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**Author:** Säterskog, K.W.P.
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Summary

Generic interacting fermions at finite density in two dimensions are described by Fermi liquid theory (FLT) and can be studied using perturbation theory in the interaction strengths. Coupling such a theory to a massless boson however results in a non-Fermi liquid that can not be studied using conventional methods. These theories show up in particular when studying quantum critical metals, metallic systems in the vicinity to a quantum critical point (QCP). Quantum critical points are known to exist in fermionic systems so there is general need for understanding of such systems. Most interestingly—though not conclusively—existence of a QCP has been suggested as a mechanisms for strange-metallicity and high-$T_c$ superconductivity.

In this thesis we study a two-dimensional spherical Fermi surface (FS) coupled to a massless boson corresponding to a QCP order parameter at zero momentum. We extend the theory to have $N_f$ flavors of fermions and a global $U(N_f)$ flavor symmetry group. The core of this thesis is that in the $N_f \to 0$ limit followed by a low-energy limit we need not rely on perturbation theory, but can non-perturbatively calculate the zero temperature fermion two-point function and the fermion density-density correlation function.

Both quantities indeed show non-Fermi liquid behavior. The pole in the two-point function gets broadened into a fractional power, $(\omega - \omega_c)^{-1/2}$, with a branch cut resulting in a continuum in the spectral function. This completely removes the discontinuity of the momentum distribution function that is found in Fermi liquids. The dispersion becomes non-monotonic, with the singular point of the two-point function crossing $\omega = 0$ at three momenta. This results in three species of low-energy excitations and a splitting of the FS. The central FS has a negative Fermi velocity equal to minus the unrenormalized boson velocity, independent
of the UV Fermi velocity.

The density-density correlator shows oscillations at momentum $2k_F$, however they die off exponentially in the separation as opposed to the power-law decay of a Fermi liquid.

A drawback of performing the $N_f \to 0$ limit first is that we are still unable to study important effects in the IR. We therefore also study the double limit $N_f \to 0$ with $N_f k_F$ constant. Some of the fermion loop diagrams survive this limit but are still amenable to a non-perturbative treatment. The non-monotonicity of the two-point function disappears and the IR is characterized by a $\omega^2 \sim k_F^3$ scaling, however, the two-point function is distinct from the RPA form. While this result is not necessarily similar to the finite $N_f$ case, it shows that the results obtained in the $N_f \to 0$-first case are specific to that limit.

An interesting finding is that the perturbative expansions of $N_f = 0$ fermion $n$-point functions have finite radii of convergence. This is not common for quantum field theories. However, it is not completely unexpected in our case since the same thing has been found for two-dimensional Fermi-liquids \[3\]. The radius is set by the energy scale so perturbation theory can nevertheless not be used to access the IR directly. We do not know whether this convergence is specific to $N_f = 0$ or if it survives to finite $N_f$. We do present a heuristic argument in Section 1.4.6 why the radius of convergence in $N_f$ is not larger than 1.

Lastly we take a step back and consider the theory of a FS interacting with a massless boson, in a more arbitrary approximation than the $N_f \to 0$ limit, but such that the boson still receives no corrections from the fermion. In addition to the $N_f \to 0$ limit this contains theories in the matrix large $N$ limit and generalizations of these theories. The symmetry properties of this class of theories is studied at the point where the UV Fermi velocity and the unrenormalized boson velocities are equal. It is found that the non-Fermi liquid behavior found in the $N_f \to 0$-first limit is rather general in such theories and is proven to exist whenever perturbation theory in the coupling constant has a finite radius of convergence.