COMMUNICATION FROM THE OBSERVATORY AT LEIDEN.

Note on some quantities connected with the precessional motion,
by F. Weeder.

1. The quantities needed for the reduction of the mean co-ordinates of stars from one epoch to another in the general case depend on both epochs. In the limiting case the derivatives with regard to the time are generally distinguished by the adjective „annual“ ¹).

In the following note ANDROYER’S notation is used ²).

ANDROYER, in the quoted paper, develops the quantities referring to two epochs in powers of the interval of time between the two epochs. The meaning of the quantities used is seen from fig. 1, where $E_o$ and $A_o$ represent the poles of the ecliptic and the equator at the earlier epoch, here taken as the origin of time, and $E_t, A_t$ the same poles at the second epoch $t$.

Further:

$\varepsilon_o = \text{arc } E_o A_o \equiv \text{inclination of ecliptic at origin of time}$,

$k = \text{arc } E_o A_t \equiv \text{angle between the two ecliptics}$,

$180^\circ - \varphi = \text{angle } A_o E_o E_t, \varphi = \text{longitude of ascending node of ecliptic at second epoch on the ecliptic at the zero epoch}$,

$P = \text{the precessional constant as defined by Newcomb, so that the motion of } A_t \text{ is perpendicular to } A_t E_t \text{ and its velocity is } P \sin A_t E_t \cos A_t E_t$.

We have:

$$\sin k \sin \varphi = pt + p' t + p'' t$$

$$\sin k \cos \varphi = qt + q' t + q'' t$$

$$P = P_o + P_t t$$

The values of $\varepsilon_o, P_o, P_t, p, p', p'', q, q', q''$ are supposed to be known from observations and theory, and the other quantities are expressed in these nine fundamental ones.

The quantities introduced by ANDROYER are:

$\varepsilon = \text{arc } E_o A_t \equiv \varepsilon_o + at + a' t + a'' t$ inclination of \n
$\varepsilon = \text{arc } E_o A_t \equiv \varepsilon_o + b + b' t + b'' t$ (ecliptic)

$\psi = \text{angle } A_o E_o E_t = ft + f' t + f'' t$ lunisolar precession

$\chi = \text{Eo Ate} = g + g' t + g'' t$ planetary precession

$\omega = \text{Eo Ete} = q + h + h' t + h'' t$ precession

¹) Newcomb preferred the use of speed of ...”


$\omega - \varphi = \text{general precession in longitude}$

$\mu + \rho = 180^\circ, + u^' t + u'' t$ general precession in $A_R$.

$\mu - \varphi = \text{right ascension and declination of C, we have}$

$A_o C = A_t C = 90^\circ - \varphi$,

$A_o S = C A_t S = 270^\circ$.

The numerical value by these formulas is $+ 0^\circ.33$.

2. After these recapitulations I pass on to consider the apparent rotation of the celestial sphere. The point $C$ in fig. 2 represents the axis of rotation and is defined by the property that its co-ordinates in the two systems $(A_o, E_o)$ and $(A_t, E_t)$ are the same. $S$ being the point of intersection of the great circles $A_o E_o$ and $A_t E_t$ (fig. 1), $\gamma$ the angle of rotation and $\alpha_c, \delta_c$, the right-ascension and declination of $C$, we have:

$\gamma = \text{V}\psi^2 + h^2$,

$\cos \alpha_c = \frac{r^2 + s^2}{c}$

where

\begin{align*}
\frac{r^2 + s^2}{c} &+ \frac{r^2 + s^2}{c} - 2 \frac{r^2 + s^2}{24c} \\
\frac{3 s u'}{c^2} &+ \frac{s' u'}{c^2} - \frac{c d'}{c^2}
\end{align*}

$d = \tan^{-1} \left(-\frac{d'}{h'}\right)$

$\frac{3 s' u'}{c^2}$

$\frac{5 s u' + s' u'}{c^2} - \frac{c d'}{c^2}$

$\frac{3 s u'}{c^2}$
\[ d' = \frac{1}{2} u' \quad e' = e' \left( \frac{r d' + \epsilon' + \frac{3}{2} \epsilon^2}{2c} \right) \]

Taking 1850.0 as the origin of time, and Newcomb’s values of \( t_0, \alpha_0, \theta_0 \), we find

\[
\begin{align*}
\gamma &= 50245'33' \quad +111'095' \quad + \theta_0'08' \\
\delta' &= 66'28'49'06' + 389'42' \quad - \theta_0'35' \\
\alpha_0 &= 270' + 39'62' \quad + \theta_0'16' 
\end{align*}
\]

*) Astr. Papers of the Am. Eph. VIII, pages 72 and 75 (1898).

The time is here expressed in units of 1000 tropical years.

The coefficients of the highest powers of \( t \) are in all cases extremely small and may be safely omitted in all computations for intervals up to 200 years.

For the reduction between two epochs \( t = t_0 - \frac{1}{2} \theta \) and \( t' = t_0 + \frac{1}{2} \theta \) the formulas become

\[
\begin{align*}
(\gamma) &= (50245'33' + 222'19' \theta_0 + 0'32' \theta_0') \theta \\
(\delta') &= 66'28'49'06' + 778'84' \theta_0 - 1'88' \theta_0' + 0'12' \theta_0' \\
(\alpha_0) &= 270' + (39'62' + 0'33' \theta_0') \theta 
\end{align*}
\]