THE DISTRIBUTION OF MASS IN M31

by M. Schmidt

A model of the distribution of mass in M31, which represents closely the radial velocities obtained in the preceding article, is presented. The total mass of the system is \(3.4 \times 10^{11}\) \(\odot\), half of which is at less than 13 kiloparsec from the axis of rotation. The mass-luminosity ratio has a practically constant value of about 27 between 3 and 16 kiloparsec from the centre. The mass-luminosity ratio for the entire nebula is 23 in the present model. Accurate observations of the distribution of light and colour over the entire nebula are urgently needed.

The rotational velocities, which were computed from the radial velocities of neutral hydrogen in M31 in the preceding article, were used in the present investigation to derive a model of the distribution of mass in the nebula. Previous models of the distribution of mass in M31 have been given by Wyse and Mayall (1942) and Schwarzschild (1954). These authors used rotational velocities based on radial velocities of hydrogen emission regions measured by Mayall (1950). Schwarzschild found that the data were not inconsistent with the assumption of a constant mass-luminosity ratio throughout the nebula.

The preceding article essentially gives the radial velocity of hydrogen clouds on the major axis of the nebula. On the assumption of circular motion in the plane of the nebula the rotational velocity is given by \(v_\text{rot} = V_\text{rot} \sin i\). We shall use the value \(i = 75.5^\circ\), which was adopted in the reductions in the preceding article. The actual value of \(i\) may be somewhat larger and will be discussed below.

The model which gave the best fit to the rotational velocities consists of two non-homogeneous spheroids. Surfaces of constant density in each of the spheroids are spheroidal surfaces with a constant axial ratio. This axial ratio was arbitrarily chosen as 0.07, which is the value found for the axial ratio in the Galactic System (Schmidt, 1956). The first non-homogeneous spheroid is composite. In the inner part, up to the surface \(a = 0.790\), the density is given by

\[
\rho_1 = -29.324 + \frac{30.888}{a},
\]

where

\[
a = \sqrt{\frac{2}{\sigma^2 + \left(\frac{z}{0.07}\right)^2}}
\]

in which \(\sigma\) is the distance to the axis and \(z\) the distance to the plane of the nebula. All densities are given in \(10^{-24}\) g/cm\(^3\). For points with \(a > 0.790\) the density in the spheroid is given by

\[
\rho_1 = \frac{3.807}{a^4}.
\]

The transition is such that density and density gradient are continuous across the surface \(a = 0.790\). The density of the second spheroid is

\[
\rho_2 = -0.494 + \frac{2.447}{a}.
\]

This spheroid is cut off at the surface \(a = 4.957\), where the density becomes zero. Formulae for potentials and forces due to non-homogeneous spheroids with density laws as used here are given in an earlier paper (Schmidt, 1956).

The apparent distance modulus (Baade, 1955, 1956) of M31 is \(24^m.25\). The total correction for absorption seems to be quite small according to Baade (private conversation). We shall assume a true distance modulus of \(24^m.0\), corresponding to a distance of 650 kpc, so \(i = 11.0\) kpc. The first two columns of Table 1 give the distance in the plane of the nebula from its centre. The next two columns give the observed rotational velocities and those computed from the model. The agreement is satisfactory. The total density in the plane is given in column 5. It should be noted that the densities are very sensitive to the assumed axial ratio of the mass distribution. The dependence is approximately such that density \times axial ratio = constant. The surface mass density \(\sigma_0'\), in a column perpendicular to the equatorial plane of M31 is given in column 6. This surface density is nearly independent of the assumed axial ratio.

The total mass of the model is \(M = 3.38 \times 10^{11}\) \(\odot\). The mass distribution given extends further outward than the region where the rotational velocities have been measured. Thus the densities outside the surface \(a = 2.5\) are not much more than a reasonable extrapolation. The mass outside this surface is 33% of the total mass. The seventh column of Table 1 gives the percentage of the total mass inside a cylinder with radius \(\sigma\). Half of the total mass is at less than 1.2 from the axis of rotation. The velocity of escape from the centre of the nebula is 703 km/sec; at \(z = \sigma\) it is 468 km/sec; at \(2z\) it is 368 km/sec.

The distribution of light along the major and minor axes of the nebula has been given by Redman and Shirley (1937). Their investigation seems to be the only one which gives the surface brightness on the major axis to a distance of \(a^2\). Their photographic magnitude system was calibrated by comparison with photoelectric work by Baum and Schwarzschild (1955) and Stebbins and Whitford (1934). Baum and Schwarzschild give magnitudes \(P_c\), an ultra-

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Table 1

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Legend: 1, 2, distance from the centre. 3, observed rotational velocity. 4, rotational velocity in the equatorial plane. 5, mass density in the equatorial plane. 6, mass surface density projected on the equatorial plane. 7, relative mass inside a cylinder with radius $\sigma$. 8, surface brightness on the sky. 9, axial ratio of the observed light distribution. 10, mass surface density projected on the sky. 11, mass-luminosity ratio.

Again Redman and Shirley's magnitudes are seen to be too bright. Application of a correction of $+1^m.406$, as found for the surface brightness, would give $4^m.54$. This would suggest that Redman and Shirley's total magnitude is too faint in their own system. Their total magnitude was obtained by integration of the surface brightness. Their values for the surface brightness in the outer parts of the nebula are undoubtedly too low, as indicated by the abrupt increase of the mass-luminosity ratio at $\sigma = 2''$ (last column of Table 1). The apparent total photographic magnitude will be taken to be $4^m.12$, equivalent to $L = 1.49 \times 10^{10}$. Combined with the mass of the present model this gives $M/L = 23$.

The mass-distribution and light-distribution given in columns 6 and 8 are not yet comparable as one is a surface density projected on the equatorial plane and the other is a surface density projected on the plane of the sky. The ratio between both quantities for a spheroidal distribution with a fixed eccentricity is equal to $b/a$, the axial ratio of the distribution projected on the sky. This follows from the fact that the same relative distribution is compressed from a projected area $\pi a^2$ to a projected area $\pi ab$. Although this rule is not exact for a distribution in which $b/a$ varies with distance, it is expected to give a good approximation. Column 9 of Table 1 gives the axial ratio $b/a$ of the light-distribution observed by Redman and Shirley. The mass surface densities projected on the sky (column 10) were obtained by dividing the surface densities in column 6 by $b/a$.

The mass-luminosity ratio obtained as the ratio $\sigma_{\text{m}}/\sigma_{\text{l}}$ is found in the last column of Table 1. The part of the nebula between $0^\circ.25$ and $1^\circ.50$, comprising about $50\%$ of the total mass, has a virtually constant mass-luminosity ratio of about 27. The ratio...