Table 2.
Corrections to be applied to the GC and FK3 for precession and motion of the equinox

<table>
<thead>
<tr>
<th></th>
<th>General Catalogue</th>
<th>Dritter Fundamentalkatalog</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \theta_1$</td>
<td>$\pm \frac{2}{11}^{a}0111$</td>
<td>$\pm \frac{2}{11}^{a}0010$</td>
</tr>
<tr>
<td>$-\Delta \epsilon - \Delta \lambda$</td>
<td>$\pm \frac{2}{11}^{a}0114$</td>
<td>$\pm \frac{2}{11}^{a}0010$</td>
</tr>
<tr>
<td>corr. to $\mu_{x}$</td>
<td>$\pm 9^{a}0040$ 90</td>
<td>$\pm 9^{a}0040$ 20 sin $\alpha$ $tg \delta$</td>
</tr>
<tr>
<td>corr. to $\mu_{y}$</td>
<td>$\pm 9^{a}0044$ $\cos \alpha$</td>
<td>$\pm 9^{a}0040$ $\cos \alpha$</td>
</tr>
</tbody>
</table>

still smaller 1). This makes it extremely unlikely that the coefficients $9^{a}0044$ and $9^{a}0040$ found in the fundamental catalogues could be due to systematic errors. The fault, then, must be with the direct determinations of $\Delta \epsilon$ or $\Delta \lambda$ 2).

K. Piliowski has expressed serious doubt about the correctness of such a conclusion 3). He argued that the necessity of introducing an empirical correction to the proper motions in right-ascention proves that it is impossible in practice to define an inertial system by means of the stars, as there might as well be systematic errors or motions equivalent to a rotation perpendicular to the equator, which could not be unravelled from the rotational motion of the stellar system. Referring to the above discussion and to previous articles we may now remark against this that, on the one hand, there is a sound a priori reason for expecting an especially large error common to all proper motions in right-ascention, while, on the other hand, the probability of other systematic errors giving rise to spurious rotations (either in the plane of the equator or in any other plane) has been shown to be extremely small.

If thus we adopt the point of view that the direct determination of $- \Delta \epsilon - \Delta \lambda$ is probably affected by considerable systematic error, the above determinations of $\Delta k$ and $\Delta n$ lead to the corrections shown in Table 2 above. As the mean epoch of the FK3 is around 1900, it follows that the FK3 positions for 1945, and thus also those given in the recent Almanacs, require corrections of $+^{a}008 -^{a}012$ sin $\alpha$ $tg \delta$ in $\alpha$, and $+^{a}18 \cos \alpha$ in $\delta$.

In practice it will sometimes be desired to apply to proper motions not only these precessional corrections, but also to eliminate galactic rotation. New tables for effecting both corrections to the GC simultaneously have been given by SMART 1); the constants used do not differ much from those derived in the present article.

It is a pleasure to thank Mr Peils for his co-operation. Most of the calculations on which the results of this Bulletin rest were made by him.

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Note about galactic precession, by A. J. van Woerkom.

In this note I have calculated the precession of the invariable plane of the planetary system, and also of the planets of Neptune and Pluto.

For this I take a co-ordinate system with the origin in the centre of gravity of the planetary system, and of which the directions of the axes are invariable. The $z$ axis is in the direction of the galactic pole and the $x$ axis in the direction of the rotation of the galactic system. It is supposed that the orbits of the planets are all in the same plane, the ecliptic. The following symbols will be used:

$m_{i}$, $a_{i}$, $n_{i}$, $e_{i}$ and $l_{i}$ are, respectively, the mass, semi-major axis, angular motion, angular momentum and mean longitude of the $i$th planet; $C$, $K_{e}$, $\frac{3 \pi}{2} \frac{\partial K}{\partial z}$, $\frac{\partial K}{\partial z}$, $\gamma$, $\Pi$, $E$ and $\frac{2 \pi}{3} - \alpha$ are, respectively, the angular mo-

1) Wilson and Raymond have shown that the value of $\Delta n$ derived from $\mu_{x}$ alone is closely comparable to that derived from $\mu_{y}$, which is an additional proof of its reality.

2) The possibility of serious errors in these quantities has been commented upon in previous articles (c.f. first footnote, p. 426, second column).


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1) M.N. 101, 40, 1941.

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From this, \( \tan \gamma \sin \Pi = \frac{c_x}{c_z} \) and \( \tan \Pi = -\frac{c_x}{c_y} \);
\[
\delta \Pi = \cos^2 \Pi \left( -\frac{\delta c_x}{c_y} + \frac{c_x \delta c_y}{c_y^2} \right),
\]
\[
\delta \gamma = \frac{\cos^2 \gamma}{\sin \Pi} \left( \frac{\delta c_x}{c_z} - \frac{c_x \delta c_z}{c_z^2} \right) - \tan \gamma \cos \Pi \delta \Pi,
\]
in which
\[
\delta c_x = \int \sum_i \left( y_i (K_x)_i - z_i (K_y)_i \right) dt
\]
\[
\delta c_y = \int \sum_i \left( z_i (K_x)_i - x_i (K_y)_i \right) dt
\]
\[
\delta c_z = \int \sum_i \left( x_i (K_y)_i - y_i (K_x)_i \right) dt,
\]
and
\[
(K_x)_i = \sin \alpha \left( -K_o - (y_i \cos \alpha + x_i \sin \alpha) \frac{\delta K}{\delta \alpha} \right) m_i
\]
\[
(K_y)_i = \cos \alpha \left( -K_o - (y_i \cos \alpha + x_i \sin \alpha) \frac{\delta K}{\delta \alpha} \right) m_i
\]
\[
(K_z)_i = -z_i \frac{\delta K}{\delta \alpha} m_i,
\]
while
\[
x_i = \cos (E + l_i) \cos \Pi - \sin (E + l_i) \sin \Pi \cos \gamma
\]
\[
y_i = \cos (E + l_i) \sin \Pi + \sin (E + l_i) \cos \Pi \cos \gamma
\]
\[
z_i = \sin (E + l_i) \sin \gamma.
\]
The secular part of the precession is
\[
\delta \Pi = + \sum_i m_i a_i^2 \left[ -2 \frac{\delta K}{\delta z} + \frac{\delta K}{\delta \alpha} \right].
\]

The following numerical values may be inserted:
\[
\gamma = 60^\circ 55, \frac{\delta K}{\delta \alpha} = (A - B) (3 A + B). \text{\(1\)}, \text{in which}
\]
\(A\) and \(B\) are Oort's constants, \(A = +0.018 \text{ km/sec.ps}\),
\[B = -0.008 \text{ km/sec.ps} \text{\(1\)},\text{ so that } \frac{\delta K}{\delta \alpha} \text{ becomes}
\]
\[1.3 \times 10^{-30} \text{ sec}^{-2}, \text{ while } \frac{\delta K}{\delta z} = 5.6 \times 10^{-30} \text{ sec}^{-2} \text{\(2\)}.\]

I thus find for the secular precession of the invariable plane \(\delta \Pi = -(3 \times 10^{-7})^2 \) per century. The effect is proportional to \(a_i^2\), so that the secular precession of the orbital plane of Neptune becomes \((10^{-6})^2\) per century, and for Pluto nearly \((2 \times 10^{-6})^2\) per century. All these values are evidently far beyond the limits of observation.

For Neptune the total shift in \(\Pi\) during the time that the solar system has existed is about \(3^\circ\), which is far smaller than the inclinations of the planetary orbits on the invariable plane. For the major planets from the Earth to Neptune the average without regard to sign of these inclinations is \(\pm 1^\circ\).

The fact that, after \(2 \times 10^7\) years, the orbital planes of Uranus and Neptune still coincide so closely with those of the other planets is in itself a proof that the effect of perturbations from the galaxy on the ecliptic cannot amount to more than about \((10^{-8})^2\) per annum. The above calculations were, however, carried out because the subject of galactic precession has recently been mentioned again in connection with determinations of galactic rotation \(3\).

I am most indebted to Prof. Oort, for his suggestion of the problem.

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1) C.f. p. 426 of this Bulletin.


3) C.f. P. van de Kamp in "Conference on the fundamental properties of the galactic system", Ann. New York Acad. of Sc. 42, Art. 2, 1941. I have only seen a set of "reprinted manuscripts", the original publication not being available in this country. The subject of galactic precession has previously been mentioned by Charlier in his memoir on "The motion and the distribution of the stars", Mem. Univ. of Cal. 7, 31, 1926. We are indebted to Prof. Lindblad for this latter reference.