COMMUNICATION FROM THE OBSERVATORY AT LEIDEN

Star-streaming among faint low-latitude stars investigated according to the dispersion method, by C. H. Hins and A. Blaauw.¹)

The dispersion method for the derivation of the direction of the principal axis and the ratio of the axes of the velocity ellipsoid is applied to the combined Radcliffe and Pulkovo proper motions in the northern Selected Areas and to the McCormick proper motions, in the zone below ± 20° galactic latitude. The galactic longitude of the vertex direction is found to be 323°5 ± 1°8 (m.e.). This confirms the coincidence with the direction of the galactic centre, found in an earlier application of the method to the Radcliffe proper motions, and indicates that the vertex deviation exhibited by bright stars is but a local phenomenon. The discrepancy between this result and those derived by means of Schwarzschild's methods is probably due to the ambiguity in the determination of the position angle of the direction toward the vertex in each area, which is inherent in the latter methods. Schwarzschild's methods are discussed in some detail and applied in a modified form to the same proper motions. It appears necessary to eliminate the influence of the accidental errors in the determination of both the ratio of the axes and the vertex longitude. The results are rather inaccurate, but seem to be no longer contradictory to the results of the dispersion method. The ratio of the axes is found to be 0.49 ± 0.04 (m.e.).

Our knowledge of the velocity distribution of the faint stars is based mainly on the proper motions in the northern Selected Areas, measured at the Radcliffe Observatory²), and in the regions covered by parallax plates measured at the Leander McCormick Observatory³). The former were first studied by Knox-­Shaw and Scott Barrett, the latter by van de Kamp and Vyssotsky, who also included results from the Selected Areas into their discussion. These investigations have shown that star-streaming is a general phenomenon within the limits of the observable region, that is up to about 1500 parsecs from the sun. Both investigations were based on the ellipsoidal hypothesis introduced by K. Schwarzschild, who developed two procedures for the derivation of the ellipsoidal elements, the "curve-fitting" and the "automatic" method ⁴). These methods start from the distribution of the position angles of the total proper motions. Knox-Shaw and Scott Barrett used the automatic method, whereas van de Kamp and Vyssotsky in the treatment of all spectral types together used the curve-fitting method. One of the important results of these investigations was, that the deviation of the vertex, found in investigations of the brighter stars, appeared to occur also at these greater distances, particularly in low galactic latitudes.

A reinvestigation of the Selected Area proper motions according to a quite different method was published by Blaauw in 1939 ⁵). (This paper will be referred to as paper I.) The dispersion of the components of the linear tangential velocities parallel to the galactic plane was computed for each area from the dispersion of the proper motion components in the direction of galactic longitude ⁶). The observed dispersion had to be corrected, of course, for the accidental errors of the proper motions and for the influence of the solar motion. From this linear velocity dispersion in areas at different galactic longitudes the direction of the component of greatest dispersion was

¹) In 1942 I suggested to Dr. Hins to determine the vertex and the ratio of the axes of the velocity ellipsoid from the Radcliffe and Pulkovo proper motions in the Selected Areas, which had recently been combined at Leiden. The suggestion was followed by frequent discussions. Because of Dr. Hins' service in the army some parts of the investigation remained unfinished when he departed for the Leibang Observatory in 1946. Dr. Blaauw undertook to finish it.

²) The entire treatment of the so-called "dispersion" results was made by Dr. Hins, who also discovered the reason why previous investigators of faint stars in low latitude had found deviating values for the longitude of the vertex. Dr. Hins added a new, and more reliable, determination of the vertex from the position angles of the Radcliffe-Pulkovo motions.

³) The satisfactory solution of discrepancies remaining in similar results from the McCormick material is due to Dr. Blaauw. He extended the discussion on the derivation of the velocity ellipsoid from position angles in the way indicated in the last two sections. He is largely responsible for these sections, as well as for the final redaction of the article. J. H. Oort

⁴) Radcliffe Catalogue of Proper Motions, 1934.
⁸) In connection with the use of the term dispersion it may be remarked that for technical reasons indicated below the quantities actually computed are not the mean squares, but the mean residual proper-motion components without regard to sign and the mean speed in the corresponding directions (see the next section). If quantitative values are given the term dispersion in this paper always refers to the root of the mean square.
solved by least squares, and this is identical to the vertex direction. Additional information on the third axis of the velocity ellipsoid was derived from the dispersion of the proper motions in galactic latitude. The most surprising result was the low longitude of the vertex direction, $322^\circ 7 \pm 3^\circ 4$ (m.e.), which coincides approximately with the direction of the galactic centre. This coincidence is what we would have to expect according to simple dynamical considerations. No explanation of the large difference between this result and the longitude found by Schwarzschild's methods was attempted at that time. It was, however, considered as an indication that the latter results were not conclusive.

A new occasion to reinvestigate the elements of the velocity distribution occurred when the proper motions in the Selected Areas had been improved by the combination of the Radcliffe material with the proper motions measured at the Pulkovo Observatory 1). The consequent reduction of the accidental errors is of considerable importance in the study of the velocity distribution of the faint stars, because the mean errors of the proper motions are of the same order of magnitude as the dispersion due to the peculiar motions.

The present paper first deals with an investigation of these improved proper motions in the zone between $\pm 20^\circ$ galactic latitude according to the dispersion method. The method is also applied to the McCormick proper motions in the same latitude zone. The vertex longitude found confirms the low value found in paper I. Some remarks are made on the procedure of Schwarzschild's methods, and it is pointed out that the ambiguity in the results for the individual areas, which is inherent to these methods, may have led to an error in the vertex longitude. Next, the automatic method is applied with some alterations (by which a.o. the ambiguity just mentioned is avoided) to the same material as used for the dispersion method. The results are rather inaccurate on account of the uncertain but necessary corrections for the accidental errors of the proper motions, which do not only affect the ratio of the axes of the ellipsoid but also appear to vitiate the vertex longitude. It can be stated that the results are no longer contradictory to those from the dispersion method, and we conclude that, as far as these faint, distant stars are concerned, the vertex direction coincides with the direction to the galactic centre.

Formulæ and notations used in the dispersion method.

The same notations are used as in paper I;

$v$ is the observed component of proper motion in galactic longitude or latitude;

$n$ is the number of stars in the area or group of areas;

$|\mu|^* = |v - \bar{v}| / n / (n - 1)$ is the average residual component of proper motion. The factor $1 / n / (n - 1)$ takes account of the statistical error of $\bar{v}$;

$r$ is the probable error of the proper motions in the area;

$|\mu|$ is the average residual component of proper motion corrected for the accidental errors by the formula

$$|\mu|^* - \bar{v}^2 = |\mu|^* - \bar{v}^2,$$

where

$n = 1.07rv$. The empirical factor 1.07 is explained on page 307 of paper I;

$\bar{v}$ is the mean parallax of the stars in the area;

$V_o$ is the sun's velocity expressed in a.u. per year. We use the value $422$ corresponding to $20$ km/sec;

$c_4$ is the cosine of the angle between the direction of the sun's motion (which is assumed to be $a = 270^\circ, \delta = +30^\circ$) and the direction of the proper-motion component;

$|\mu|$ is the average residual proper-motion component obtained by eliminating the influence of the solar motion from $|\mu|^*$:

$$|\mu|^* = |\mu|^* - \left( \frac{56 c_4 V_o}{\bar{v}} \right)^2 \frac{\bar{v}}{\bar{v}}.$$

This formula is derived from the exact formula

$$|\mu|^* = |\mu|^* - \left( c_4 V_o \right)^2 \frac{\bar{v}}{\bar{v}},$$

by substituting the value 0.56 for the approximately constant ratio $|\bar{v}| / \bar{v}$; see page 307 of paper I;

$u_1, u_6$ is the average peculiar velocity component in the direction of the proper-motion component in longitude or latitude, defined by

$$u_{1,6} = |\mu| / \bar{v};$$

$l, b$ are the galactic co-ordinates of the centre of the area or group of areas, based on OHLSSON's table 1);

$l$ is the galactic longitude of the direction of the vertex, the latitude of which is assumed to be $0^\circ$;

$s_1$ is the average peculiar velocity component in the direction of the vertex;

$s_2$ is the average component perpendicular to the vertex direction in the galactic plane;

$s_3$ is the average component perpendicular to the galactic plane.

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1) A. N. Deutsch, Publ. Pulkovo, Série 2, Vol. LV, 1940. For the combination of the Pulkovo and Radcliffe proper motions see B.A.N. No. 362, 1943.

1) Land Annals No. 3, 1932.
For a given area we have

\[ u_i^* = s_i^* \sin^2 (l - l\rho) + s_i^* \cos^2 (l - l\rho) \]
\[ u_b^* = s_i^* \cos^2 (l - l\rho) \sin^2 b + s_i^* \sin^2 (l - l\rho) \sin^2 b + s_i^* \cos^2 b. \]

These formulae are equivalent to the expression of the mean radial velocity (regardless of sign) as a function of the principal axes of the velocity ellipsoid and the direction cosines of the area. See, for instance, Eddington’s Stellar Movements and the Structure of the Universe (1914), page 146. Instead, we now use, of course, the direction cosines of the proper-motion component. As in the study of the radial velocities, it is preferable to use the observed values \( u_i \) and \( u_b \) and not \( u_i^* \) and \( u_b^* \) for the reasons set forth by Eddington and Hartley \(^1\). We accordingly base the solution on the equations

\[ u_i = s_i \sin^2 (l - l\rho) + s_i \cos^2 (l - l\rho) \]
\[ u_b = s_i \cos^2 (l - l\rho) \sin^2 b + s_i \sin^2 (l - l\rho) \sin^2 b + s_i \cos^2 b, \]

which can be written

\[ u_i = P \sin^2 l + Q \cos^2 l - R \sin l \cos l \]
\[ u_b = P \cos^2 l \sin^2 b + Q \sin^2 l \sin^2 b + R \sin l \cos l \sin^2 b + s_i \cos^2 b, \]

where

\[
\begin{align*}
P = s_i \cos^2 l\rho + s_i \sin^2 l\rho \\
Q = s_i \sin^2 l\rho + s_i \cos^2 l\rho \\
R = (s_i - s_j) \sin 2 l\rho.
\end{align*}
\]

\[ P, Q, R \text{ and } s_i \text{ are solved by least squares and } s_i \text{ and } l\rho \text{ are derived from the relations} \]
\[ s_1 + s_2 = P + Q, \quad s_1 - s_2 = \frac{P - Q}{\cos 2 l\rho}, \quad \tan 2 l\rho = \frac{R}{P - Q} \]

Errors in the mean parallaxes affect \( u_i \) and \( u_b \), and if these errors vary systematically with the galactic longitude they may vitiate the resulting longitude of the vertex \( l\rho \). This can be avoided by using only the quotients \( u_i/u_b \) which, apart from the minor influence of errors in the last term of (2), are independent of the mean parallaxes. As we are only concerned with the low latitudes in the present paper, \( u_b \) is approximately equal to \( s_j \) and we have

\[ u_i/u_b = (s_j/s_2) \sin^2 (l - l\rho) + (s_j/s_2) \cos^2 (l - l\rho) \]
\[ = p \sin^2 l + q \cos^2 l - r \sin l \cos l, \]

where

\[ p = (s_j/s_2) \cos^2 l\rho + (s_j/s_2) \sin^2 l\rho \\
q = (s_j/s_2) \sin^2 l\rho + (s_j/s_2) \cos^2 l\rho \\
r = (s_j/s_2 - s_2/s_2) \sin 2 l\rho, \]

and from these equations \( s_j/s_2, s_j/s_2, \text{ and } l\rho \) are solved.

\[ \log \tilde{p} = \log \tilde{p}_{12} - 1.15 (m - 12), \]
\[ \tilde{p}_{12} \text{ being tabulated for different latitudes.} \]

On the other hand, the mean parallax can be represented independently of the galactic latitude as a function only of the number of stars per square degree brighter than the apparent magnitude \( m \):

\[ \log \tilde{p}_m = -0.98 - 0.36 \log N(m). \]

It is important for our purpose to use the most reliable values of the mean parallaxes and especially to take into account a possible dependence on the galactic longitude. This is possible by making use of the relation (15). However, in applying it to the McCormick stars for which the mean magnitude is 12.2, this formula gave too small mean parallaxes as compared to those derived by (14). The numbers \( N(m) \) were counted directly in the McCormick regions. It was decided to use the combined formula

\[ \log \tilde{p} = \log \tilde{p}_{12} - 1.15 (m - 12) + 36 (\log N_{m,b} - \log N_{m,b,l}), \]

where \( \log \tilde{p}_{12} \) is given in Binnendijk’s Table 10 and \( \log N_{m,b} \) and \( \log N_{m,b,l} \) have been taken from Groningen Publications No. 43, Tables 6 and 10, respectively. Thus local fluctuations in the surface density are taken into account, but these do not exclusively determine the mean parallaxes. Except when otherwise stated, this formula has been used in the computations for both the Selected Areas and the McCormick regions.

**Mean parallaxes.**

Binnendijk \(^2\) has computed mean parallaxes of faint stars from the combined Radcliffe-Pulkovo proper motions, following a method devised by Oort \(^3\). Within each zone of galactic latitude the mean parallax as a function of the apparent photographic magnitude can be represented by the formula

\[ \log \tilde{p}_m = \log \tilde{p}_{12} - 1.15 (m - 12), \]

\[ \tilde{p}_{12} \text{ being tabulated for different latitudes.} \]

On the other hand, the mean parallax can be represented independently of the galactic latitude as a function only of the number of stars per square degree brighter than the apparent magnitude \( m \):

\[ \log \tilde{p}_m = -0.98 - 0.36 \log N(m). \]

**Application of the dispersion method to the Selected Areas.**

All areas at galactic latitudes below \( \pm 20^\circ \) were used, except areas 63 and 110 for which the limiting magnitude is considerably brighter than for the remaining areas. All stars between \( m = 11'5 \) and \( 14'4 \)

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2) B.A.N. No. 362, 1943.
3) B.A.N. No. 299, 1936.
### Table 1. Selected Areas \( |b| < \pm 20^\circ \).

The unit of proper motion and of parallax is \( \cdot 001 \).

| Area | \( l \) | \( b \) | \( n \) | \( \bar{\mu}_p \) | \( \bar{\mu} \) | \( \mu \) | \( \xi \) | \( C \) | \( O - C \) | \( \bar{\mu} \) | \( \bar{\xi} \) | \( \mu \) | \( O - C \) |
|------|-----|-----|-----|--------|--------|-----|-----|-----|-----|--------|--------|-----|-----|-----|
| 109* | 333 | +14 | 101 | 135 | 161 | 353 | | | | | | | |
| 111* | 6   | +11 | 102 | 136 | 147 | 449 | | | | | | | |
| 86*  | 10  | +13 | 178 | 126 | 137 | 407 | | | | | | | |
| 87   | 17  | 0   | 107 | 135 | 554 | 293 | | | | | | | |
| 88   | 24  | +12 | 199 | 136 | 147 | 362 | | | | | | | |
| 64   | 35  | +1   | 130 | 136 | 133 | 311 | | | | | | | |
| 38   | 42  | +18 | 135 | 135 | 143 | 398 | | | | | | | |
| 65   | 43  | -12 | 101 | 136 | 139 | 284 | | | | | | | |
| 39   | 47  | +9  | 172 | 134 | 133 | 327 | | | | | | | |
| 40   | 53  | 0   | 151 | 133 | 124 | 302 | | | | | | | |
| 17   | 59  | +19 | 104 | 134 | 136 | 399 | | | | | | | |
| 41   | 61  | +8  | 288 | 137 | 129 | 342 | | | | | | | |
| 18   | 68  | +6  | 130 | 134 | 142 | 275 | | | | | | | |
| 42   | 79  | -13 | 113 | 135 | 139 | 214 | | | | | | | |
| 43   | 78  | -17 | 151 | 134 | 147 | 258 | | | | | | | |
| 19   | 81  | +1  | 188 | 134 | 142 | 236 | | | | | | | |
| 2    | 88  | +13 | 61 | 132 | 205 | 356 | | | | | | | |
| 20   | 89  | -17 | 120 | 134 | 142 | 287 | | | | | | | |
| 8    | 92  | -2  | 131 | 134 | 153 | 323 | | | | | | | |
| 21   | 99  | -17 | 134 | 135 | 161 | 324 | | | | | | | |
| 3    | 102 | +18 | 189 | 137 | 192 | 428 | | | | | | | |
| 9    | 106 | +3  | 73 | 139 | 199 | 253 | | | | | | | |
| 22   | 111 | +14 | 137 | 136 | 205 | 356 | | | | | | | |
| 10   | +18 | +13 | 177 | 201 | 492 | 294 | | | | | | | |
| 23   | 120 | -10 | 100 | 135 | 168 | 336 | | | | | | | |
| 24   | 128 | 0   | 75 | 136 | 169 | 273 | | | | | | | |
| 25   | 133 | +9  | 82 | 136 | 163 | 271 | | | | | | | |
| 48   | 136 | -12 | 62 | 136 | 163 | 292 | | | | | | | |
| 26   | 138 | +18 | 115 | 134 | 182 | 449 | | | | | | | |
| 49   | 145 | -2  | 100 | 135 | 178 | 303 | | | | | | | |
| 50   | 151 | +10 | 178 | 135 | 146 | 311 | | | | | | | |
| 73   | 156 | -11 | 70 | 137 | 192 | 293 | | | | | | | |
| 74   | 163 | +1 | 99 | 134 | 142 | 293 | | | | | | | |
| 75   | 170 | +14 | 134 | 137 | 133 | 239 | | | | | | | |
| 97   | 174 | -11 | 58 | 132 | 187 | 460 | | | | | | | |
| 98   | 181 | +1 | 380 | 135 | 124 | 396 | | | | | | | |
| 99   | 188 | +18 | 153 | 134 | 145 | 321 | | | | | | | |

* In Radcliffe Catalogue only.

with proper motions of full weight were used. Selected Areas which only occur in the Radcliffe catalogue (areas 3, 10, 26, 86, 97, 98, 99, 109, and 111) were also included. The number of areas used is 37. The data from the individual areas are in Table 1. In the accompanying diagram the values of \( u_1 \) are plotted against the longitude of the area increased or decreased by 90°. The gain in accuracy due to the improvement of the proper motions is apparent from a comparison of the corrected mean residual proper motions \( \bar{\mu} \) with the quantities \( \lambda \). Whereas in the original Radcliffe material the values of \( \bar{\mu} \) for the \( l \)-component for many areas were smaller than the average error of the proper motions and hence extremely uncertain, we now find among the areas with improved proper motions only two cases where \( \bar{\mu} \) is smaller than \( \lambda \). For the \( b \)-component we find four cases with \( \bar{\mu} \) smaller than \( \lambda \), but the other values of \( \bar{\mu} \) are nearly equal to \( \lambda \) and hence considerably less accurate than for the \( l \)-component. (In paper I the large accidental errors and the resulting doubtful corrections to \( \bar{\mu} \) * even led to imaginary values of \( \bar{\mu} \) for the \( b \)-component in some areas.)

As a consequence of the uncertainty of the values of \( \bar{\mu} \) for the \( b \)-component and, hence, for the quotients \( u_1/u_2 \), the best results have been obtained from a solution based on equations (8) and (9). The areas were weighted roughly proportional to the square root of the numbers of stars, but lower weight was given to those areas for which no improved proper motions are available. The residuals of the solution are in the first column of the first division of Table 3. The run of the \( O-C \)'s is in Table 1, where the areas are arranged in the order of increasing galactic longitude, is satisfactory for the \( l \)-component. This proves that the observations are well represented by the formula (6). In the case of the \( b \)-component the \( O-C \)'s are negative for the greater part of the areas at high longitudes. This could not be improved by using formula (4) instead of (6), but may be due to systematic errors in the mean parallaxes.
The mean peculiar tangential velocity component parallel to the galactic plane, \( u_\| \), plotted against the galactic longitude of the area plus or minus 90°, for the Selected Areas (left) and the McCormick regions (right).

That these are not yet well enough determined follows also from the correlation between the \( O-C \)'s of the \( l \)- and the \( b \)-component.

The second column of the first division of Table 3 shows the results from a solution based on the quotients \( u_\| /u_b \) according to formula (12). Only the 28 areas with combined proper motions were used. That even for these areas the proper motions are not yet sufficiently accurate, so that the quotients \( u_\| /u_b \) are very uncertain, follows from the large mean error of \( l \). As only the ratios \( s_l /s_b \), and \( s_b /s_b \) are solved, \( s_b \) has been put equal to unity. Weights were given inversely proportional to the square of the mean error of the quotients, the latter being computed from the mean errors of \( u_\| \) and \( u_b \).

In the third column are the results from a preliminary solution based on the same 28 areas and on the components \( u_\| \), using formula (8), so that only \( l \), \( s_l \), and \( s_b \) were solved. The parallaxes used in this solution are the mean of values computed by equations

**Table 2.** McCormick regions \( |b| < \pm 20^\circ \).

The unit of proper motion and of parallax is \( "001 \).

| group | number of regions | \( l \) | \( |b| \) | \( n \) | \( \bar{m}_p \) | \( \bar{\rho} \) | \( n \) | \( l \)-component | \( b \)-component |
|-------|------------------|------|-----|------|-------|-------|------|----------------|----------------|
|       |                  |      |     |      |       |       |      | \( |\mu| \) \( +5\epsilon_4V_\mu \bar{\rho} \) | \( |\mu| \) \( +5\epsilon_4V_\mu \bar{\rho} \) |
| 1     | 1                | 318° | 14  | 21   | 11    | 1     | 1    | 107†1 | 385 †3 | 339 †3 + 0 †44 |
| 2     | 5                | 336  | 115 | 144  | 11    | 2     | 2    | 392 †3 | 392 †3 | 392 †3 + 0 †44 |
| 3     | 5                | 343  | 13  | 266  | 11    | 7     | 2    | 845 †3 | 845 †3 | 845 †3 + 0 †44 |
| 4     | 6                | 355  | 75  | 336  | 11    | 9     | 20   | 1076†3 | 1076†3 | 1076†3 + 0 †44 |
| 5     | 5                | 372  | 10  | 232  | 11    | 8     | 20   | 729 †3 | 729 †3 | 729 †3 + 0 †44 |
| 6     | 6                | 393  | 5   | 865  | 11    | 8     | 1     | 579 †3 | 579 †3 | 579 †3 + 0 †44 |
| 7     | 7                | 414  | 8   | 865  | 11    | 8     | 1     | 150 †3 | 150 †3 | 150 †3 + 0 †44 |
| 8     | 8                | 435  | 6   | 972  | 11    | 6     | 1     | 729 †3 | 729 †3 | 729 †3 + 0 †44 |
| 9     | 8                | 466  | 6   | 850  | 11    | 5     | 1     | 150 †3 | 150 †3 | 150 †3 + 0 †44 |
| 10    | 5                | 55   | 9   | 427  | 11    | 6     | 1     | 729 †3 | 729 †3 | 729 †3 + 0 †44 |
| 11    | 5                | 66   | 8   | 310  | 11    | 5     | 1     | 150 †3 | 150 †3 | 150 †3 + 0 †44 |
| 12    | 6                | 74   | 9   | 270  | 11    | 6     | 1     | 150 †3 | 150 †3 | 150 †3 + 0 †44 |
| 13    | 7                | 85   | 5   | 521  | 11    | 8     | 1     | 729 †3 | 729 †3 | 729 †3 + 0 †44 |
| 14    | 8                | 94   | 9   | 380  | 11    | 9     | 1     | 729 †3 | 729 †3 | 729 †3 + 0 †44 |
| 15    | 9                | 107  | 3   | 83   | 11    | 8     | 1     | 729 †3 | 729 †3 | 729 †3 + 0 †44 |
| 16    | 11               | 114  | 105 | 190  | 11    | 8     | 1     | 729 †3 | 729 †3 | 729 †3 + 0 †44 |
| 17    | 12               | 124  | 105 | 533  | 11    | 9     | 1     | 729 †3 | 729 †3 | 729 †3 + 0 †44 |
| 18    | 13               | 133  | 10  | 288  | 11    | 4     | 1     | 729 †3 | 729 †3 | 729 †3 + 0 †44 |
| 19    | 14               | 166  | 10  | 98   | 11    | 2     | 1     | 729 †3 | 729 †3 | 729 †3 + 0 †44 |
| 20    | 15               | 184  | 7   | 85   | 11    | 9     | 1     | 729 †3 | 729 †3 | 729 †3 + 0 †44 |
| 21    | 16               | 215  | 7   | 289  | 11    | 4     | 1     | 729 †3 | 729 †3 | 729 †3 + 0 †44 |
| 22    | 17               | 276  | 6   | 310  | 11    | 9     | 1     | 729 †3 | 729 †3 | 729 †3 + 0 †44 |
| 23    | 18               | 285  | 5   | 98   | 11    | 2     | 1     | 729 †3 | 729 †3 | 729 †3 + 0 †44 |
| 24    | 19               | 304  | 7   | 330  | 11    | 4     | 1     | 729 †3 | 729 †3 | 729 †3 + 0 †44 |
| 25    | 20               | 327  | 7   | 289  | 11    | 4     | 1     | 729 †3 | 729 †3 | 729 †3 + 0 †44 |
The value for \( s_5 \) given in brackets in the table is the weighted mean of \( u_5 \); with this value \( s_5/s_4 \) has also been computed.

**Application of the dispersion method to the McCormick proper motions.**

The McCormick regions below \( \pm 20^\circ \) galactic latitude were combined into one group for each \( 10^\circ \) interval of galactic longitude. Only the stars between photovisual magnitudes \( 10^\circ \) and \( 13^\circ \) were used. For each region residual proper-motion components in longitude and latitude with respect to the mean proper motion for the region were computed and averaged, and the weighted mean of these average residuals was calculated for each group of regions. The correction term \( \kappa \) was computed with the aid of the information concerning the accidental errors given by Van de Kamp and Vyssotsky in the introduction of the catalogue. For the computation of the mean parallaxes as a function of the photovisual magnitude, \( 0^\circ 6 \) was added to the mean photovisual magnitude.

The data for the individual groups are in Table 2. In the accompanying diagram the values of \( u_i \) are plotted against the longitude of the group increased or decreased by \( 90^\circ \). Results of the solutions are in the second division of Table 3. The first column is the solution from equations (8) and (9), and is based on the mean parallaxes. Weights were given according to the square root of the numbers of stars. The solution in the second column, based on the ratios \( u_i/u_5 \) and therefore practically unaffected by errors in the mean parallaxes, gives the most valuable results. Contrary to the corresponding solution from the Selected Areas, the accidental errors in \( u_5 \) and hence in \( u_i/u_5 \) are not so great now as to undo the advantage of eliminating the mean parallaxes. That an improvement of the mean parallaxes would lead to better results follows again from the correlation between the \( O-C \)'s for the \( l \)- and the \( b \)-components in Table 2, which have been computed with the first solution based on \( u_5 \) and \( u_i \). The last column shows the preliminary solution based on the components \( u_i \) only and on formula (8); mean parallaxes derived from (14) and (15) were used. \( s_5 \) is again the weighted mean of \( u_5 \) and with this value \( s_5/s_4 \) was computed.

**Discussion of results from the dispersion method.**

The longitudes \( l_V \) found in the four final solutions agree fairly well. The weighted mean from Selected Areas and McCormick regions,

\[
l_V = 323^\circ \cdot 5 \pm 1^\circ \cdot 8 \text{ (m.e.)},
\]

may be considered as representative for the stars between photographic magnitudes \( 10^\circ 5 \) and \( 14^\circ 5 \) and below \( \pm 20^\circ \) galactic latitude. It confirms the earlier result for this zone found in paper I, \( l_V = 320^\circ \cdot 2 \pm 6^\circ \cdot 0 \), and forms a satisfactorily accurate determination of the direction of preferential motion for these stars. Contrary to the solutions obtained up to now by Schwarzschild's methods for low latitudes, the preferential direction found from the dispersion method appears to coincide nearly exactly with the direction of the galactic centre. The question naturally arises how it is possible that such a large difference between the results can exist, the material of proper motions being practically the same. This point is considered more in detail in the next section; it is pointed out there that the deviating longitude of the vertex obtained by Schwarzschild's methods may well be spurious and due to the unsuitability of this method for the low-latitude zone.

We observe good agreement between the results for \( s_5 \) in the first columns of the two divisions of Table 3, the difference being small compared to its mean error.

### Table 3

Results from the dispersion method.

\( s_1, s_2, s_3 \) are average velocities along the axes of the velocity ellipsoid, expressed in a.u./year.

<table>
<thead>
<tr>
<th></th>
<th>Selected Areas</th>
<th>McCormick regions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>final solutions</td>
<td>preliminary sol.</td>
</tr>
<tr>
<td></td>
<td>form. (8) and (9)</td>
<td>form. (12)</td>
</tr>
<tr>
<td>( l_V ) = longitude of vertex</td>
<td>( 318^\circ 6 \pm 2^\circ 7 ) (m.e.)</td>
<td>( 314^\circ \pm 10^\circ ) (m.e.)</td>
</tr>
<tr>
<td>( s_1 ) = major axis</td>
<td>( 5^\circ 14 \pm 2^\circ 3 )</td>
<td>( 1^\circ 70 \pm 2^\circ 2 )</td>
</tr>
<tr>
<td>( s_2 ) = second axis in galactic plane</td>
<td>( 1^\circ 92 \pm 2^\circ 3 )</td>
<td>( 1^\circ 26 \pm 3^\circ 0 )</td>
</tr>
<tr>
<td>( s_3 ) = third axis</td>
<td>( 2^\circ 40 \pm 1^\circ 3 )</td>
<td>( 1^\circ 00 ) adopted</td>
</tr>
<tr>
<td>( s_2/s_1 )</td>
<td>( 37 \pm 0^\circ 5 )</td>
<td>( 74 \pm 2^\circ 0 )</td>
</tr>
<tr>
<td>( s_3/s_1 )</td>
<td>( 47 \pm 0^\circ 3 )</td>
<td>( 59 \pm 0^\circ 7 )</td>
</tr>
<tr>
<td>number of areas or groups</td>
<td>37</td>
<td>28</td>
</tr>
</tbody>
</table>
This means that the formula (16) for the mean parallaxes correctly represents the ratio between the mean parallaxes of the Selected Areas and the McCormick stars. The mean, \( s_1 = 5'21 \text{ a.u.}/\text{year} \pm '16 \) (m.e.) = 24.7 km/sec cannot, however, be considered as a new determination of the mean linear velocity in the direction of the principal axis, because the determination of the mean parallaxes which we used rests on the very assumption that the mean linear velocity has this value. (According to Figure 1 of Oort's paper, L.c., page 85, we find 147 and 550 a.u./year, respectively for the average velocities of the stars of photographic magnitudes 12.2 and 13.5 at galactic latitude 10\(^\circ\).) The determination of \( s_1 \) is not seriously affected by errors in the mean errors of the proper motions. These, however, do strongly affect \( s_2 \) and \( s_3 \) and hence \( s_2/s_1 \) and \( s_3/s_1 \). For the Selected Area proper motions the probable errors have been checked by the comparison of the two entirely independently determined sets of proper motions of Radcliffe and Pulko-vo. Such a check cannot be made for the McCormick material. The fact that the ratios \( s_2/s_1 \) and \( s_3/s_1 \) found from the principal solution for the Selected Areas are smaller than those from the McCormick regions may be due, therefore, to an underestimate of the probable errors of the McCormick proper motions. In this connection we draw attention to the rather large difference between the results for \( s_3 \) in brackets in Table 3; both being the weighted mean of the corrected latitude components reduced to unit of distance, this difference strongly suggests either an insufficient reduction to the unit of distance or an underestimate of the McCormick accidental errors. As the former is excluded by the better agreement for the values of \( s_3 \), the latter possibility needs careful consideration. It is, however, also possible that the small values of \( s_2/s_1 \) and \( s_3/s_1 \) for the Selected Areas are due to some extent to the accidental errors in the values of \( \eta \). Consider, for instance, two areas for which the \( \eta \)'s as well as the \( |\mu|'s \) are equal. If the computations are performed with two erroneous values of \( \eta \), the first one too small and the second one too large (by the same amount, so that the mean \( \eta \) is correct), then the mean of the resulting values of \( |\mu|' \) is not correct but too small. A few too large values of \( \eta \) may thus cause very low values of \( |\mu| \) which are not compensated by areas where the error in \( \eta \) works in the other direction. Furthermore, the uncertainty in the results of the Selected Areas solution is illustrated by the fact that here \( s_3 \) is found to be considerably smaller than \( s_1 \), whereas for the McCormick regions we find the reverse, and this cannot be due to an underestimate of the mean errors of the latter. Assuming \( s_2 \) and \( s_3 \) to be equal and giving double weight to the Selected Area results we find the mean value

\[
s_2/s_1 = s_3/s_1 = 0.47 \pm 0.04 \text{ (m.e.)}
\]

**Remarks on Schwarzschild's methods.**

The procedures proposed by Schwarczschild for the determination of the elements of the velocity ellipsoid differ from the dispersion method essentially in two respects:

1) absolute proper motions are used instead of relative proper motions;

2) the directions of the total proper motions are used and not their amounts.

One way to explain the difference between the results from the dispersion method and those from Schwarzschild's methods would be, to assume that the true distribution of the space velocities deviates from the representation by the velocity ellipsoid on the basis of which Schwarzschild's formulae have been derived. Let us suppose that the higher velocities, say those exceeding 10 km/sec, can be represented by an ellipsoidal distribution with the major axis in the direction of the galactic centre, but that the smaller velocities have a different preferential direction. This might be the case, for instance, if there is a surplus of small velocities in a certain direction not coinciding with that of the galactic centre or anti-centre and due to an unequal density distribution of the stars in the galactic system. The dispersion of the velocity components in any direction would be practically unaffected by the distribution of the directions of these small velocities, and hence the direction of greatest dispersion, found from the dispersion method, would not change appreciably. On the other hand, the surplus stars with small velocities and deviating preferential direction would enter with full weight into the Schwarzschild solution where only the directions of the proper motions are counted. Thus, different directions of the vertex would have to be found from the two methods.

We do not, however, believe that it is necessary to assume this special distribution of velocities for the stars considered in this paper, and it occurs to us that the origin of the discrepancy is to be sought in the unsuitability of Schwarzschild's procedure when applied to the low latitude stars. We shall first consider this procedure in some detail, and next apply it with some alterations to the improved proper motions in the Selected Areas and to the McCormick proper motions.

In Schwarzschild's methods the distribution of the numbers of proper motions in intervals of position angle (usually 10\(^\circ\)) forms the basis from which the following quantities are derived for each area:

- \( \theta_4 \), the position angle of the great circle through the centre of the area and the solar apex;
- \( \theta_5 \), the position angle of the great circle through the centre of the area and the vertex;
- \( \alpha/V_\alpha \sin \lambda \) and \( \beta/V_\beta \sin \lambda \), where \( \alpha \) and \( \beta \) are the major
and the minor axes of the apparent velocity ellipse expressed in the same unit as the solar velocity, \( V_\odot \), and \( \lambda \) is the angular distance of the area from the solar apex \(^1\).

\[ \alpha \text{ and } \beta \text{ are related to the axes of the velocity ellipsoid by the formula} \]

\[ \left( \frac{\alpha^2}{\beta^2} - 1 \right) = \left( \frac{\sigma_1^2}{\sigma_2^2} - 1 \right) \sin^2 \chi, \]  

(17)

where

\[ \sigma_1 \text{ is the major galactic axis of the velocity ellipsoid;} \]

\[ \sigma_2 \text{ is the minor galactic axis of the velocity ellipsoid;} \]

\[ \sigma_3 \text{ is the third axis.} \]

In SCHWARZSCHILD's formulae the two smallest axes are supposed to be equal: \( \sigma_2 = \sigma_3 \).

\[ \chi \text{ is the angular distance of the area from the vertex.} \]

According to the definition of the velocity ellipsoid

\[ X = \sqrt{\frac{\sigma_1^2}{\sigma_2^2} - 1} \cos l \cos b \]

\[ Y = \sqrt{\frac{\sigma_1^2}{\sigma_2^2} - 1} \sin l \cos b \]

\[ Z = \sqrt{\frac{\sigma_1^2}{\sigma_2^2} - 1} \sin b, \]

from which \( \sigma_1/\sigma_2 \) and \( l, b \), the co-ordinates of the vertex direction, are found.

Two points are especially of importance in connection with the application of these procedures to the faint low latitude stars. There is in SCHWARZSCHILD's methods always a freedom of choice in the determination of \( \theta_\nu \). We may, namely, choose between the four values \( \theta_\nu, \theta_\nu + 90^\circ, \theta_\nu + 180^\circ, \) and \( \theta_\nu + 270^\circ \). As to the choice between \( \theta_\nu \) and \( \theta_\nu + 180^\circ \) on the one hand and \( \theta_\nu + 90^\circ \) and \( \theta_\nu + 270^\circ \) on the other hand, the condition is that the first mentioned direction should coincide with the major axis of the velocity ellipse for the area. This condition fails, however, if the axis of the ellipse which is directed towards the vertex is the smaller one, and this may well occur. Consider, for instance, an area at the galactic longitude of the vertex and at latitude, say, \( 10^\circ \). One axis of the velocity ellipse of this area is parallel to the galactic circle and the other one nearly perpendicular to it, and the ratio of these axes is approximately equal to that of the minor galactic axis and the third axis of the velocity ellipsoid. If, now, this latter axis is

\[ X \sin l + Y \cos l \]

\[ -X \cos l \sin b - Y \sin l \sin b + Z \cos b = \sqrt{\frac{\sigma_1^2}{\sigma_2^2} - 1} \sin \theta_\nu. \]

(20)

As \( b \) is small for the areas considered, the quantities \( X \) and \( Y \), which define \( l, b \), are solved mainly from the first equation. An error of \( 180^\circ \) in \( \theta_\nu \) means an error of sign in the right-hand member, and such errors will lead to erroneous quantities \( X \) and \( Y \), especially if they occur in areas at some distance from the vertex where \( \theta_\nu \) is \( 90^\circ \) or \( 270^\circ \), and hence \( \sin \theta_\nu = \pm 1 \) or \( -1 \).

To illustrate this point, we consider some results from the treatment of the improved Selected Area proper motions. Values of \( \sqrt{\sigma_1^2/\sigma_2^2 - 1} \sin \theta_\nu \) are determined

\[ \text{in SCHWARZSCHILD's paper, } \sigma_1 \text{ and } \sigma_2 \text{ are equal to } \sqrt{2} \text{ times the dispersion of the velocity components in the directions of the major and the minor axis, respectively.} \]

For details concerning the application of SCHWARZSCHILD's method we refer to his original papers (l.c.). The automatic method is also described at length in the Radcliffe Catalogue of Proper Motions, p. xxv--xxxiii.

The usual procedure in the determination of the elements of the velocity ellipsoid from the results for the individual areas is to combine the quantities \( \theta_\nu \) and \( \alpha/\beta \) into the expressions

\[ \sqrt{\frac{\alpha^2}{\beta^2} - 1} \sin \theta_\nu \text{ and } \sqrt{\frac{\alpha^2}{\beta^2} - 1} \cos \theta_\nu, \]

and to solve by least squares the quantities

\[ \text{the smaller one, which may well be the case, the major axis of the velocity ellipse for this area is the one which is perpendicular to the direction } \theta_\nu. \]

Similarly, if we consider an area on the galactic circle differing some degrees in galactic longitude from the vertex direction, the determination of \( \theta_\nu \) may go wrong if the third ellipsoidal axis happens to be greater than the minor galactic axis. Summarizing, we can state that there must be always a considerable uncertainty in the determination of \( \theta_\nu \) for the areas near the vertex. However, in case we are dealing with a zone of low latitudes it is only in these areas that the angle \( \theta_\nu \) contributes to the determination of \( l, b \). The remaining areas, because of their low latitude, should always have values of \( \theta_\nu \) near \( 90^\circ \) or \( 270^\circ \), and if the observed values of \( \theta_\nu \) deviate considerably from these values, they are simply useless and violate the expressions (18).

A second, and undoubtedly very important, point is the arbitrariness of the choice between \( \theta_\nu \) and \( \theta_\nu + 180^\circ \). That this choice has a definite meaning is immediately clear from the equations which are used in solving \( X, Y, Z \). They are:

\[ \begin{align*}
X &= \sqrt{\frac{\alpha^2}{\beta^2} - 1} \sin \theta_\nu \\
Y &= \sqrt{\frac{\alpha^2}{\beta^2} - 1} \cos \theta_\nu.
\end{align*} \]

(20)

According to the automatic SCHWARZSCHILD method. The areas are situated between galactic longitudes \( 17^\circ \) and \( 170^\circ \). If we suppose \( l, b \) to exceed \( 350^\circ \), the sign of \( \sin \theta_\nu \) nowhere changes when we proceed along the galactic circle, and if these values of \( \sin \theta_\nu \) are used in a solution of \( X \) and \( Y \) based on equations (20), the result is \( l, b = 351^\circ \). If, however, we assume the vertex to be near longitude \( 325^\circ \), we must change the sign of the right-hand member of the equations (20) for the four areas at longitudes exceeding \( 325^\circ \), and the result is \( l, b = 321^\circ \). If the critical areas are omitted, we find \( l, b = 331^\circ \). Evidently the wrong choice of the sign of \( \sin \theta_\nu \) — we might also say: negli-
gence of the change of sign of \( \sin \theta_V \) at the right longitude — causes an error in the longitude \( l_V \), the sign and amount of which depend on the distribution of the areas according to the galactic longitude.

It is, therefore, desirable to avoid the use of the observed quantity \( \theta_V \) in the solution of the co-ordinates of the vertex. It is not necessary to introduce both \( \theta_V \) and \( \alpha/\beta \) in the equations of condition, as was proposed by Schwarzschild. The angular distances \( \chi \) from the true vertex will satisfy equation (17), which contains only \( \alpha/\beta \), better than angular distances from any other point in the sky, and we may determine the vertex co-ordinates from the \( \alpha/\beta \)'s exclusively by this condition. It is true that in deriving \( \alpha/\beta \) for the individual areas the direction of the axes of the velocity ellipse \( \theta_V \), must be known before \( \alpha \) and \( \beta \), and hence \( \alpha/\beta \), can be computed. It may be obtained from Schwarzschild's methods; in that case there is a chance that the axes of the ellipse are interchanged for areas near the vertex or opposite to it, as was explained above. As in such areas \( \alpha \) and \( \beta \) are nearly equal, the errors thus introduced in \( \alpha/\beta \) will not affect seriously the solution, though the values of \( \alpha/\beta \) smaller than unity will be reversed. We also may assume some probable set of co-ordinates for the vertex, compute \( \theta_V \) for each area and use this value in calculating \( \alpha/\beta \). The solution of the vertex from these \( \alpha/\beta \)'s will only slightly depend on the assumed co-ordinates, and any dependence can be checked by using different sets of assumed co-ordinates.

Equation (17) can be written:

\[
(\alpha/\beta)^* = (\sigma_1/\sigma_3)^* \sin^2 \chi + \cos^2 \chi.
\]

In the case of those low-latITUDE areas for which \( \chi \) is about 90°, \( \alpha/\beta \) is nearly equal to the ratio of the principal galactic and the third axis. We therefore substitute \( \sigma_1/\sigma_3 \) for \( \sigma_1/\sigma_2 \) in this formula. For the smallest values of \( \chi \), the \( \alpha/\beta \) varies between \( \sigma_2/\sigma_3 \) and \( \sigma_3/\sigma_2 \) according to the position of the area. In order to take into account the possibility that the average value of \( \alpha/\beta \) for these areas differs from unity we write the equation in the form

\[
(\alpha/\beta)^* = (\sigma_1/\sigma_3)^* \sin^2 \chi + \gamma^* \cos^2 \chi,
\]

where \( \gamma^* \) is to be considered as an additional unknown. As in the case of the dispersion method, we use first powers instead of squares of the observed and unknown quantities:

\[
\alpha/\beta = \frac{\sigma_1}{\sigma_3} \sin^2 \chi + \gamma' \cos^2 \chi.
\]

This equation can be written:

\[
\alpha/\beta = \frac{\sigma_1}{\sigma_3} \sin^2 b + p' \sin^2 l \cos^2 b + q' \cos^2 l \cos^2 b - r' \sin l \cos l \cos^2 b,
\]

where

\[
p' = \frac{\sigma_1}{\sigma_3} \cos^2 l v + \gamma' \sin^2 l v
\]

\[
q' = \frac{\sigma_1}{\sigma_3} \sin^2 l v + \gamma' \cos^2 l v
\]

\[
r' = \left( \frac{\sigma_1}{\sigma_3} - \gamma' \right) \sin 2 l v.
\]

As before, the latitude of the vertex has been supposed to be zero. If \( b = 0^\circ \), i.e., for areas on the galactic circle, the first term of the right-hand member of (21) vanishes and the equations are similar to equations (12) and (13) used in the dispersion method. The term with \( \sin^2 b \) is always very small for the areas considered here; it may be evaluated with an estimated value of \( \sigma_1/\sigma_3 \) and subtracted from the observed left-hand term, so that the unknowns to be solved are \( p', q' \), and \( r' \), and from these \( \sigma_1/\sigma_3 \), \( \gamma \) and \( l \) are derived.

Instead of using the ratios \( \alpha/\beta \), we can also base the solution on the quantities \( \alpha/V_o \sin \lambda \) obtained by Schwarzschild’s methods. From these we compute values \( \alpha/V_o \) with the known apex distances \( \lambda \), and these are related to \( \sigma_1/V_o \) by the equation:

\[
(\alpha/V_o)^* = (\sigma_1/V_o)^* \sin^2 \chi + (\gamma/V_o)^* \cos^2 \chi,
\]

\( \gamma \) being between \( \sigma_2 \) and \( \sigma_3 \). The equation of condition now becomes, similarly to the preceding case,

\[
\alpha/V_o = \frac{\sigma_1}{V_o} \sin^2 b + P' \sin^2 l \cos^2 b + Q' \cos^2 l \cos^2 b - R' \sin l \cos l \cos^2 b,
\]

where

\[
P' = \frac{\sigma_1}{V_o} \cos^2 l v + \frac{\gamma}{V_o} \sin^2 l v
\]

\[
Q' = \frac{\sigma_1}{V_o} \sin^2 l v + \frac{\gamma}{V_o} \cos^2 l v
\]

\[
R' = \frac{\sigma_1 - \gamma}{V_o} \sin 2 l v,
\]

and which for \( b = 0^\circ \) is similar to equation (8) for \( u_t \).

The quantities \( \beta/V_o \) would be equal to \( \sigma_2/V_o \) if \( \sigma_2 \) and \( \sigma_3 \) were equal. They may be used to determine the values of \( \sigma_2/V_o \) and \( \sigma_3/V_o \) by means of the equation

\[
\frac{\beta}{V_o} = \frac{\sigma_2 - \sigma_3}{V_o} \frac{\sin^2 b}{\cos^2(l-l v) \sin^2 b + \sin^2(l-l v) + \sigma_3} + \frac{\sigma_3}{V_o},
\]

where the longitude \( l_v \) obtained from the preceding solutions is to be substituted in the coefficient of the first term.
Application of the modified Schwarzschild method to the Selected Areas and the McCormick regions.

The material to which the dispersion method was applied has been treated also according to the Schwarzschild automatic method with some modifications as indicated in the preceding section. The grouping of the McCormick regions was now taken the same as in the treatment by Van de Kamp and Vysotsky, with the exceptions that groups 15 and 43 were omitted because in these groups the angles $\theta_A - \theta_V$ differ considerably for the various regions, while these groups could not be subdivided because of the small number of stars. Further, the regions in groups 25, 26 and 27 were rearranged into three somewhat different groups. The relative proper motions were made absolute by means of the parallactic motion computed with the known mean parallax and the solar motion 20 km/sec toward $\alpha = 270^\circ$, $\delta = +30^\circ$. Quantities $\alpha / V_o \sin \lambda$ and $\beta / V_o \sin \lambda$ were derived for each area or group, their ratio being fixed by the procedure of conjugate diameters. From these, $\alpha / V_o$ and $\beta / V_o$ were obtained by multiplication by $\sin \lambda$, computed for the standard apex.

In a provisional solution these quantities derived from the Selected Areas were substituted as known terms in the equations (6) and (7). These equations do not exactly represent the quantities $\alpha / V_o$ and $\beta / V_o$ because the areas are not at zero latitude, but the solution will be approximately correct as far as $\ell_P$ is concerned. Only the Radcliffe-Pulkovo areas were used. Area 17, which gave quite abnormal, large values of $\alpha / V_o$ as well as $\beta / V_o$, was excluded; its inclusion would not have seriously affected $\ell_P$ but might give an incorrect value for the mean error. The result is in the first division of Table 4.

Next, a solution was made from the ratios $\alpha / \beta$ according to equation (21), for both the Selected Areas and the McCormick regions. The Selected Areas occurring only in the Radcliffe Catalogue were now also included. Values of $\theta_V$ were calculated with the assumed vertex coordinates $\ell_P = 325^\circ$, $b_V = 0^\circ$, and $\theta_A$ was computed with the standard apex. The results are in the second division of Table 4. An increase of the assumed value of $\ell_P$ by $10^\circ$ would have slightly decreased the resulting $\ell_P$ for the Selected Areas. However, these solutions for $\ell_P$ are still not entirely reliable. We have not taken into account the influence of the accidental errors of the proper motions. These increase $\alpha / V_o$ and $\beta / V_o$, but $\beta / V_o$, which is the smaller of the two, more than $\alpha / V_o$, and hence the accidental errors affect the ratios $\alpha / \beta$ too. The consequence is not only that $\sigma_2 / \sigma_1$ will come out too low, but the longitude $\ell_P$ is also vitiated, because the ratio between the mean error of the proper motions and $\beta / V_o$ depends on the galactic longitude. This can be shown as follows. From the data referring to the Selected Areas in Table 1 we compute the quotients $\eta / [\mu]$ for the $b$-components in the longitude intervals $10^\circ$ to $55^\circ$ and $55^\circ$ to $100^\circ$, which is the region where $\alpha / \beta$ is most sensitive to the influence of the accidental errors. The mean values of $\eta / [\mu]$ are 97 and 76, respectively. Hence the values of $\beta$ are increased in the ratio 1.39 and 1.26. The increase of $\alpha$ is much smaller and more nearly equal in the two regions. Consequently $\alpha / \beta$ decreases by approximately 10% more in the interval $10^\circ$ to $55^\circ$ than in the interval $55^\circ$ to $100^\circ$, and the longitude of maximum $\alpha / \beta$, if the true value is about $55^\circ$, will be found at too high a longitude. This explains at least partly the high longitude found from the Selected Areas. For the McCormick proper motions a similar effect is present.

In order to eliminate the influence of the accidental errors we proceed as follows. The distribution of the accidental errors in a certain area, with mean parallax $\bar{\beta}$, corresponds to a circular velocity distribution with "axes" $\alpha / V_o = \beta / V_o = \mu V^2 / \bar{\beta} V_o$, where $\mu$ is the mean error of the proper motions. Hence the true values of $\alpha / V_o$ and $\beta / V_o$ are obtained from the relations

$$\text{(true } \alpha / V_o) = \left( \text{observed } \alpha / V_o \right) - 2 \left( \frac{\mu}{\bar{\beta} V_o} \right)^2,$$

and

$$\text{(true } \beta / V_o) = \left( \text{observed } \beta / V_o \right) - 2 \left( \frac{\mu}{\bar{\beta} V_o} \right)^2.$$


Table 4

<table>
<thead>
<tr>
<th>Selected Areas preliminary solution from uncorrected $\alpha / V_o$ and $\beta / V_o$</th>
<th>Solutions from uncorrected $\alpha / \beta$</th>
<th>Selected Areas from corrected $\alpha / V_o$</th>
<th>McCormick regions from uncorrected $\alpha / V_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ell_P = 318^\circ \pm 6^\circ$</td>
<td>$\ell_P = 340^\circ \pm 5^\circ$</td>
<td>$\ell_P = 330^\circ \pm 6^\circ$</td>
<td>$\ell_P = 316^\circ \pm 10^\circ$</td>
</tr>
<tr>
<td>$\sigma_1 / V_o = 2.41 \pm 2^\circ$</td>
<td>$\sigma_1 / V_o = 1.33 \pm 0.04$</td>
<td>$\gamma / V_o = 1.11 \pm 0.17$</td>
<td>$\gamma / V_o = 1.81 \pm 0.19$</td>
</tr>
<tr>
<td>$\sigma_2 / V_o = 1.48 \pm 2^\circ$</td>
<td>$\gamma = 97^\circ \pm 0.07$</td>
<td>$\gamma = 97^\circ \pm 10^\circ$</td>
<td>$\gamma = 97^\circ \pm 10^\circ$</td>
</tr>
<tr>
<td>$\sigma_3 / V_o = 1.44 \pm 1^\circ$</td>
<td>$\gamma = 97^\circ \pm 0.07$</td>
<td>$\gamma = 97^\circ \pm 10^\circ$</td>
<td>$\gamma = 97^\circ \pm 10^\circ$</td>
</tr>
</tbody>
</table>

27 areas | 37 areas | 29 groups | 36 areas | 29 groups

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In many areas the corrected $\beta/V_0$ is smaller than the correction term and hence very unreliable. The corrected $\alpha/V_0$ are also rather unreliable, but in the case of the Selected Areas it nevertheless seemed worth while to base on them a solution according to equation (22). The result is in the third division of Table 4. Additional solutions based on values of $\delta\nu$ computed with $\lambda_\nu = 315^\circ$ and $335^\circ$ showed that the solution $\lambda_\nu = 316^\circ$ is practically independent from these assumed values, and that the same holds for $\sigma_1/V_0$ and $\gamma/V_0$. The corrected values of $\beta/V_0$, though very uncertain, were used for a solution of $\sigma_2/V_0$ and $\sigma_3/V_0$, according to equation (23); for $\lambda_\nu$ we substituted $316^\circ$. We found

$$\sigma_2/V_0 = 1.37 \pm 0.16 \text{ and } \sigma_3/V_0 = 1.09 \pm 0.06.$$  

Finally, for the McCormick regions a solution was made from the uncorrected $\alpha/V_0$. These quantities are less affected by the accidental errors than $\beta/V_0$, and the solution for $\lambda_\nu$ will be shifted slightly in the reverse direction compared to the solution from the uncorrected $\alpha/\beta$; the true value of $\lambda_\nu$ therefore must be somewhat higher than the result given in the last division of Table 4.

The various solutions for $\lambda_\nu$, especially those in which the influence of the accidental errors is minimized, seem to point definitely in favour of the low longitude of the vertex found from the dispersion method. We believe that the conclusion is justified that the so-called vertex deviation for the faint low latitude stars, found from earlier investigations, is spurious and due to systematic errors to which the unmodified Schwarzschild methods may lead.

The ratio of the axes, $\gamma/\sigma_1$, found from the corrected Selected Areas is $52 \pm 11$. The uncorrected ratio from the McCormick regions (last division of Table 4) is $612$, and if corrected values $\sigma_1/V_0 = 2.53$, and $\gamma/V_0 = 1.29$ derived by means of the quantities $\psi$ and $\bar{p}$ in Table 2 are used, we find for the ratio of the axes $\gamma/\sigma_1 = 51 \pm 10$. These results agree satisfactorily with those from the dispersion method and we adopt as a final value

$$\frac{\sigma_2}{\sigma_1} = 49 \pm 0.04 \text{ (m.e.)}.$$  

The quantities $\sigma_1/V_0$, which are $\sqrt{2}$ times the velocity dispersion expressed in the sun’s velocity as a unit, may be compared with the results from the dispersion method. Supposing $V_0 = 20 \text{ km/sec}$ and the dispersion of the velocity components to be $1.253 \times$ the mean speed (the factor valid for a normal distribution) we find for the average linear velocity component, $s_1$, in the direction of the major axis $5.08$ and $6.03 \text{ a.u.}/\text{year}$, respectively, from the Selected Areas and the McCormick regions, compared to $5.14$ and $5.29$ from Table 3. The disagreement for the McCormick regions is not serious in view of the somewhat uncertain mean errors of these proper motions. The quantities $\alpha/V_0$ may prove useful in the study of stars at intermediate and high galactic latitudes. For instance, they may afford a check on the adopted linear velocity dispersion used in the determination of mean parallaxes according to the method applied by Oort (1937) which is based on the comparison of linear and angular velocity dispersions.