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Title: Adapted deformations and the Ekedahl-Oort stratifications of Shimura varieties
Date: 2017-10-18
Summary

Let \((G, X) \hookrightarrow (GSp, S_\pm)\) be a Shimura datum of Hodge type. Let \(p > 2\) be a prime number and assume \(G_{\mathbb{Q}_p}\) has a reductive model \(\mathfrak{G}\) over \(\mathbb{Z}_p\). Let \(K \subset G(\mathbb{A}_f)\) be an open compact subgroup, hyperspecial at \(p\), and \(\text{Sh}_K(G, X)\) the associated Shimura variety over the reflex field \(E\) of \((G, X)\). Fix a place \(v\) of \(E\) above \(p\). Denote by \(\mathfrak{O}_{E,v}\) the ring of integers of the completion of \(E\) at \(v\), and by \(\kappa\) the residue field of \(\mathfrak{O}_{E,v}\). Denote by \(S_K(G, X)\) the canonical integral model of \(\text{Sh}_K(G, X)\) over \(\mathfrak{O}_{E,v}\) constructed by Kisin, which is a quasi-projective and smooth scheme over \(\mathfrak{O}_{E,v}\). Denote by \(S\) the special fibre of \(S_K(G, X)\), over \(\kappa\).

In her paper *Truncations of level 1 of elements in the loop group of a reductive group* (Annals of Math. 2014), E. Viehmann introduced and studied some new invariants, called “truncations of level 1”, for elements in the loop group of \(G\), where \(G\) is the special fibre of \(\mathfrak{G}\). It follows from her results that such invariants can be used to parametrize Ekedahl-Oort strata of \(S\), and to give a criteria for the inclusion relations of these strata. But such a parametrization is quite indirect. In this thesis, using the classification result of \(p\)-divisible groups in terms of filtered Breuil-Kisin modules (Breuil-Kisin windows, in our term), we give a more conceptual explanation of Viehmann’s results.

Via constructing Breuil-Kisin windows, we obtain a morphism of schemes \(I_+ \rightarrow \mathcal{D}_1\), where \(I_+\) is an fpf torsor over \(S\) and \(\mathcal{D}_1\) is a quotient of a double coset scheme inside the loop group of \(G\). From this morphism we finally obtain a morphism of fpqc sheaves \(\eta : S \rightarrow \mathcal{D}_1/\mathcal{K}\), where \(\mathcal{D}_1/\mathcal{K}\) is a quotient sheaf of \(\mathcal{D}_1\), whose geometric points correspond to Viehmann’s new invariants (truncations of level 1). We show that the fibres of \(\eta\) on geometric points are exactly the Ekedahl-Oort strata of \(S\) as defined by C. Zhang and D. Wortmann in their theses.