{T} \frac{a \sin i}{\sqrt{1 - e^2}} \left[ \cos u + e \cos \omega \right].$

If $A$ and $-B$ are the maximum and minimum values of $\frac{dz}{dt}$ we have the relation:

$e \cos \omega = \frac{A - B}{A + B}.$

A second relation between $e$ and $\omega$ is derived from a consideration of the curve representing $\frac{dz}{dt}$ as function of $t$; this relation connects the values $z_1$ and $z_2$ of $z$ in the points $C$ and $D$ with the quantity $e \sin \omega$ by the equation:

$\frac{z_1}{z_2} = -\frac{\sin u_2 + e \sin \omega}{\sin u_2 - e \sin \omega},$

or solved with regard to $e \sin \omega$:

$e \sin \omega = \frac{z_1 + z_2}{z_1 - z_2} \sin u_2.$

As $\sin u_2$ is determined by the condition

$\frac{dz}{dt} = 0 \cos u_2 = -e \cos \omega$

we have

$e \sin \omega = 2 \frac{z_1 + z_2 \sqrt{AB}}{z_1 - z_2} A + B.$

Thus everything depends on the factor $\frac{z_1 + z_2}{z_1 - z_2}$ or on the ratio $\frac{z_1}{z_2}$. Now we may pass from the value of $\frac{dz}{dt}$ to the value of $z$ either by integration or by differentiation, as the second derivative of $z$ is connected by the equations of motion with the coordinates. Lehmann-Filhés chooses the first way and determines $\frac{z_1}{z_2}$ as the ratio of the surfaces $B B', C$ and $A A', D$. However the same ratio may be derived from the relation:

$\frac{d^2 z}{dt^2} = \frac{z_1}{r_1^2} \frac{z_2}{r_2^2} = -z_2^2 : z_1^2$

as results from the equations

$z = r \sin t \sin u \quad \sin u_2 = -\sin u_2,$

where $r$ denotes the radius vector.

As $\frac{d^3 z}{dt^3}$ is the tangent of the angle formed by the tangent at the curve with the axis of $t$ we have:

$\frac{d^3 z}{dt^3} = -\tan \alpha_2 : \tan \alpha_2$

from these relations we may derive $e \sin \omega$ by (6). Equations (10), (6) and (6) constitute our solution of the problem.