The distribution of the apparent Major Axes of the Binary stars with known orbits, by E. A. Kreiken.

**Introduction.** In the Monthly Notices of November 1927, LXXXVII, page 101, the space distribution of the major axes of the binary stars with known orbits was discussed. As the sign of \(i\), the inclination of the orbital plane, is generally unknown, we had to take into account two possible directions of these axes. In order to discriminate between the true axis and the spurious one, the sense of the rotation of the companion round the main star was taken into consideration.

It appeared that if the sign of the rotation was taken equal to case \(L\) the major axes corresponding to this rotation showed a strong tendency to concentrate towards a point with the galactic coordinates \(\lambda = 320^\circ; \beta = 0^\circ\). On the other hand a very improbable distribution was obtained if the rotation was taken equal to \(R\) (in case \(L\) the galactic longitude of the companion increases when it passes between us and the main star).

So we concluded, that it is probable that case \(L\) actually occurs. If this be true, not only the distribution of the true major axes should deviate from the accidental one, but also that of the apparent major axes, which are given by direct observations. It will be evident, that if the true axes were all pointing towards one and the same point of the celestial sphere, the same would also be the case with the apparent major axes.

Each apparent axis defines a great circle and the different circles, each corresponding to a binary, will then intersect each other in one and the same point. It is of importance to note that this should be so, whatever the sense of rotation may be. From the different diagrams in the paper quoted before, we see that the true major axes have a large spread round the main direction and so the great circles will not intersect in one point, but we must expect that the intersection points will converge towards one definite point of the sphere.

It is very apparent that, if this latter truly occurs, this would be a strong proof of the correctness of the views expressed in the previous paper.

I have investigated the question in the following way. In the first place we determined from the orbits with well defined apparent major axis i.e. from those pairs, which have a large apparent axis and a large excentricity, the direction towards which the intersection points are clustering. This problem is almost identical with the problem of determining the Solar Apex from the position angles of the proper motions of the stars. This problem has been treated by Argelander. Comparing the observed position angles with those derived from an approximate apex Argelander derives the following equation:

\[
(q - \phi_0) \sin \Delta = [\cos \delta \sin D_0 \cos (z - A_0) - \sin \delta \cos D_0] \cos D_0 \cdot dA + \cos \delta \cdot \sin (z - A_0) \cdot dD
\]

In this equation \(A_0\) and \(D_0\) denote the coordinates of the approximate apex, \(\Delta\) the angular distance of the stars from this apex, \(z, \delta\) the coordinates of the star, \(q\) the observed position angle of the proper motion, \(\phi_0\) the computed one. Finally \(dA\) and \(dD\) denote the corrections which must be applied to the approximate apex in order to find the true one.

At first sight it might appear that Argelander's equation is very suitable for our problem, but his equation is fundamentally based on the assumption, that the differences \(q - \phi_0\) are small and this condition will surely not be fulfilled in the present problem. This appears at once from our diagrams (I.e.).

Therefore I have applied a slight modification of Bessel's method. By the position angle of the apparent major axis a great circle is defined. We computed the pole of each, always using the one situated in the direction \((q - 90^\circ)\). If there is a certain apex these poles should not be distributed arbitrarily over the celestial sphere, but should show a tendency to concentrate towards a great circle, the pole of which co-incides with the apex. So in the first instance our
problem is to determine this circle. Mathematically it is defined by the condition
\[ \Sigma d^2 = \text{minimum}, \] (1)
where \( d \) is the distance from the pole of an apparent axis to the circle.

In my solution I confined myself to those stars for which we may expect the position angle of the apparent major axis to be well determined i.e. the binaries with a large apparent axis and a large excentricity.

From the list of van den Bos (B. A. N. Vol. III, 101, page 159) the two following groups of double stars were chosen

1\(^{a}\) those with \( a > 0^\circ.400; \ e > 0.200 \) and \( \sin \Delta > 0.50 \),
\( \Delta \) being the distance to the approximate apex

2\(^{a}\) those with \( a > 0^\circ.700; \ e > 0.200 \) and
\( 0.50 > \sin \Delta > 0.30 \)

These limits are rather arbitrarily chosen, but give some confidence in the observed position angle of the apparent axis.

**The fundamental equation.**
We started from the approximate apex \( L_o = 320^\circ; \ B_o = 0^\circ. \) The coordinates of the true apex will be indicated by \( L = L_o + dL \) and \( B = B_o + dB. \) The distance of a pole to the great circle derived from the approximate apex is called \( d_o, \) the distance to the correct circle is indicated by \( d = d_o + \delta d. \)

If \( A \) and \( D \) indicate the galactic coordinates of the pole of an apparent axis, the distance \( d \) is found from the equation
\[ \sin d = \sin D \sin B + \cos D \cos B \cos (A - L). \] (2)

For this equation we write
\[ \sin\delta d = \sin D \sin B + \cos D \cos B \cos (A - L)\]
\[ \sin\delta d = \sin D \cos B_o \cos (A - L_o) + \cos D \cos B \cos (A - L) - \cos D \sin B \sin (A - L) dL. \]

After expanding the different terms into series and omitting the quadratic terms this reduces to
\[ \sin\delta d = \sin D \sin B + \cos D \cos B \cos (A - L_o) + \sin D \cos B_o \sin B \sin (A - L) - \cos D \sin B \sin (A - L) dL. \]

For \( B_o \) we insert the value \( B_o = 0^\circ \) and for \( \sin D \sin B + \cos D \cos B \cos (A - L_o) \) we write \( \sin d_o. \)
Then we obtain
\[ \delta d = \frac{\sin D}{\cos d_o} \sin B + \frac{\cos D}{\cos d_o} \sin (A - L_o) dL. \] (3)

So the distance \( d = d_o + \delta d \) will be equal to
\[ d = \frac{\sin D}{\cos d_o} \sin B + \frac{\cos D}{\cos d_o} \sin (A - L_o) dL \]
\[ + \text{arc sin} \left( \frac{\cos D}{\cos (A - L_o)} \right) \]

In this way a value \( d \) is found for each pair and from (1) we now obtain our fundamental equation
\[ \Sigma d^2 = \Sigma \left[ \frac{\sin D}{\cos d_o} \sin B + \frac{\cos D}{\cos d_o} \sin (A - L_o) dL \right]^2 = \text{minimum} \] (5)

From the equation (5) the values of the correction terms \( dB \) and \( dL \) are found by least squares.

In this way we obtained the values \( dB = -9^\circ; \ dL = -8^\circ. \) The probable error in each coordinate being
\[ \pm \frac{22^\circ.5}{\sqrt{73}} \]

A second solution, in which all binaries with \( a > 0^\circ.400 \) and \( e > 0.200 \) were used with weights equal to \( \sin \Delta, \) gives almost the same values. This was to be expected as only a very small number of stars was excluded.

With a view to the rather scanty material no high accuracy can be claimed for the values derived here.

In Table 3 the different quantities used here have been enumerated.

Table 1 contains the distribution of the distances \( d \) on which our solution is based. From this table fig. 1 was constructed. The observed distribution is indicated by a full drawn line, the accidental one by a dotted curve. From this table we see that the distribution curve of the distances \( d \) is not quite symmetrical. This is due to a number of binaries which were already noted in the \( M. N., \) and which have a major axis nearly perpendicular to the plane of the galaxy. At present there is however no apparent reason why they should be excluded from our solution.

| Table 1. Distribution of the distances \( d \) used for determining the Apex. |
|-----------------|-----------------|
| +90  +60  +40  +20  0  -20  -40  -60  -90  |
|  5   10   9   14   17   10   8   0   |

| Table 2. Distribution of the differences \( \varphi - \psi. \) |
|-----------------|-----------------|
| +90  +60  +40  +20  0  -30  -60  -90  |
|  16   11   17   18   13   8   |

It is of interest to see how the directly observed quantities, i.e. the apparent position angles of the apparent major axes, are affected by the apex.

Therefore I have also given in Table 3 the observed position angles and the ones computed from the
The corrected apex (6). The differences $\varphi - \psi$ are always reduced to an angle between $-90^\circ$ and $+90^\circ$.

$$B = -8^o, \quad L = 312^o \text{ (galactic)} \quad (6)$$

$$\alpha = 260, \quad \delta = -45$$

The distribution of the differences $\varphi - \psi$ is examined in Table 2, from which fig. 2 is obtained.

Again there is a marked asymmetry in the distribution curve, due to the same reason as before.

The maximum near the zero value, however, is very well defined.

We conclude that there is an evidence that the distribution of the position angles of the apparent major axes is not an accidental one. The apex derived from this distribution almost coincides with the apex of the true axes derived in the $M.N$. This makes it probable that the way in which we discriminated between the true axes and the spurious ones was correct.

**Table 3. List of binary stars used in the present investigation.**

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<th>$\alpha$</th>
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