Remark on the mean proper motion of stars of a given apparent magnitude,*

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In Groningen Publ. 30, Table 21, p. 99 KAPTEYN and VAN RHIJN have given the mean proper motion of stars of different apparent visual magnitude for 3 zones of different numerical galactic latitude.

If the star density were constant in space and the distribution of peculiar motions the same everywhere, the relative frequencies of \( m + 5 \log \mu \), where \( m \) is the apparent magnitude and \( \mu \) the annual proper motion, would be the same for all magnitudes. The determination of these would-be relative frequencies is of importance in statistics of proper motions. The real frequencies of different proper motions for stars of a given apparent magnitude are in comparison characterized by the incompleteness of small proper motions owing to the deficiency of remote stars caused by the falling off of the stardensity with increasing distance. It will now be of interest to see, how far the observed data agree with the simple assumption, that the missing stars are those of smallest possible proper motion.

In A. N. 4883, 204, 187; 1917, Tab. II, column 7, logarithms of number of stars in the whole sky, between definite limits of proper motion are given for the interval 5\(^m\) to 6\(^m\) visual, representing a mean magnitude of say 5\(^m\)5. It is seen that 1/15 of the stars, showing the greatest proper motions, yield about half the arithmetical sum of all proper motions of these stars. Adopting the numbers l. c. it is easy by the aid of the results of star counts given in Groningen Publ. 27, Table V, p. 63 to calculate in the way just described the expected mean proper motion of stars of different visual magnitude and galactic latitude.

To illustrate this I have chosen the representative magnitudes 4\(^m\), 8\(^m\) and 12\(^m\) for each of the 3 galactic regions. The different mean proper motions are

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\begin{array}{cccccc}
\text{KAPTEYN and VAN RHIJN} & \text{log N}^m_{m-\frac{5}{35}} & \frac{\mu \text{ observed}}{\mu \text{ true}} & m.e. & \mu \text{ calculated} \\
\text{Galactic latitude} & \pm 0^\circ & \pm 20^\circ & & & \\
4 & -1.775 & 1.115 & 1.115 & \pm 0.0074 & 0.092 \\
8 & -2.588 & 0.398 & 0.398 & \pm 0.0027 & 0.031 \\
12 & -1.988 & 0.017 & 0.008 & \pm 0.0065 & 0.014 \\
\text{Galactic latitude} & \pm 20^\circ & \pm 40^\circ & & & \\
4 & -1.906 & 1.160 & 1.160 & \pm 0.0142 & 0.124 \\
8 & -0.33 & 0.48 & 0.41 & \pm 0.0022 & 0.046 \\
12 & -1.620 & 0.32 & 0.19 & \pm 0.0052 & 0.024 \\
\text{Galactic latitude} & \pm 40^\circ & \pm 90^\circ & & & \\
4 & -2.019 & 1.196 & 1.196 & \pm 0.016 & 0.154 \\
8 & -0.83 & 0.54 & 0.46 & \pm 0.0022 & 0.055 \\
12 & -1.356 & 0.33 & 0.26 & \pm 0.003 & 0.033 \\
\end{array}
\]

The agreement between the mean proper motions of KAPTEYN and VAN RHIJN and those derived in the present paper is, I think, all that could be expected of this provisional calculation. The aim of the present note is to call attention to a simple relation, which appears to be in better agreement with observation than would perhaps be anticipated.

CORRECTION.

In B. A. N. 25, p. 149 a table has been given of 21 binary systems near our sun. Mr. G. P. KUIPER called my attention to the fact that the well known pair \( \gamma \) Cassiopeiae is not contained in that table, as it ought to be. The general conclusions of the note remain unaltered.

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