COMMUNICATION FROM THE OBSERVATORY AT LEIDEN.

The longitudes of Jupiter's Satellites derived from photographic plates taken at Leiden in 1922, by W. de Sitter.

The photographic observations of Jupiter's Satellites, which were begun at the Cape Observatory in 1913 and continued at Greenwich when Jupiter's declination had become positive in 1916, were originally intended to cover a period of 8 years. Owing to various circumstances the observations at Greenwich were interrupted in 1917, and not continued after the series of 1918–19. Unfortunately, in consequence of my absence from Leiden on account of ill health, the fact that no observations were made at Greenwich in the years 1920 and 1921 did not come to my knowledge in time to arrange for the observations to be made at Leiden in these years. Under these circumstances it appeared desirable to take one further series of plates here at the opposition of 1922, although the southern declination of Jupiter made the epoch rather unfavourable, and also the focal length of the Leiden telescope is only three fourths of that of the telescopes used at the Cape and Greenwich.

The aperture of the telescope is 33 cm, and its focal length 516 cm, so that 1 mm on the plate is equal to 39°37'. The observations were all made by Dr. W. H. van den Bos, who secured in all 20 plates on 20 nights from March 13 to May 27. On plate 348, taken through haze on March 17, the images of the satellites are invisible, and plate 374 taken on May 23 proved to be so dark that it was not measurable. At first the weather was very bad, and only a few plates could be secured at long intervals. It was therefore decided to reject the four plates taken from March 13 to April 2. Of the remaining plates the numbers 361, taken through clouds, and containing no standard stars, and 373 were rejected. It would have been better to reject only Satellite II on the latter plate, the image of which is very near to the planet, so that there is danger for systematic error due to photographic effects, and to retain the other satellites. As however the solution had already been made excluding this plate, it was not thought necessary to alter the normals so as to include it.

The final results thus depend on 12 plates, taken on 12 nights from April 19 to May 27.

The standard stars used are:

A. G. Wien Ott. 4766 and 4785,
of which the adopted distance and position-angle are:

\[ s = 4075''59, \quad \rho = 88° 32' 14''. \]

The scale-value was also determined from the satellites for every plate, but not applied. The orientation from the satellites was also not applied to every plate individually, but a constant correction of \(-000 357\) was adopted, which has, however, no effect on the resulting values of the unknowns.

The measures were all made by Mr. G. Pels, who is also responsible for all computations and reductions. The average errors \(\sigma, \epsilon, \zeta\) were derived in the same way as for the Cape and Greenwich series (see B. A. N. 50, p. 62 and 61, p. 141). We find, in units of \(\frac{1}{5}\) of a micron

\[ \sigma : \quad \pm 2'8 \]
\[ \epsilon : \quad \pm 9'9 \]
\[ \zeta \text{ from } \sigma \text{ and } \epsilon : \quad \pm 5'5 \]
\[ \zeta \text{ from } I - II : \quad \pm 9'1 \]
\[ \zeta \text{ from plates measured twice:} \quad \pm 11'9 \]

We adopt again \(\zeta = \pm 7'5\), to which now corresponds the probable error of unit weight \(\rho = \pm '00048\). The comparison with the value of \(\rho\) derived from the residuals of Sol. I gives the plate error. We find;

<table>
<thead>
<tr>
<th>Series</th>
<th>(\rho) from residuals</th>
<th>(\rho) from plates</th>
</tr>
</thead>
<tbody>
<tr>
<td>1922</td>
<td>(\pm '00051)</td>
<td>(\pm '00017)</td>
</tr>
</tbody>
</table>

The plate error is very small indeed, which is very satisfactory, having regard to the unfavourable circumstances under which the plates were taken.

Jupiter being south of the equator, the plates were necessarily taken at rather large zenith-distances, between 54° and 58°. The effect of refraction consequently is much larger than in any of the other
series, and this series is an exceptionally favourable one to investigate the question in how far the colour of the satellites affects their measured position on the plate. If the constant of refraction for the satellite \( i \) requires the correction \( \delta k_i \), then the resulting effect on the coordinates of the satellite is

\[
\begin{align*}
\delta x_i &= -\delta k_i \tan \zeta \sin (P - q) = -h_x \delta k_i \\
\delta y_i &= \delta k_i \tan \zeta \cos (P - q) = h_y \delta k_i.
\end{align*}
\]

Since Jupiter's equator is always nearer to a horizontal than to a vertical position, the effect on the \( y \) coordinate is larger than on \( x \). We denote by \( \delta k_o \) the mean of the four values \( \delta k_i \), and we put

\[
\delta k_i - \delta k_o = \rho_i, \quad y_i - y_o = y_i, \quad y_{obs} - y_{tab} = n'.
\]

Further putting

\[
\delta \rho_i = k_i \pi_i, \quad \delta q_i = k_i \zeta_i,
\]

where \( k_i \) are the same numerical factors as are used for the corrections to the longitudes (see B. A. N. 50, p. 63), and

\[
\begin{align*}
b_i &= \frac{a_i}{l_i} k_i \sin i \cos B \cos \nu_i \\
c_i &= \frac{a_i}{l_i} k_i \sin i \cos B \sin \nu_i,
\end{align*}
\]

and denoting by \( B \) a constant correction to the adopted position-angle of the standard stars, then the equation of condition for the satellite \( i \) becomes, for a plate containing \( n \) satellites:

\[
\frac{n-1}{n} (b_i \pi_i + c_i \zeta_i) - \frac{1}{n} \sum_{j} (b_j \pi_j + c_j \zeta_j) + x_i B + h_y \rho_i = n_i,
\]

where \( j \neq i \) refers to the other satellites occurring on the plate, and \( \rho_i = \rho_i - \rho_o \), \( \rho_o \) being the mean of the values of \( \rho_i \) for the satellites occurring on the plate.

The coordinates \( y \) were measured in two positions of the plate, differing by 180°, and the tabular values were computed to five decimal places, as for the \( x \)'s. Normal equations were formed from the equations of condition, and three different solutions were made. It should be kept in mind that the correction \( B \) cannot be separated from the unknowns \( \pi_i \) and \( \zeta_i \) in a short series during which the jovicentric longitude of the earth does not change appreciably. Consequently, when the unknowns \( \pi_i \) and \( \zeta_i \) are introduced, the term with \( B \) must be transferred to the right hand member, and the unknowns determined as linear functions of it. A first approximation was first made, with \( B \) as the only unknown, which gave \( B = -0^\circ 000357 \). Then in sol. \( A \) the unknowns \( \pi_i \) and \( \zeta_i \) were introduced, in sol. \( B \) \( \delta B \) and \( \rho_i \), and in sol. \( C \) \( \pi_i \) and \( \rho_i \). The values of \( \pi_i \) and \( \zeta_i \) resulting from the solutions \( A \) and \( C \) were practically identical. They are of no interest for our present purpose, and need not be given here. The result of the solutions \( B \) and \( C \) as to the other unknowns was:

\[
\begin{align*}
\delta B &= +000005 \quad - \quad 000027 + 12 \delta B &= -00029 \\
\rho_o &= +22 + 16 + 72 + 19 \\
\rho_0 &= +17 + 21 - 03 + 19 \\
\rho_o &= -9 - 10 - 122 - 9
\end{align*}
\]

The agreement between the two solutions is all that could be expected. We adopt the values given in the last column above. The probable error of each of these may be taken to be

\[
\pm 00011.
\]

It is well known that satellite I is red. The order of brightness is visually III, I, II, IV and photographically III, I, IV. Beyond this order, which seems to be invariable, it is very difficult to make any exact statement regarding the brightness of the satellites, on account of their variability. We may perhaps adopt the following visual magnitudes:

\[
\text{Satellite:} \quad \text{I} \quad \text{II} \quad \text{III} \quad \text{IV} \\
\text{vis. mag.:} \quad 5'55 \quad 5'65 \quad 5'1 \quad 6'0
\]

The photographic magnitudes were determined by Mr. SHULT on one of the Leiden plates (356) with the microphotometer. Assuming the visual and photographic magnitudes to be the same for II and III, we find from his measures:

\[
\text{phot. mag.:} \quad 5'9 \quad 5'65 \quad 5'1 \quad 6'15
\]

The colour indices would thus be:

\[
\text{colour index:} \quad +15 \quad 0 \quad 0 \quad +15
\]

According to HERTZSPRUNG, A. N. 192, 4603, p. 309 (1912) the difference in the constant of refraction for photographic rays between the types AO and KO, corresponding to \( m = 50 \) in colour-index, is \( 0'18 \), which gives \( 0'00200 \) per magnitude. The differences of the constant of refraction for the different satellites can be derived from the values \( \rho_i \) found above, if we assume a value \( \delta k_i \) for any one satellite. Assuming \( \delta k_i = 0 \), we find:

\[
\delta k_i: \quad -00048 \quad 0 \quad 0 \quad -00028
\]

corresponding to the colour-indices:

\[
\text{col. ind.:} \quad +024 \quad 0 \quad 0 \quad +14.
\]

The agreement with the expected values is satisfactory.

We now return to the coordinates \( x \) and the corrections to the longitudes derived from them. After the first approximation \( O \) which was based on all
B. A. N. 62.

LEIDEN

plates, a solution was made (Sol. I) based on the 12 not rejected plates as explained above. In sol. II the constant correction $b = -000357$ to the orientation and the corrections $-k_x p_t$ were first applied to the right-hand members. It will be seen that the values of the unknowns in the two solutions are very nearly the same. The coefficients $k_x$ ranged from $+3.240$ to $+8.82$ for the different plates, the mean being $+5.2$. The effect of the introduction of the corrections $-k_x p_t$ should thus be a change in the constants $c_i$ of $d_i = -5.2 p_t$, which for the four satellites is:

$+00015$, $-00010$, $-00010$, $+00005$.

The values actually found are:

$+00014$, $-00009$, $-00011$, $+00006$,

in exact agreement with the expectation.

The values of $\lambda_i$ in the two solutions are practically identical. The series 1922 is a comparatively unfavourable one as regards the effect of differential refraction depending on the colour, owing to the large zenith-distances. We may thus safely conclude that also in the other series the effect of colour on the resulting corrections to the longitudes has been negligible.

The different solutions are given below.

Solution II was adopted. To the average residual of sol. II corresponds the probable error of weight unity

$\rho = \pm 00050$.

This is of the same order as for the Cape and Greenwich series, notwithstanding the fact that the focal length of the telescope is only three quarters of those used in the other series. Returning to the original unknowns, we find

$1922.35$

$\Delta \lambda = +000520 \pm 00057$

$\Delta \lambda = +0236 \pm 07$

$\Delta \lambda = +0279 \pm 023$

$\Delta \lambda = +0124 \pm 014$

The following tables are entirely similar to those in B. A. N. 50 and 61. In the column of the date are given the factors $k_x$ of the corrections $p_t$ for differential refraction. The plates were all taken by Mr. Van den Bos. The remarks printed in italics were made by him during the observation, the others by Mr. Pels during the measurement.

<table>
<thead>
<tr>
<th>Plate No.</th>
<th>Date 1922 G.M.T.</th>
<th>Satellite</th>
<th>$v$</th>
<th>$\rho$</th>
<th>$x$</th>
<th>$y$</th>
<th>$a$</th>
<th>$\chi$</th>
<th>$n$</th>
<th>Residuals</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>347</td>
<td>March 13 1922.35</td>
<td>I</td>
<td>58.4172</td>
<td>1.002061</td>
<td>-1.43143</td>
<td>+0.08361</td>
<td>-6.419</td>
<td>+0.15048</td>
<td>-0.00274</td>
<td>-0.00175</td>
<td>Slight haze at end</td>
</tr>
<tr>
<td></td>
<td>March 13 1922.35</td>
<td>II</td>
<td>120.8915</td>
<td>1.002281</td>
<td>-3.33096</td>
<td>+0.01721</td>
<td>+1.148</td>
<td>-1.72647</td>
<td>+0.2927</td>
<td>+0.3235</td>
<td>Very steady and transparent</td>
</tr>
<tr>
<td></td>
<td>March 13 1922.35</td>
<td>III</td>
<td>240.4989</td>
<td>1.000886</td>
<td>+3.26652</td>
<td>-2.50485</td>
<td>+4.535</td>
<td>+5.35193</td>
<td>+0.279</td>
<td>+0.4157</td>
<td>Images faint, especially of sat. IV.</td>
</tr>
<tr>
<td>350</td>
<td>March 22 1922.35</td>
<td>I</td>
<td>82.4200</td>
<td>1.000168</td>
<td>-1.90907</td>
<td>+0.04500</td>
<td>-6.222</td>
<td>-3.99207</td>
<td>+0.00024</td>
<td>-0.00128</td>
<td>Haze; sharp and steady.</td>
</tr>
<tr>
<td></td>
<td>March 22 1922.35</td>
<td>II</td>
<td>307.9074</td>
<td>0.993878</td>
<td>+3.16394</td>
<td>+0.11851</td>
<td>+1.18109</td>
<td>-1.0894</td>
<td>+1.5553</td>
<td>+0.4124</td>
<td></td>
</tr>
<tr>
<td></td>
<td>March 22 1922.35</td>
<td>III</td>
<td>331.5182</td>
<td>0.999290</td>
<td>+3.81425</td>
<td>+2.18852</td>
<td>+4.559</td>
<td>+3.8218</td>
<td>+0.5750</td>
<td>+0.7916</td>
<td></td>
</tr>
<tr>
<td>353</td>
<td>March 23 1922.35</td>
<td>I</td>
<td>289.0052</td>
<td>1.001524</td>
<td>+2.16362</td>
<td>+0.00577</td>
<td>-5.036</td>
<td>-2.45604</td>
<td>+0.00885</td>
<td>+0.00533</td>
<td>Steady and trans. parent.</td>
</tr>
<tr>
<td></td>
<td>March 23 1922.35</td>
<td>II</td>
<td>252.8949</td>
<td>0.995299</td>
<td>-2.05481</td>
<td>+1.59471</td>
<td>+2.8676</td>
<td>-1.75161</td>
<td>+0.297</td>
<td>+0.3216</td>
<td></td>
</tr>
<tr>
<td></td>
<td>March 23 1922.35</td>
<td>III</td>
<td>221.9988</td>
<td>0.998659</td>
<td>-0.87976</td>
<td>-2.93982</td>
<td>+6.343</td>
<td>-0.3281</td>
<td>+0.2106</td>
<td>+0.3216</td>
<td></td>
</tr>
<tr>
<td>356</td>
<td>April 19 1922.35</td>
<td>I</td>
<td>156.3471</td>
<td>0.995344</td>
<td>-1.35730</td>
<td>-0.08383</td>
<td>+4.424</td>
<td>-2.82618</td>
<td>+0.00500</td>
<td>-0.00102</td>
<td>Slight haze. I hear planet.</td>
</tr>
<tr>
<td></td>
<td>April 19 1922.35</td>
<td>II</td>
<td>340.7833</td>
<td>0.990279</td>
<td>+1.88833</td>
<td>+1.27487</td>
<td>-5.005</td>
<td>-0.95412</td>
<td>+0.363</td>
<td>+0.7650</td>
<td></td>
</tr>
<tr>
<td></td>
<td>April 19 1922.35</td>
<td>III</td>
<td>163.7336</td>
<td>0.990460</td>
<td>-2.29140</td>
<td>-2.58354</td>
<td>+5.542</td>
<td>-3.71948</td>
<td>+0.390</td>
<td>+0.6250</td>
<td></td>
</tr>
<tr>
<td>357</td>
<td>April 19 1922.35</td>
<td>I</td>
<td>537.4193</td>
<td>1.000342</td>
<td>+0.54656</td>
<td>+0.10636</td>
<td>-5.505</td>
<td>-3.39496</td>
<td>+0.00555</td>
<td>+0.00230</td>
<td></td>
</tr>
<tr>
<td></td>
<td>April 19 1922.35</td>
<td>II</td>
<td>254.7103</td>
<td>0.998680</td>
<td>+0.90444</td>
<td>+0.80666</td>
<td>+2.735</td>
<td>-0.08438</td>
<td>+0.383</td>
<td>+0.7650</td>
<td></td>
</tr>
<tr>
<td></td>
<td>April 19 1922.35</td>
<td>III</td>
<td>294.7996</td>
<td>1.000001</td>
<td>+3.35061</td>
<td>+0.79666</td>
<td>+1.430</td>
<td>+1.4127</td>
<td>+0.64</td>
<td>+0.3150</td>
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</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th>Plate No.</th>
<th>Date 1922</th>
<th>G.M.T. $\delta$&lt;sub&gt;x&lt;/sub&gt;</th>
<th>Satellite</th>
<th>$v$</th>
<th>$p$</th>
<th>$x$</th>
<th>$y$</th>
<th>$a$</th>
<th>$x_{obs}$</th>
<th>$n$</th>
<th>Residuals</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>359</td>
<td>April 20</td>
<td>10&lt;sup&gt;th&lt;/sup&gt; 20&lt;sup&gt;th&lt;/sup&gt; 33&lt;sup&gt;rd&lt;/sup&gt; + 67</td>
<td>II</td>
<td>3584226</td>
<td>0991699 + 018406 + 015263 + 579</td>
<td>-312010 + 00305 + 00665</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Slight haze, a little unsteady.</td>
<td></td>
</tr>
<tr>
<td>360</td>
<td>April 21</td>
<td>10&lt;sup&gt;th&lt;/sup&gt; 23&lt;sup&gt;rd&lt;/sup&gt; 37&lt;sup&gt;th&lt;/sup&gt; + 64</td>
<td>I</td>
<td>414205</td>
<td>0101076 - 122778 + 008852 - 465</td>
<td>-188463 + 000099 + 00059</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Second star exp. interrupted by haz.</td>
<td></td>
</tr>
<tr>
<td>361</td>
<td>April 22</td>
<td>9&lt;sup&gt;th&lt;/sup&gt; 23&lt;sup&gt;rd&lt;/sup&gt; 50&lt;sup&gt;th&lt;/sup&gt; + 90</td>
<td>I</td>
<td>844209</td>
<td>0999240 - 521934 + 7555 + 788</td>
<td>-746095 + 165 + 232</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Clouds, stars invisible.</td>
<td></td>
</tr>
<tr>
<td>362</td>
<td>May 2</td>
<td>9&lt;sup&gt;th&lt;/sup&gt; 26&lt;sup&gt;th&lt;/sup&gt; 50&lt;sup&gt;th&lt;/sup&gt; + 67</td>
<td>III</td>
<td>3021875</td>
<td>0993592 + 894640 + 14580 - 195</td>
<td>714093 - 642 - 561</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>No stars.</td>
<td></td>
</tr>
<tr>
<td>364</td>
<td>May 4</td>
<td>10&lt;sup&gt;th&lt;/sup&gt; 40&lt;sup&gt;th&lt;/sup&gt; 14&lt;sup&gt;th&lt;/sup&gt; + 34</td>
<td>II</td>
<td>1674686</td>
<td>0986003 - 507285 - 009777 + 531</td>
<td>216585 + 000332 + 00057</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Strong wind, haze, unsteady.</td>
<td></td>
</tr>
<tr>
<td>365</td>
<td>May 7</td>
<td>9&lt;sup&gt;th&lt;/sup&gt; 54&lt;sup&gt;th&lt;/sup&gt; 33&lt;sup&gt;rd&lt;/sup&gt; + 47</td>
<td>II</td>
<td>2787715</td>
<td>0991359 + 172879 + 16844 - 497</td>
<td>397743 - 157 - 87</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Haze, steady.</td>
<td></td>
</tr>
<tr>
<td>366</td>
<td>May 10</td>
<td>9&lt;sup&gt;th&lt;/sup&gt; 52&lt;sup&gt;nd&lt;/sup&gt; 43&lt;sup&gt;rd&lt;/sup&gt; + 42</td>
<td>II</td>
<td>3086551</td>
<td>0100349 + 184038 + 004670 - 265</td>
<td>189658 + 000042 + 00063</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Some clouds. Full moon near.</td>
<td></td>
</tr>
<tr>
<td>367</td>
<td>May 12</td>
<td>9&lt;sup&gt;th&lt;/sup&gt; 46&lt;sup&gt;th&lt;/sup&gt; 58&lt;sup&gt;th&lt;/sup&gt; + 42</td>
<td>II</td>
<td>3543586</td>
<td>0103659 + 053538 + 004970 - 527</td>
<td>156333 + 00135 + 00336</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Clouds.</td>
<td></td>
</tr>
<tr>
<td>368</td>
<td>May 15</td>
<td>8&lt;sup&gt;th&lt;/sup&gt; 39&lt;sup&gt;th&lt;/sup&gt; 11&lt;sup&gt;th&lt;/sup&gt; + 63</td>
<td>III</td>
<td>2359911</td>
<td>0096682 + 149813 + 006792 + 373</td>
<td>474206 + 000010 + 00045</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Unsteady.</td>
<td></td>
</tr>
<tr>
<td>369</td>
<td>May 16</td>
<td>9&lt;sup&gt;th&lt;/sup&gt; 60&lt;sup&gt;th&lt;/sup&gt; 10&lt;sup&gt;th&lt;/sup&gt; + 50</td>
<td>III</td>
<td>211906</td>
<td>0100145 + 103162 + 23655 + 566</td>
<td>218944 + 106 + 313</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Unsteady.</td>
<td></td>
</tr>
<tr>
<td>373</td>
<td>May 22</td>
<td>9&lt;sup&gt;th&lt;/sup&gt; 22&lt;sup&gt;nd&lt;/sup&gt; 14&lt;sup&gt;th&lt;/sup&gt; + 35</td>
<td>III</td>
<td>2264073</td>
<td>0100409 + 122892 + 007572 + 423</td>
<td>209566 + 000254 + 00089</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>II near planet.</td>
<td></td>
</tr>
<tr>
<td>379</td>
<td>May 24</td>
<td>9&lt;sup&gt;th&lt;/sup&gt; 20&lt;sup&gt;th&lt;/sup&gt; 32&lt;sup&gt;th&lt;/sup&gt; + 32</td>
<td>II</td>
<td>2728788</td>
<td>0103022 + 201622 - 001132 + 058</td>
<td>-177202 - 00093 - 00093</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Steady.</td>
<td></td>
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