ordinates has been applied to $\bar{m}$ so that the maximum and the minimum coincide. On the whole the agreement between the curves is satisfactory, although the difference just after the maximum seems incompatible with the accuracy of the curves.

I have also compared STEBBINS's G-curve ($\lambda_{\text{eff}} = \mu^{-0.57}$) with my photovisual light-curve in B.A.N. 10, 86, and again STEBBINS's curve was found higher than mine just after the maximum.

The lower curve in Figure 3 shows a comparison with STEBBINS's results of the light-curve derived from the photo-electric measures by GUTHNICK and SMART (B.A.N. 10, 89, 94). The effective wavelength of GUTHNICK and SMART's curve is $\mu^{-0.44}$.

The light-curve derived from STEBBINS's measures $\frac{1}{2}(V + B)$ has very nearly this same wavelength and is represented by open circles. The curves have been adjusted in phase and brightness in such a way that the rising branches and the maximum and minimum coincide. Again the agreement between the curves is satisfactory on the whole, though there seems to exist a small systematic difference on the descending branch in such a way that STEBBINS's light-curve is the brighter.

More photo-electric observations will be needed to settle the question whether there are real changes in the light-curve of $\delta$ Cephei or not.

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A redetermination of the mean radius of $\delta$ Cephei, by A. J. Wesseling.

A redetermination of the mean radius of $\delta$ Cephei has been made, in which the recent photo-electric measures by STEBBINS, discussed in the preceding paper, have been used. The result is $R = 52 R_0 \pm 2 R_0$ (m.e.). If this result is combined with that obtained in B.A.N. No. 368 we find as the best value now available: $R = 48 R_0 \pm 1 R_0$ (m.e.). A discussion is given of the principles of the method. The retardation of the phase of maximum brightness of the light-curves mentioned by STEBBINS is in agreement with these principles and has been predicted by A. VAN HOOF.

The determination of the mean radius of $\delta$ Cephei made in B.A.N. No. 368 has been repeated with the extremely accurate photo-electric data published by J. STEBBINS. STEBBINS's results were discussed in the preceding article. We will show below that these measures give additional support to the hypothesis underlying the determination.

As in B.A.N. No. 368 it is assumed that $\delta$ Cephei is at any moment a spherical star, the radius and surface-brightness (at some wave-length) of which vary periodically in time. Let us call this hypothesis "the pulsation hypothesis" and denote it by $(A)$.

The brightness of the variable at any moment is proportional to the surface-brightness and the area of the stellar disc. If brightness and surface-brightness are expressed in magnitudes (with an arbitrary zero-point), respectively denoted by $m$ and $\sigma$, we have:

$$m = \sigma + m_i$$  \hspace{1cm} (1)

where $m_i$ is the variation in magnitude the variable would show as a consequence of the varying size alone, thus if the surface-brightness were constant.

The formation of the curve $D$ from the radial-velocity curve by integration has been fully described in B.A.N. No. 368. The displacement-curve, describing the variation in kilometres in the radius is $pD$, where $p$ is a factor between $3/2$ and $4/3$, which depends on the degree of darkening towards the limb (compare B.A.N. No. 368).

The problem of determining the mean radius of $\delta$ Cephei is equivalent with the problem of separating the observed $m$ into $\sigma$ and $m_i$, with this difference that for the determination of the mean radius the coefficient of limb-darkening $(\beta)$ has to be known, whereas the latter problem is independent of the darkening.

On the other hand the coefficient of limb-darkening to be adopted, introduces through $p$ an error of only a few per cent in the result for the mean radius $R$.

If a value for $S = R_{\text{min}}/p$ is adopted, $m$, follows from:

$$m = -5 \log (S + D)$$  \hspace{1cm} (2)

The surface-brightness corresponding to this adopted value $S$ then follows from (1).

It is clear that no definite results for $\sigma$ and $m_i$ can be obtained without the introduction of a new element into the problem. This new element has been called "basic assumption" in B.A.N. No. 368.

In agreement with our discussion in the preceding article we formulate this hypothesis as follows: If at two phases the relative distributions of energy from $\mu^{-0.42}$ to $\mu^{-0.03}$ in the spectrum of $\delta$ Cephei are the same, then the absolute distributions are also equal.

We denote this hypothesis by $(B)$. In particular $(B)$ means that the surface-brightness at $\lambda_{\text{eff}} = \mu^{-0.56}$ (this is the effective wave-length of $\bar{m}$) is a single-valued function of $\epsilon$ (see preceding article).

Consider the plot of $\sigma$ against $\epsilon$ for some adopted value of $S$. Since both $\sigma$ and $\epsilon$ vary periodically, the result will be a closed curve in general, which corresponds to a double-valued relation between $\sigma$ and $\epsilon$.

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This is at variance with (B). $S$ is varied so long that the corresponding plot is reduced to a line of which each point is reached twice (from opposite directions) during a cycle. For the corresponding value of $S$ the relation between $\sigma$ and $\epsilon$ is single-valued in agreement with (B). Then $R_{\min} = \rho S$ and $\overline{R} = \rho (S + \frac{1}{2} \text{amplitude of } D)$.

The data that have been used are the curves $\overline{m}$ and $\epsilon$ derived in the preceding article from Stebbins's photo-electric observations and the curve $D$ as given in B.A.N. No. 368. The phases given there for $D$ have been diminished by $P \cdot 019$ to bring them in agreement with the phase used in the preceding article for $\overline{m}$ and $\epsilon$.

The best single-valued relation between $\sigma$ and $\epsilon$ was found to occur for $S = 25 \times 10^6$ km. The relation is shown in Figure 1 and is seen to be practically linear. We consider this linearity quite accidental and note only that for black bodies radiating according to Wien's law a linear relation does apply.

The points that correspond to the rising branch of the curve $\epsilon$ (phases between $P \cdot 300$ and $P \cdot 600$) are represented by open circles, whereas the rest is shown by dots. It is seen that no systematic differences exist between open circles and dots, in agreement with (B). The mean error of $S$ was found from a least-squares solution similar to the one made in B.A.N. No. 368, in which the relation between $\sigma$ and $\epsilon$ was assumed to be linear.

The results with their mean errors are:

$$S = 25 \pm 1 \text{ (in } 10^6 \text{ km as unit).}$$

Hence $\overline{R}/\rho = 26.2 \pm 1$ in the same unit.

With $\rho = 1.4$, corresponding to a coefficient of darkening $\beta = 2/3$, we find:

$$\overline{R} = (367 \pm 14) \times 10^6 \text{ km}$$

$$= 52R_\odot \pm 2R_\odot \text{ (m.e.)}$$

The difference between the present result and that obtained in B.A.N. No. 368 ($\overline{R}/\rho = 18.8$) is (also in $10^6$ km as unit): $74 \pm 14$ (m.e.). Though this is four times the mean error it should be noticed that the mean errors of both determinations are relatively uncertain as they were determined from only 25 residuals each. It seems therefore possible that the difference $74$ is actually three times the mean error. Although at the limit, it seems just possible that the divergence between the results is due to chance.

If the result of the present paper is combined with that obtained in B.A.N. No. 368 we find for the best value of the mean radius now available:

$$\overline{R} = 48R_\odot \pm 17R_\odot \text{ (m.e.)}.$$
In the preceding paper it was found that the various colours that may be formed out of the five magnitudes \( V, B, G, R \) and \( I \) are single-valued functions of one another. If the various colour-indices were double-valued functions of one another, this would be a fatal argument against \((B)\) as it has been used in B.A.N. No. 368. For if any \( \sigma \) were a single-valued function of some colour-index, it would necessarily be a double-valued function of another one and the use of \((B)\) would be meaningless.

It is true that the ultra-violet magnitude \( U \) was found to be exceptional in this sense that colour-indices based on it and another magnitude are not single-valued functions of \( \sigma \), the deviations being real and of the order of a few hundredths of a magnitude. We nevertheless consider \((B)\) to have sense if the surface-brightness which it claims to be a single-valued function of \( \sigma \) has an effective wave-length in the interval \( \nu'42 - 1\nu'03 \).

The ratio between simultaneous changes in surface-brightness and colour-index may be found from Figure 1, from which we find \( \frac{\Delta \nu}{\Delta \sigma} = 929 \). This corresponds to a ratio 1'93 if the colour-index is measured on the International Scale of colour-indices. There does exist as yet no theoretical argument why this ratio is found so much smaller than the ratio 4'0, which holds for black-body radiation and the effective wave-lengths we deal with. It has been remarked in B.A.N. No. 368 that ordinary dwarf stars show nearly the ratio valid for black-body radiation. Further investigations are necessary in order to decide whether we have here an argument in favour of or against the principles of the method.

Although nowhere in the present article it is assumed that \( \delta \) Cephei radiates as a black body, \((B)\) holds for black bodies. In B.A.N. No. 368 I have given an argument for \((B)\) for non-variable stars, which are comparable in density. If as regards \((B)\) a Cepheid may be considered as a non-variable star, the argument applies also to such a variable.

Instead of \( m' \) we might as well have made our analysis with any of the five magnitudes \( V, B, G, R \) or \( I \). It is clear that as a consequence of the single-valued mutual dependence of the colour-indices based upon them (see preceding article) the same result would necessarily ensue for \( S \).

Stebbins has remarked\(^1\) that the phases of maximum brightness of his six light-curves show an increase with the effective wave-length. From Stebbins's observations I find the following values:

\[
\begin{align*}
&\text{P}^{610}(c); P^{610}(U); P^{612}(V); P^{612}(B); P^{624}(G); P^{639}(R); P^{640}(I).
\end{align*}
\]

This phenomenon may be explained satisfactorily on the basis of \((A)\) and \((B)\). For if it is assumed that any surface-brightness in the interval \( \nu'42 - 1\nu'03 \) is a single-valued function of \( \sigma, \sigma_V, \sigma_B, \sigma_G, \sigma_R \) and \( \sigma_I \) attain their maximum value simultaneously at the phase of maximum of \( \sigma \). At the phase of maximum of \( \sigma \) and of the various surface-brightnesses, the radius and thus \( r_r \) are increasing. As is seen from (1), the maximum of the light-curve is later than that of \( \sigma \), which equals \( P^{610} \). The retardation will be greatest for the effective wave-length at which the corresponding \( \sigma \) varies least. This is in agreement with the expectation since the amplitude of \( \sigma \) diminishes with increasing wave-length.

It is interesting that this effect has been predicted before being observed by A. van Hoof\(^2\) by a reasoning resembling that given here.

**Correction to B.A.N. No. 368, by A. J. Wesselink.**

Professor A. van Hoof called my attention to the fact that in B.A.N. No. 368, p. 99, I have erroneously compared his result for the minimum radius with my result for the mean radius. The correct comparison is:

\[
\begin{align*}
\bar{R}/\bar{p} (\delta \text{ Cephei}) &= 18'4 \pm 4'0 \text{ (m.e.) (van Hoof)} \\
&= 18'8 \pm 1'6 \text{ (m.e.) (Wesselink)}
\end{align*}
\]

and not \( \bar{R}/\bar{p} = 17'4 \pm 4'0 \) (van Hoof), as quoted. The unit is \( 10^6 \) km.

\(^{1}\) Ap. J. 101, 1, 1944.

\(^{2}\) Koninklijke Vlaamse Acad. etc. Jaargang V, 12, 1943.