COMMUNICATION FROM THE OBSERVATORY AT LEIDEN.

Note on the mean parallax of faint stars, by A. S. Raimond.

The present paper contains a discussion of the mean parallaxes, derived from the proper motions of stars in the zone +45° of KAPTEYN's Selected Areas, as determined by Mr. O. J. Lee 1). By the kindness of Prof. Dr. P. J. van Rhijn I could start the computations with the \( \nu \) and \( \tau \) components (i.e. the components of the proper motion in the direction of the ant-apex and in the direction perpendicular to it), which had already been computed at the laboratory at Groningen. For the apex the following coordinates were adopted:

\[
\alpha_{1929} = 17^h 59^m \text{ and } \delta_{1929} = +31^\circ.
\]

The coordinates of the centres of the areas were taken from the Third Report on the Progress of the Plan of Selected Areas by Prof. Dr. P. J. van Rhijn, p. 36.

The general idea is that in the mean the \( \nu \) components are the reflection of the sun's motion and that in the mean the \( \tau \) components have to be zero.

Because the proper motions and parallaxes, as determined by Mr. O. J. Lee 1), are relative to the standard stars used, all computed mean parallaxes are also relative to these stars. No attempt was made to reduce them to absolute proper motions and parallaxes.

First the 24 areas have been arranged according to galactic latitude. 13 areas have a galactic latitude between 0° and 20° with a mean latitude of 11°; 4 areas lie between 20° and 40° with a mean of 34° and 7 areas between 40° and 90° with a mean of 61°.

In each area the stars were divided into 5 groups according to magnitude. For each area and each magnitude group the mean \( \nu \) component \( \langle \nu \rangle \), the mean \( \tau \) component \( \langle \tau \rangle \) and the mean of the \( \tau \) components without regard to sign \( \langle \tau^2 \rangle \) have been computed.

We have tried to utilize the mean \( \tau \) components for finding out if the proper motions are affected by a systematic magnitude error in the following way.

\[
\langle \frac{\tau}{\rho} \rangle = \frac{\sum \rho \langle \tau \rangle}{\sum \rho}
\]

where: \( h \) = the sun's velocity, \( \rho \) = the distance of a star of determined magnitude, \( \lambda \) = the distance from the apex to the centrum of the area, \( \rho \) = the weight.

The areas are arranged according to \( \chi \), i.e. the angle between the direction of the great circle from the centrum of the area to the north pole and the direction to the apex. If \( \chi \) lies between 150° and 210° the direction of the \( \tau \) component agrees approximately with the direction of increasing \( \alpha \); for the areas where \( \chi \) lies between 330° and 30° the direction of the \( \tau \) component agrees with the direction of decreasing \( \alpha \).

If, after reversing the sign of the mean \( \tau \) components in these latter areas, we combine them with the areas in the first group and compare the mean \( \tau \) components, averaged over all these areas, for stars of different magnitude, we can find out whether the \( \mu_2 \)'s are affected by a considerable magnitude equation which is systematic for all areas.

It appears that there is no appreciable error of this kind and in a similar way we may conclude the same for the \( \mu_2 \)'s.

There is no evidence that other systematic errors are existing in the proper motions considered.

The \( \langle \tau \rangle \) have been used as a measure for the average deviations to be expected in the \( \nu \) components, and from it I derived factors by which the numbers of stars have been multiplied to reduce these numbers in each magnitude group and for each galactic latitude to the same unit of weight. The unit of weight adopted corresponds to an average deviation of \( \pm 0.0100 \).

A first solution was made:

\[
\langle \frac{\tau}{\rho} \rangle = \frac{\sum \rho \langle \tau \rangle}{\sum \rho}
\]

The results are given in Table 1.
A second solution was made by means of the formula:

\[
\frac{\nu}{\sin \lambda} = a + b \frac{m}{m}
\]

By the method of least squares \( b \) was computed for each galactic latitude. When all magnitude groups are considered we find:

\[
\begin{array}{ccc}
\text{average} & \text{gal. lat.} & b \\
11 & -"0026 \pm "0006 & (m. e.) \\
34 & 42 & 18 \\
61 & 56 & 24 \\
\end{array}
\]

If I omit the brightest stars (i.e. the first magnitude group only) I find:

\[
\begin{array}{ccc}
\text{average} & \text{gal. lat.} & b \\
11 & -"0017 \pm "0007 & (m. e.) \\
34 & 037 & 21 \\
61 & 100 & 36 \\
\end{array}
\]

The rapid increase of \( b \) with galactic latitude is somewhat surprising.

Dr. P. van de Kamp finds \(^1\) for the visual magnitude \(11^m.5\) (which is about comparable with the average magnitude of the stars investigated in this note) the following values of \( b \):

\[
\begin{array}{ccc}
\text{average} & \text{gal. lat.} & b \\
6^o & -"0032 & \\
14 & 40 & \\
28 & 36 & \\
72 & 77 & \\
\end{array}
\]

From these values he finds for an average latitude of 30°:

\[
b = -"00405 \pm "0004 \text{ (p. e.)}.
\]

The agreement is satisfactory.

In Table 1 I have also indicated the mean trigonometric parallaxes for each magnitude group; these were combined for all galactic latitudes with the same relative weights as were used for combining the \( v \) components. To compare the mean secular parallaxes with the mean trigonometric parallaxes we have to divide the first by \(4.10 \) (supposing the sun's velocity to be independent of the magnitudes of the stars and equal to 195 km/sec).

For all galactic latitudes combined we thus find

\[
\text{the following comparison between the two series of mean parallaxes (both relative to the same standard stars).}
\]

<table>
<thead>
<tr>
<th>( m )</th>
<th>mean secular parallax</th>
<th>mean trigonometric parallax</th>
</tr>
</thead>
<tbody>
<tr>
<td>9'5</td>
<td>(+&quot;0027)</td>
<td>(+&quot;0116)</td>
</tr>
<tr>
<td>11'1</td>
<td>(+02)</td>
<td>(+053)</td>
</tr>
<tr>
<td>12'7</td>
<td>(+07)</td>
<td>(+051)</td>
</tr>
<tr>
<td>12'9</td>
<td>(-05)</td>
<td>(+041)</td>
</tr>
<tr>
<td>13'8</td>
<td>(-06)</td>
<td>(+022)</td>
</tr>
</tbody>
</table>

It is highly improbable that the large deviations may be explained by a different value of the sun's velocity for stars of different magnitude. It seems more probable that they are due to a small magnitude error in the trigonometric parallaxes.

It is a little surprising why the mean trigonometric parallaxes do not become negative for the faintest stars. To find out the reason of this unexpected fact, I have chosen the two magnitude groups which contain the largest number of stars, namely the group with \( m=11^m.51-12^m.51 \) and that with \( m=12^m.51-13^m.51 \). These two groups have been subdivided with respect to total proper motion. For these sub-groups I computed the mean trigonometric parallaxes for each galactic latitude and combined them with the relative weights already mentioned. The results are as follows:

\[
\begin{array}{ccc}
\text{m} & \text{m} & \text{m} \\
11^m.51-12^m.51 & 12^m.51-13^m.51 \\
\mu = "000-"010 & +"0054 & +"0025 \\
\mu = "010-"020 & +55 & +37 \\
\mu = "020-"030 & +21 & +85 \\
\mu \geq "030 & +70 & +60 \\
\end{array}
\]

There appears to be no variation with the total proper motion. The large values of the mean trigonometric parallaxes for the stars with a total proper motion between "000 and "010 are very surprising, because among these stars are most of the standard stars. One should expect to find for these sub-groups a mean trigonometric parallax which is nearly zero.

To clear up this point I computed the mean trigonometric parallax of all stars with a total proper motion between "000 and "010 which have not been used as standards by Mr. Lee. Then I find for the two magnitude groups considered:

\(+"0111\) and \(+"0034\).

The differences between these values and those in the first line of the above table would seem to indicate that for the final selection of the comparison stars other circumstances beside proper motion and magnitude have played a part and that these circumstances have worked in such a way that the observed parallaxes

\(^1\) P. van de Kamp: De zonsbeweging met betrekking tot apparent zwakke sterren, p. 29.

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### Table 1.

<table>
<thead>
<tr>
<th>magnitude group</th>
<th>galactic latitude $0^\circ - 20^\circ$</th>
<th>galactic latitude $20^\circ - 40^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean secular par.</td>
<td>mean trigon. par.</td>
</tr>
<tr>
<td>$m \leq 10.50$</td>
<td>+0.012 ± 0.003 (m. e.)</td>
<td>+0.007</td>
</tr>
<tr>
<td>$m = 10.51 - 11.51$</td>
<td>+0.002 ± 0.007</td>
<td>+0.034</td>
</tr>
<tr>
<td>$m = 11.51 - 12.51$</td>
<td>+0.004 ± 0.009</td>
<td>+0.055</td>
</tr>
<tr>
<td>$m = 12.51 - 13.51$</td>
<td>-0.017 ± 0.008</td>
<td>+0.042</td>
</tr>
<tr>
<td>$m \geq 13.51$</td>
<td>-0.021 ± 0.015</td>
<td>+0.022</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>magnitude group</th>
<th>galactic latitude $40^\circ - 90^\circ$</th>
<th>all galactic latitudes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean secular par.</td>
<td>mean trigon. par.</td>
</tr>
<tr>
<td>$m \leq 10.50$</td>
<td>+0.033 ± 0.080 (m. e.)</td>
<td>+0.030</td>
</tr>
<tr>
<td>$m = 10.51 - 11.51$</td>
<td>+0.012 ± 0.003</td>
<td>+0.031</td>
</tr>
<tr>
<td>$m = 11.51 - 12.51$</td>
<td>-0.018 ± 0.0045</td>
<td>-0.0023</td>
</tr>
<tr>
<td>$m = 12.51 - 13.51$</td>
<td>-0.0044 ± 0.0046</td>
<td>+0.035</td>
</tr>
<tr>
<td>$m \geq 13.51$</td>
<td>-0.022 ± 0.0142</td>
<td>-0.0237</td>
</tr>
</tbody>
</table>

### Table 2.

<table>
<thead>
<tr>
<th>magnitude group</th>
<th>galactic latitude $0^\circ - 20^\circ$</th>
<th>galactic latitude $20^\circ - 40^\circ$</th>
<th>galactic latitude $40^\circ - 90^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean secular par.</td>
<td>mean trigon. par.</td>
<td>number of stars</td>
</tr>
<tr>
<td>$m \leq 10.50$</td>
<td>-0.005 ± 0.008 (m. e.)</td>
<td>+0.007 ± 0.0134 (m. e.)</td>
<td>-0.001 ± 0.0288 (m. e.)</td>
</tr>
<tr>
<td>$m = 10.51 - 11.51$</td>
<td>+0.011 ± 0.0035</td>
<td>+0.027 ± 0.0145</td>
<td>+0.0019 ± 0.0059</td>
</tr>
<tr>
<td>$m = 11.51 - 12.51$</td>
<td>+0.008 ± 0.0085</td>
<td>-0.010 ± 0.0200</td>
<td>-0.001 ± 0.0200</td>
</tr>
<tr>
<td>$m = 12.51 - 13.51$</td>
<td>-0.0050 ± 0.0133</td>
<td>+0.0041 ± 0.0084</td>
<td>-0.0132 ± 0.0191 (m. e.)</td>
</tr>
<tr>
<td>$m \geq 13.51$</td>
<td>+0.032 ± 0.0047</td>
<td>+0.0047 ± 0.0042</td>
<td>+0.0120 ± 0.0147</td>
</tr>
</tbody>
</table>

1) $m$ = mean international photographic magnitude.

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of the stars selected as standards are systematically smaller than those of the stars of the same proper motion which were not selected.

Finally I have tried to use the colour-indices, as determined by the late J. A. Parkhurst 1); in order to see how the mean parallax for stars of determined magnitude varies with the colour-index 5 sub-groups were formed within each magnitude group. Table 2 shows the results of this attempt. No conclusions can be drawn from them. In Table 3 some results have been combined.

There may be some evidence of a variation, but for the present we must conclude that this material is too small to give reliable results.

The whole work was done under the direction and with the assistance of Dr. Oort, to whom I wish to express my warmest thanks for his valuable help. I am much indebted also to Prof. Van Rhijn for allowing me to use his computations.

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1) Publ. of the Yerkes Obs. Vol. IV, Part VI.

<table>
<thead>
<tr>
<th>colour-index group</th>
<th>$m \leq 10^\circ 50$</th>
<th>$m = 10^\circ 51 - 12^\circ 51$</th>
<th>$m \geq 12^\circ 51$</th>
</tr>
</thead>
<tbody>
<tr>
<td>col. ind. $\leq 00$</td>
<td>$-00006 \pm 00052$ (m. e.)</td>
<td>$-0021 \pm 00020$ (m. e.)</td>
<td>$-0001 \pm 00020$ (m. e.)</td>
</tr>
<tr>
<td>col. ind. $= 00 - 30$</td>
<td>$+0186 \pm 00045$</td>
<td>$-0001 \pm 00012$</td>
<td>$-0017 \pm 00012$</td>
</tr>
<tr>
<td>col. ind. $= 30 - 100$</td>
<td>$+0084 \pm 00059$</td>
<td>$+0116 \pm 00016$</td>
<td>$-0022 \pm 00013$</td>
</tr>
<tr>
<td>col. ind. $\geq 100$</td>
<td>$+0188 \pm 00055$</td>
<td>$-0026 \pm 00021$</td>
<td>$-0030 \pm 00013$</td>
</tr>
</tbody>
</table>

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