COMMUNICATION FROM THE OBSERVATORY AT LEIDEN.

Discussion of a list of stars with common motion published in the Greenwich Astrographic Catalogue, by D. Veldt.

In the fifth volume of the Greenwich Astrographic Catalogue (pages 145 to 148), which contains the proper motions of faint stars with declinations between $+64^\circ$ to $+72^\circ$, figures a list of stars having nearly equal and parallel motions. It cannot be judged whether there is any physical relation between the components of pairs or between groups of such stars, as long as no inquiry has been made into the number of pairs with apparently common motion that must be expected by reasons of probability. If this last number comes out to be nearly equal to the number observed, the list of stars with common motion, as published in the Astrographic Catalogue, is not very interesting.

In the following communication an account is given of an investigation that aims to compare the observed number of pairs with common motion with the expected number of spurious, non-physical pairs. This investigation leads to the conclusion that the occurrence of a relatively high number of stars with common motion is only accidental.

The list occurring in the catalogue mentioned above contains pairs and groups of stars, of which the proper motions differ no more than some thousandth parts of a second in size and some degrees in direction. The direction has been defined by the angle $\Phi$, the right-ascension of the point of intersection of the proper motion circle with the equator. When the mutual distance of the components of a pair is somewhat large, for instance 5 or 10 degrees, the equality of the angle $\Phi$ does not mean very much. The larger part (about $60^\circ/3$) of the pairs given have a distance of less than $2^\circ$. A considerable percentage has, however, too large a mutual distance for concluding anything about the parallelism of their motions on the ground of equal $\Phi$.

Therefore a more systematic definition of equality of motion is necessary.

In the first place the errors of observation bring about an inequality of the observed components of proper motions of the stars that move with common motion in space. Moreover, the separation of the components of a pair is a reason why equal motions are not projected as equal motions on the celestial sphere. Now the mean error of the proper motions, published in the catalogue, is $\pm 0075$. Thus the difference of the proper motions of the components

![Distribution of proper motions in the region right-ascension $0^h$ to $3^h$, declination $+64^\circ$ to $+72^\circ$. The curves are curves of equal density.](image)
of a pair has a mean error of \( \pm 0.010 \). With a separation of 2\( ^{\circ} \) the average difference to be expected between the proper motions of the components of a physical pair is of the order of 4\( ^{\prime \prime} \) of the proper motion.

Led by these considerations we have tried to answer the following question:

How many pairs are to be expected that have components with a mutual distance less than 2\( ^{\circ} \) and with proper motions \( \mu_\alpha \) and \( \mu_\delta \) that differ by less than 0.020? Of the zone +64\( ^{\circ} \) to +72\( ^{\circ} \) we have investigated the region between \( \omicron^b \) and \( \omicron^3 \) right-ascension, including the stars in the immediately neighbouring regions of right-ascension, so far as their distance to the boundaries of the region investigated is less than 2\( ^{\circ} \). The diagram on the preceding page shows the distribution of the proper motions of the stars in these regions. Each star has been represented by a point with co-ordinates \( \mu_\alpha \) and \( \mu_\delta \). The points lying in the densest regions (all proper motions smaller than 0.050, as well as those in the shaded part of the diagram) have been omitted. Their inclusion would rather have spoiled the argument.

In this manner there remained 123 stars between \( \omicron^b \) and \( \omicron^3 \) right-ascension and 28 stars in the neighbouring regions. These 28 stars have been represented by open circles in the diagram.

Curves of equal density have been drawn in the diagram. The inner curves have been constructed by picking out points that were centres of circles with the same radius containing equal numbers of points. The outer ones have been drawn less systematically, but taking into account the form of the inner curves.

For each of the regions of equal density the mean density was determined, that is, the number of stars in a square with sides of 0.040. For four succeeding regions these mean densities were 1.3, 3.5, and 13. With the aid of these numbers we interpolated for each star the number of neighbouring stars in a square with sides of 0.040 parallel to the arcs of \( \mu_\alpha \) and \( \mu_\delta \) around the proper motion of the star as centre.

Where such a square covered more than one density-region (and this happened nearly always, the density-gradient being very large) a determination was necessary of the fractions of the square that were covered by the different density-regions. With these fractions as weights the mean density in the square around each star could be calculated.

From the data published in Volume I of the Greenwich Astorgraphic Catalogue I have derived very rough positions of all stars considered. The right-ascension is accurate only to one or two minutes of time, the declination to one or two minutes of arc.

| No. | \( \alpha \) | \( \delta \) | \( \mu_\alpha \) | \( \mu_\delta \) | \( \rho \) | No. | \( \alpha \) | \( \delta \) | \( \mu_\alpha \) | \( \mu_\delta \) | \( \rho \) | No. | \( \alpha \) | \( \delta \) | \( \mu_\alpha \) | \( \mu_\delta \) | \( \rho \) |
|-----|--------|--------|--------|--------|-----|-----|--------|--------|--------|--------|-----|-----|--------|--------|--------|--------|
| 35  | 60  | 60  | 60  | 60  | 60  | 60  | 60  | 60  | 60  | 60  | 60  | 60  | 60  | 60  | 60  |
| 60  | 60  | 60  | 60  | 60  | 60  | 60  | 60  | 60  | 60  | 60  | 60  | 60  | 60  | 60  | 60  |
| 72  | 72  | 72  | 72  | 72  | 72  | 72  | 72  | 72  | 72  | 72  | 72  | 72  | 72  | 72  | 72  |
| 251 | 251 | 251 | 251 | 251 | 251 | 251 | 251 | 251 | 251 | 251 | 251 | 251 | 251 | 251 | 251 |
| 385 | 385 | 385 | 385 | 385 | 385 | 385 | 385 | 385 | 385 | 385 | 385 | 385 | 385 | 385 | 385 |
| 567 | 567 | 567 | 567 | 567 | 567 | 567 | 567 | 567 | 567 | 567 | 567 | 567 | 567 | 567 | 567 |
| 700 | 700 | 700 | 700 | 700 | 700 | 700 | 700 | 700 | 700 | 700 | 700 | 700 | 700 | 700 | 700 |
| 808 | 808 | 808 | 808 | 808 | 808 | 808 | 808 | 808 | 808 | 808 | 808 | 808 | 808 | 808 | 808 |
| 935 | 935 | 935 | 935 | 935 | 935 | 935 | 935 | 935 | 935 | 935 | 935 | 935 | 935 | 935 | 935 |
| 1061| 1061| 1061| 1061| 1061| 1061| 1061| 1061| 1061| 1061| 1061| 1061| 1061| 1061| 1061| 1061|

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In the preceding table \( \dot{p} \) shows the expected number of stars with proper motion components differing less than \( \dot{0.20} \) from those of the star in question. The sum of all the numbers \( \dot{p} \) for the 123 stars is 530, so that we might say that there should be in this region 265 different pairs whose proper motion components differ by less than \( \dot{0.20} \).

If the additional condition is imposed that the components of a pair must have mutual distances less than 2\( \circ \), this number is considerably reduced. The proportion of the surface of a circle with radius 2\( \circ \) to the total surface of the region between 0\( h \) and 3\( h \) right ascension augmented by the neighbouring regions containing the 28 stars mentioned above, is 0:08. Thus the number of pairs to be expected is 0:08 \times 265 = 21.2.

This number requires a small correction, as part of the circles of radius 2\( \circ \) for the stars with declinations between +64\( ^{\circ} \) and +65\( ^{\circ} \) and between +71\( ^{\circ} \) and +72\( ^{\circ} \) lie outside the zone. This can be taken into account by multiplying the numbers \( \dot{p} \) for the stars with declinations between +64\( ^{\circ} \) and +65\( ^{\circ} \) and between +71\( ^{\circ} \) and +72\( ^{\circ} \) by a factor \( \gamma / 3 \), and for the stars with declinations between +65\( ^{\circ} \) and +66\( ^{\circ} \) and between +70\( ^{\circ} \) and +71\( ^{\circ} \) by a factor \( \gamma / 5 \). After applying this correction we find that the average number of accidental pairs with components separated by less than 2\( \circ \) and with proper motion components \( \mu_x \) and \( \mu_y \) differing less than \( \dot{0.20} \) is 19.

Now the number of such pairs actually observed is 20, viz.:

\[
\begin{array}{cccccccc}
64 & 567 & - & 65 & 565 & 67 & 441 & - & 68 & 592 \\
65 & 100 & - & 66 & 36 & 68 & 379 & - & 69 & 324 \\
65 & 776 & - & 66 & 585 & 68 & 592 & - & 69 & 643 \\
66 & 36 & - & 67 & 88 & 69 & 188 & - & 69 & 324 \\
66 & 59 & - & 66 & 95 & 69 & 190 & - & 69 & 191 \\
66 & 334 & - & 67 & 313 & 69 & 995 & - & 70 & 1200 \\
66 & 355 & - & 67 & 390 & 70 & 1200 & - & 70 & 1259 \\
66 & 493 & - & 67 & 441 & 70 & 1496 & - & 71 & 1522 \\
67 & 390 & - & 67 & 494 & 70 & 1496 & - & 71 & 1896 \\
\end{array}
\]

(Of these pairs there is only one, viz. 69\( ^{\circ} \), 190 - 69\( ^{\circ} \), 191, of which the components have too small a separation to be an accidental pair).

Had the limiting difference in \( \mu_x \) and \( \mu_y \) for stars counted as a pair been taken \( \dot{0.10} \) instead of \( \dot{0.20} \) the number of pairs found would have been 6, whereas the expected number would have been decreased to 4:8.

As the number of pairs observed turns out to be equal to the number of spurious pairs expected, there seems to be no reason to assume that there is any physical relation between the components of the great majority of the stars with common motion as published in the Greenwich Astrographic Catalogue.