Do the galaxies expand with the universe?, by W. de Sitter.

In *B. A. N.* 200, p. 276, I derived the equations of motion (15) and (16) of a particle, or star, in the expanding universe. To the right hand members of these equations must be added the terms due to the mutual attraction of the stars. The equation for the radius vector thus becomes

\[
\frac{d^2r}{ds^2} = \left(\frac{d^2r}{ds^2}\right)_o + A \left(\frac{dt}{ds}\right)^2 \cdot r,
\]

where

\[
A = \frac{1}{R} \frac{d^2R}{dt^2}
\]

can with sufficient approximation be treated as a constant, and

\[
\left(\frac{d^2r}{ds^2}\right)_o = -Br
\]

is the ordinary gravitational term for motion inside a homogeneous sphere. Further \(dt/ds = 1\), since the pressure \(p_o\) can be taken to be zero. The complete equation for the motion of a star in the gravitational field of a homogeneous spherical galaxy is thus:

\[
\frac{d^2r}{dt^2} = -(B - A) r.
\]

According to the equation (15) of *B. A. N.* 200 the integral of areas is not affected by the expansion of the universe. Since \(A\) is much smaller than \(B\) (the ratio \(B/A\) being of the order of a million), the nature of the motion is not changed, the only effect being a slight diminution of the effective density. The orbits of the stars remain ellipses and the galaxies do not expand.

Actually \(A\) is not a constant, but increases slowly with the time. Consequentially the period of the star’s motion is slowly lengthened, the size of the orbit remaining the same. This change of the period is, however, very much slower than the expansion of the universe, and is practically entirely negligible.