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BELLE STARR RECONSTRUCTION

Light thinks it travels faster than anything but it is wrong. No matter how fast light travels, it finds the darkness has always got there first, and is waiting for it

Terry Pratchett, Reaper Man

The Belle Starr reconstruction is a set of algorithms developed to reconstruct a two shower signature caused by tau neutrino interactions. Additionally, it provides discriminators to distinguish two shower signatures from other signatures in the KM3NeT detector. To achieve this goal the reconstruction is divided into four steps, namely:

- **Prefit**: Reconstruct position, direction and energy based on single shower hypothesis
- **Scan**: Evaluate two shower likelihood along the trajectory established by Prefit
- **Peak**: Apply a peak finding algorithm on the likelihood scan
- **Refit**: Perform a two shower likelihood fit starting at the two positions established by Peak

The Refit step uses all the information present in the event by applying the two shower likelihood. Ideally, one would evaluate the likelihood in each point in space and time, but such a complete scan over the whole volume of the KM3NeT detector is computationally too demanding. Therefore, Refit uses a minimizer to evaluate the two shower likelihood. The performance of such a minimizer in finding the correct minimum is strongly dependent on the starting values of the parameters. Hence, such starting parameters are provided by employing the algorithms Prefit, Scan and Peak.

Dividing the reconstruction into consecutive steps provides practical advantages in the application of the reconstruction, for instance, by allowing optimal CPU usage.

In this chapter the signal and background signatures for the Belle Starr reconstruction are discussed, followed by a description of the different reconstruction steps and their performances. The results presented are based on an isotropic astrophysical neutrino flux of

\[
E^{2.46} \Phi_\nu(E) = 4.11 \times 10^{-6} \left( \frac{E}{\text{GeV}}^{-0.46} \right) \text{GeV cm}^{-2} \text{s}^{-1} \text{sr}^{-1}
\]
with a 3 PeV cut-off (see Chap. 6), unless stated otherwise. Only neutrino events are considered, since at energies exceeding 10 TeV no differences between neutrino and anti-neutrinos are expected.

5.1 Tau “Double Bang” Event Signature

“Double Bang” events are one possible outcome of the CC interactions of tau neutrinos with a nucleon. In the “Double Bang” case, two showers are present, namely: One at the tau neutrino interaction vertex and another at the tau decay vertex for 82.4% of the tau decays (see Sec. 1.2.4.3). The “Double Bang” events are also referred to as “tauCCshow” events in this work. Henceforth, the shower at the neutrino interaction is referred to as the neutrino shower and the shower at the tau decay as the tau decay shower.

While most “Double Bang” events look similar to electron neutrino CC interactions, for tau lepton energies in excess of \(100 \text{ TeV}\) the tau can fly visible distances before its decay. In this case, the two showers of “Double Bang” events can be independently identified. The likeliness of identifying these events mainly depends on the energy distribution between the two showers and the length of the tau flight path.

The energy distribution between the two showers is expressed in the Bjorken \(y\) variable which, in the detector frame, is given by:

\[
y = \frac{E_\tau}{E_\nu},
\]

where \(E_\tau\) is the energy of the tau lepton and \(E_\nu\) the energy of the neutrino that produced it. By definition, Bjorken \(y\) gives the energy distribution between the two showers of a “Double Bang” event not accounting for energy carried away by neutrinos. Since the energy of a shower approximately scales with the number of photons produced, Bjorken \(y\) is a measure for the relative luminosity of the showers. For “Double Bang” events with values of Bjorken \(y\) close to one or zero either of the showers can be so luminous as to make the other shower indistinguishable, causing the event to resemble a single shower signature. This effect is shown in Fig. 51 for event displays of the same event with different Bjorken \(y\) values. As can be seen, with Bjorken \(y\) approaching one the neutrino shower produces less light, to such a degree that it is not identifiable anymore by eye for Bjorken \(y = 0.99\). Although these event displays cannot portray the full event information (e.g. they lack the time of all hits) they give a good indication on the likeliness of reconstructing two showers.

The second important characteristic is the spatial separation between the two showers, which is given by the tau flight length. In order to distinguish “Double Bang” events from single shower events, they have to be separated by visible distances. For most “Double Bang” events the two showers are mainly overlapping due to the short lifetime of the tau lepton. However, at relativistic
Figure 51: Event displays for a toy MC simulation of a “Double Bang” event with different Bjorken $y$ values but otherwise identical kinematics; initial neutrino energy of 500 TeV and a tau flight length of 200 m; the red line indicates neutrino flight path; colored spheres show hit DOMs, the size of the spheres indicates the number of hits and the color indicates the time of the first hit.
energies, the tau lepton can fly visible distances. The relation between tau flight length $L_\tau$ and tau energy at mean lifetime is given by:

$$L_\tau = 4.9 \text{ m} \times \frac{E_\tau}{100 \text{ TeV}} .$$

The distribution of the tau flight lengths as a function of the neutrino energy is shown in Fig. 52. As anticipated, the tau flight length increases with simulated energy. The Belle Starr reconstruction is expected to resolve the two showers if they are separated by a distance larger than its position resolution. Typical position resolutions for shower events in KM3NeT are around one meter, corresponding to a tau lepton energy of around 100 TeV and higher. Tau events with shorter flight length cannot be distinguished from single shower events using a position reconstruction. An example of the apparent distance between showers is shown in Fig. 53. In the figure, the same “Double Bang” event is shown for varying tau flight length. As can be seen, the two showers are easily identified at a flight length of 300 m, while they are hard to be identified at 150 m and at 70 m the signature looks like a single shower by eye.

The requirements for Bjorken $y$ and tau flight length for a “Double Bang” event to be identifiable are contradictory. While large Bjorken $y$ enhances the energy of the tau lepton and thereby the tau flight length, it can cause the tau decay shower to be too luminous to identify the neutrino shower. This conflict is especially significant at neutrino energies of $\mathcal{O}(100 \text{ TeV})$, since the tau lepton needs a significant fraction of the energy to achieve visible flight length. The energy dependence is shown in Fig. 54, by requiring a minimum flight length of 5 m and looking at the tau decay shower energy versus the neutrino shower energy. As can be seen, the distribution at energies below $\mathcal{O}(100 \text{ TeV})$ favors
Figure 53: Event displays for a toy MC simulation of a “Double Bang” event with different simulated tau flight length but otherwise identical kinematics; initial neutrino energy of 2 PeV and a Bjorken $y$ of 0.7; the red line indicates the neutrino flight path; colored spheres show hit DOMs, the size of the spheres indicates the number of hits and the color indicates the time of the first hit.
the tau decay shower to have more energy than the neutrino shower while no such effect is visible for larger energies. If one would require a larger tau flight length, the asymmetry would increase and extend to higher energies. Due to this conflict, identifying a clear threshold value of Bjorken y at which “Double Bang” events are identifiable is not straightforward. One should keep in mind, that large Bjorken y values benefit the tau flight length and thereby tau identification, but extremely large y values can be detrimental to the “Double Bang” identification.

In addition to the Bjorken y and tau flight length, it is more practical for both vertices to be located within the instrumented detector volume. A “Double Bang” event is considered contained, if both the neutrino interaction vertex and the tau decay vertex are located within the instrumented detector volume.

For the above reasons, a subclass of ideal “Double Bang” events is selected. These events are required to have a minimum tau flight length of 5 m and to be contained. They are referred to as “tau signal events” in this work. This selection causes the overall Bjorken y distribution to be peaked at one as shown in Fig. 55, which is dominated by events at $O(100 \text{ TeV})$. 

Figure 54: Visible energy of the tau decay shower plotted against the visible energy of the neutrino shower shower for “Double Bang” events which fly at least 5 m and have both vertices contained in the detector; Z-axis: the rate per year per block.
This section addresses how the track and single shower events can mimic a “Double Bang” signature. In the following, the most likely scenarios leading to “Double Bang” like signatures are illustrated.

**Tracks:** The muons can lose a significant fraction of their energy due to Bremsstrahlung. This will create showers along the track. These showers can cause “Double Bang” like signatures, since they are separated in time and space by the propagation of a relativistic particle. For neutrino induced track events, one shower is produced at the neutrino interaction vertex, requiring a single Bremsstrahlung shower to mimic a “Double Bang” signature. For atmospheric muons, two Bremsstrahlung showers or multiple muons are required.

Some event displays of track events which produce a signature that can mimic “Double Bang” events are shown in Fig. 56.

**Showers:** There are three scenarios which can result in a “Double Bang” like signature: High energetic single showers, production of muons in the showers or different shower evolutions for showers with the same origin.

In the case of highly energetic showers, a single such shower can emit $O(10^6)$ or more photons. This causes PMTs up to hundreds of meters away to be illuminated. In addition, a large amount of photons may scatter. Since scattered light arrives later at the PMTs, they can produce signals that mimic the presence of a second shower. An event display of such an event is shown in Fig. 57a.

In the rare case of a high energy secondary muon, the muon will propagate through the detector. It can then cause shower signatures by Bremsstrahlung leading to “Double Bang” like signals as discussed for track events. An event display of such an event is shown in Fig. 57b.

For two showers originating at the same vertex, different shower evolutions can be caused by a difference in particle content or energy. The effect can
(a) A numuCC with 7.2 PeV and 3.5 TeV muon energy.  
(b) An atmospheric muon event with a single muon of 0.88 PeV at can level.

Figure 56: Event displays for track events; the red line indicates neutrino flight path; colored spheres show hit DOMs, the size of the spheres indicates the number of hits and the color indicates the time of the first hit.

cause variations between shower maxima positions of up to 0(10 m), thereby mimicking a “Double Bang” signal.
Figure 57: Event displays for nueCC events; the red line indicates neutrino flight path; colored spheres show hit DOMs, the size of the spheres indicates the number of hits and the color indicates the time of the first hit.

5.3 BELLE STARR PREFIT

The Prefit performs a fit of the position followed by a simultaneous fit of the direction and energy of a single shower to the data. The routine is adapted from the AAShowerFit package developed by A. Heijboer [112] by changing the hit selection for the position fit.

5.3.1 Position and time fit

The position and time fit is performed using an M-estimator fit method. M-estimators are generalized least-squares estimators which allow for a modification of the normal distribution [113]. These modifications can have multiple purposes. In this case they are used to reduce the effect of outliers on the results of the fit, thereby making the fit more robust. Robustness in a statistical sense was coined in 1953 as: “insensitive to small departures from the idealized assumptions for which the estimator is optimized” [114]. Such a reduction is needed due to the optical background present.

The M-estimator is used to fit the position and time (4 parameters) assuming a single shower vertex. As a starting point of the minimization, the position and time of the earliest hit on the DOM with the most hits is used. The minimization is based on the score-function $m$, given by

$$m = A_h \times \sqrt{0.5 + \Delta T_{ih}^2} ,$$

where $A_h$ denotes the number of hits and $\Delta T_{ih}$ the difference between the hit time and the expected arrival time (so-called hit time residual) for a shower assumption. By minimizing $m$, the reconstructed position and time correspond
to that of the shower maximum position. The hit time residual for the shower assumption is given by:

$$\Delta T_h = \text{dst}(\text{pos}_h, \text{pos}_{\text{vertex}})/c_{\text{water}} + T_{\text{vertex}} - T_h,$$

where \(\text{dst}(\text{pos}_h, \text{pos}_{\text{vertex}})\) is the distance between the assumed shower vertex and the hit PMT, \(c_{\text{water}}\) is the speed of light in water, \(T_{\text{vertex}}\) is the assumed time of the shower vertex and \(T_h\) is the time of the hit. The speed of light in water is defined as \(c_{\text{light}}/n_{\text{water}}\) with \(n_{\text{water}} = 1.35\) the refractive index corresponding to KM3NeT like sea water.

If multiple shower vertices are contained within the detector, the fit is biased towards the shower with the most hits. As the number of hits a shower produces approximately scales with the shower energy, the prefit is biased to reconstruct the position of the most energetic shower. This bias can be adjusted by employing a hit selection. For “Double Bang” events, biasing the hit selection towards the neutrino shower is found to be beneficial for the reconstruction performance: The direction reconstruction gets improved if the tau decay shower is downstream because it causes the maximal amount of hits point in the tau flight direction.

In order to create a sample of hits which are dominated by the neutrino shower the relative timing of the two showers is used. Since the neutrino interaction is always earlier in time, one would intuitively expect the neutrino shower hits to arrive first on a PMT. However, the assumption does not hold for the arrival time of the hits on all PMTs. While light travels at the speed of light in water, the tau lepton travels at the speed of light in vacuum which is a factor \(n_{\text{water}}\) faster. This enables the tau to “overtake” the light emitted by the first shower. Some light from the tau decay can actually arrive earlier than the light from the first shower. However, this effect only applies to a relatively small subset of the hits. Therefore, selecting the first hits on DOMs and PMTs produces a selection biased towards the neutrino vertex. For the prefit all hits on the same PMT within 500 ns are merged into the first hit. The effect of the hit merging is shown in Fig. 58. As can be seen, without a hit selection, the prefit typically yields the position of the tau decay vertex. Once the hit selection is applied, the prefit reconstructs a position on the tau flight path which is most likely inbetween the two showers. Such a position on the axis between the two showers is beneficial for the later steps of the reconstruction compared to the large spread of positions without the hit selection.

While this hit selection, on average, leads to reconstructing the neutrino vertex, one problematic case remains, namely: A selection of approximately equal number of hits from both showers. In such cases, the M-estimator score function does not have a global minimum at either shower position, but rather at a position somewhere else. This position is typically far outside of the detector, resulting in a low efficiency reconstruction.

This unwanted feature can be counteracted by adding another step to the hit selection, namely clusterizing. Clusterizing uses the correlation between hits given a certain hypothesis and finds the largest sub sample of correlated hits.
Given a shower hypothesis, hits are matched as follows: Per hit the number of hits with which this hit is correlated is determined and then the hit with the smallest number of correlated hits is discarded. This process is repeated until the hit with the lowest number of correlated hits has as many correlated hits as there are hits left in the selection. For showers, hits are correlated using the hit time differences assuming maximal spatial distance $D_{\text{max}}$. For the “Double Bang” events $D_{\text{max}} = 500$ m is chosen in order to select all hits. The maximal allowed time difference for two correlated hits from the same shower vertex depends on the distance between the two hits $d$ in relation to the maximal distance $D_{\text{max}}$ as given by:

\[
\begin{align*}
\text{if } d &\leq 0.5 \times D_{\text{max}} : \\
\Delta T(\text{hit}_i, \text{hit}_j) - T_{\text{extra}} &\leq d/c_{\text{water}} \\
\text{if } 0.5 \times D_{\text{max}} &\leq d \leq D_{\text{max}} : \\
\Delta T(\text{hit}_i, \text{hit}_j) - T_{\text{extra}} &\leq (D_{\text{max}} - d)/v_{\text{water}} .
\end{align*}
\]

Considering that $D_{\text{max}}$ represents the typical distances between hits from a single shower, the reason for the two different cases is given by geometrical limitations: If the two hits are separated by more than half of the specified $D_{\text{max}}$, the shower vertex has to be inbetween the two hits, thereby limiting the maximal time difference the hits can have. The quantity $T_{\text{extra}}$ ensures that hits with extremely small distances are correlated independent of their distance. It is set to be 30 ns.

In the case of multiple showers, the algorithm starts by discarding the hit with the smallest number of correlated hits. If there is more than one such hit, one of them will randomly be discarded. As a consequence, the number of correlated hits will be reduced for the cluster to which the discarded hit belongs, causing clusterize to continually discard hits from that cluster. This results in a sub-sample of hits from one shower with a high purity. This effect
is shown for a single “Double Bang” event with approximately equal hits from both showers in Fig. 59. In the top row, the distribution of hits for the event is shown before clusterize is applied. As can be seen, two clusters of similar sizes are present above a constant background of noise hits. These clusters correspond to the neutrino interaction and the tau decay vertices, respectively. In the bottom row, the same event is shown after applying clusterize. As can be seen, only one cluster of hits remains.

For the other “Double Bang” events the hit selection already selected a sample of hits biased towards one of the vertices. Starting on such a sample, the cluster algorithm will enhance the purity of that selection.

The added effect of applying the cluster algorithm to all “Double Bang” events is shown in Fig. 60. As can be seen, applying the cluster algorithm improves the performance of the prefit to find either of the two vertex positions at all simulated tau flight lengths. Therefore it is used as a final step of the prefit hit selection.

In addition to the hits, the used starting values could influence the prefit performance. Therefore, the prefit was tested by using either the simulated neutrino interaction or the simulated tau decay vertex as a starting point. It was found that the reconstructed prefit position is independent of the used starting
position, proving the position fit result to be independent of the starting values as long as they are within reasonable limits (e.g. the instrumented detector volume). Therefore, the position of the DOM with the most hits and the time of the first hit on that DOM are used as starting values (as done in AAshowerfit).

**Position reconstruction**

The achieved precision of the single shower position reconstruction is around 1 m for KM3NeT \cite{km3net}. The performance for single showers can be considered as a benchmark for events with multiple showers.

For multiple showers events the determination of the precision of the position reconstruction is ambiguous, as multiple true shower positions exist. A solution is to scrutinize the position resolution by using the minimal distance between the reconstructed position and all shower maxima positions as shown in Fig. 61. As can be seen, the minimal median distance achieved by the Prefit in the “Double Bang” case is around 3 m. The performance is worse than for single showers due to the presence of two showers. This also causes the long tails in the distribution. The distance between the reconstructed positions and either shower being larger than 10 m for approximately 10% of the events.

For a better understanding of the position reconstruction performance for “Double Bang” events, the dependence of the position resolution on the tau flight length is helpful. In Fig. 62 the distance between the reconstructed position...
and the neutrino shower maximum is shown as a function of the tau flight length. There are two populations: one at distances close to zero and another at distances equal to the tau flight distance. The population at distances smaller than 5 m corresponds to the Prefit reconstructing the neutrino interaction vertex and the population at distances equal to tau flight length corresponds to the Prefit reconstructing the tau decay vertex. Only a small number of events is not found within the two populations. These are the events in which the Prefit did not reconstruct either of the two vertices.

Given the median resolution and the robustness of the starting values, the prefitted position is used as input to the subsequent reconstruction steps.

**Time reconstruction**

Minimizing the score function recovers also the time of the interaction. In order to check the performance of the time reconstruction, the reconstructed time is compared to the simulated time of that position as given by:

\[
T_{\text{res}} = T_{\text{MC}_\nu} + \text{dst} (\text{pos}_\nu, \text{pos}_{\text{rec}})/c_{\text{light}} - T_{\text{rec}},
\]

where \(T_{\text{MC}_\nu}\) is the simulated neutrino interaction time, \(\text{dst} (\text{pos}_\nu, \text{pos}_{\text{rec}})\) is the distance between the neutrino interaction and the reconstructed position. Therefore, \(T_{\text{MC}_\nu} + \text{dst} (\text{pos}_\nu, \text{pos}_{\text{rec}})/c_{\text{light}}\) is the simulated time of the reconstructed position, assuming the shower propagates with \(c_{\text{light}}\). In most cases, this position will correspond to the shower maximum. A plot of the time resolution \(T_{\text{res}}\) for the different neutrino channels is shown in Fig. 63a. As can be seen, all neutrino channels have an positive offset corresponding to the reconstructed...
time being too early. The offset is especially pronounced for the tau signal events.

This offset is found to be increasing with simulated neutrino energy as shown in Fig. 62b for single showers and the effect is even larger for “Double Bang” events as shown in Fig. 62c. This causes the larger offset of approximately 5 ns for the tau signal events as shown in Fig. 62a, as they are events with high energies. As a more refined fit will be made afterwards, this offset is simply corrected for by adding 5 ns to the reconstructed time after the Prefit step.

5.3.2 Energy and direction fit

The energy and direction fit is adopted from the AAShowerfit reconstruction. It is a likelihood minimization based on the probability of a PMT being not hit or hit given a shower energy and direction. This section will illustrate the reconstruction procedure and the performance for “Double Bang” events.

The energy and direction fit takes the result of the position Prefit as a starting point to reconstruct the direction and energy (4 parameters). In order to evaluate the $4\pi$ phase space evenly, the likelihood is minimized for different starting directions. These are chosen as the 12 corners of an icosahedron around the prefit position. For the energy the start parameter is arbitrarily set to $10^{10}$ TeV.

For each starting direction a likelihood minimization is performed based on a multidimensional probability density function (PDF). The PDF describes the probability of a PMT being not hit or hit with one or more photons from a shower given the PMT alignment, shower direction and shower energy. The likelihood is evaluated for all hits with $|\Delta T_h| \leq 800$ ns given the reconstructed
(a) Distribution of time difference between simulated neutrino interaction time and reconstructed time.

(b) Time resolution as function of simulated neutrino energy for nueCC single shower events.

(c) Time resolution as function of simulated neutrino energy for tauCC-show events.

Figure 63: Time resolution performance studies.
position. From the 12 fit results, the one with the best likelihood value is selected as the result of the minimization.

The performance of the direction fit depends on the energy of an event. A minimal energy of 10 TeV is required for the shower to produce sufficient hits in the ARCA detector to be properly reconstructed. Above this energy, the direction reconstruction performance for single showers changes with energy as shown in Fig. 64 on the right. The median angular resolution varies between 1.5° to 2° and is worst at low and high energies.

A similar performance is to be expected for “Double Bang” events because the two showers give comparable photon distributions to a single shower event. The resulting angular resolution of the tau lepton (which closely matches that of the neutrino) and the reconstructed energy resolution are shown in Fig. 65 for tau signal events. As can be seen in Fig. 65a a median angular resolution of around 2° is achieved which is indeed comparable to the resolution for nueCC events.

Figure 65b and Fig. 65c show the reconstructed energy as a function of the simulated visible energy for contained nueCC and tau signal events respectively. The distributions show the same correlation between simulated and reconstructed energy, demonstrating a successful energy reconstruction for contained “Double Bang” events. The underestimation of energies in the region below $O(100 \text{ TeV})$ is a known bias which can be correct. This effect is negligible for a “Double Bang” reconstruction, due to the characteristic energies of the tau signal events.

Directly inferring the neutrino energy from the reconstructed energy in the “Double Bang” case is not possible, since the tau decays produces at least one neutrino. The neutrino, on average, carries away around one third of the tau energy for three body decays. Since the tau lepton decays into three particles about 60% of the time, the neutrino can be estimated to have about 25% of the tau energy for all tau decays. Therefore, the reconstructed energy can be considered a lower bound of the true tau energy in the “Double Bang” case.
5.4 **BELLE STARR SCAN**

The Scan algorithm utilizes the results of the Prefit to scan a sub-region of the detector volume for a second shower vertex. For this purpose, a two shower position likelihood is evaluated along the Prefit trajectory. In theory, the evaluation of such a likelihood for the whole detector volume could optimally reconstruct the two shower parameters. But, as discussed before, such a scan requires significant computational resources. Restricting the evaluation of the likelihood to a trajectory results in a statistically robust and computationally less demanding procedure. As discussed in Sec. 5.3, the Prefit yields a good position resolution for one of the showers and good tau lepton direction resolution. Therefore, the search for the second shower position can be limited to points along the Prefit trajectory.

In the following, first the characterization of the “Double Bang” likelihood is discussed and then the scan procedure is elaborated upon.

“**Double Bang**” likelihood

A two shower position likelihood for “Double Bang” events has a total of seven free parameters, namely: Six position parameters and one interaction time. The
second time can be concluded from the distance between the positions, as the
two vertices are connected by the tau flight duration.

The “Double Bang” likelihood for a given hit, is defined as the sum of the
probabilities of the hit originating from either shower vertex and the probability
of the hit being background:

\[ \mathcal{L}_i = c_1 \times P(\Delta T_{1,i}) + c_2 \times P(\Delta T_{2,i}) + c_3 \times P(\text{hit}_i | \text{bkg}), \quad (28) \]

where \( \Delta T_{1,2,i} \) is the hit time residual given shower 1/2, \( P(\Delta T_{1,2,i}) \) is the
probability that hit \( i \) is a hit originating from shower vertex 1/2, \( P(\text{hit}_i | \text{bkg}) \) is the
probability of the hit being a background hit and \( c_i \) are weighting parameters.
The different probabilities are added in the likelihood since they are indepen-
dent hypotheses: a hit can only be caused by one of the shower vertices or
background. In theory, a fourth probability should be added for the hit to
be caused by the tau lepton. The tau lepton is minimum ionizing and hence
the amount of light produced by the tau lepton as it traverses the medium
is negligible. The total likelihood is given by the product of the likelihood of
all hits. The characterization of the different hit probabilities and weighting
parameters is discussed in the following paragraphs.

The probability of a hit being produced by a shower (signal hit) is parama-
eterized by a one-dimensional probability distribution function (PDF) of the hit
time residuals given a single shower. In general, the PDF is a multi-dimensional
function depending on other parameters such as shower energy, shower di-
rection and the relative position and orientation of the PMTs and shower. In
order simplify the reconstruction, only a one-dimensional PDF of the hit time
residuals is used.

The PDF is obtained by averaging the hit time residual distribution of 100
nueCC single shower events obtained from MC truth. These events are sim-
ulated with a shower energy of 1 PeV and contain no optical background.
The resulting distribution is stored in a histogram and normalized such that
its integral is one in order to represent a probability. Since the histogram is
binned in hit time residuals, the PDF can suffer from binning effects. To avoid
these, the cumulative histogram is fitted with a monotonously increasing spline
function to achieve a continuous function. The spline fit is developed by A.
Heijboer \[112\] in order to avoid irregularities which may be caused by empty
bins in the residual histogram.

For the used hit selection a binned representation of the used spline function
is shown in Fig. 66. Considering that the shape of the PDF depends on the used
hit selection, this representation is only applicable to the specific hit selection
of the scan procedure. The distribution shows a peak at zero with a FWHM of
around 20 ns and a tail towards higher residuals. The peak and tail are caused
by direct and scattered light respectively. Between them is a slight dip around
hit time residuals of 30 ns which is caused by the merging of hits on the same
PMT as discussed in Sec. 4.3

Also shown in Fig. 66 are the PDF for different shower energies. As can be
seen, the PDFs have different peak to tail ratios. With increasing shower
energy, the tail of the distribution increases since the amount of scattered light
increases. Since the distributions are scaled to have an integral of one, this causes a decrease in peak height. Except for this shift in the peak to tail ratio the histograms show little variation with energy in the region of interest for “Double Bang” events. This supports the simplification to ignore the dependence of the PDF on shower energy. Otherwise, a much more complex reconstruction would be necessary. For the likelihood the PDF obtained for an electron energy of 1 PeV is used.

The probability of a hit being a optical background hit is given by the hit selection and the expected rate. For the used trigger time window of approximately 5 µs, hit merging within 500 ns on a PMT and an assumed rate of 5 kHz per PMT, the relative probability of a hit being background is around 0.008%. This probability depends to some degree on the energy of the showers event, since more energetic showers produce more hits and thereby decrease the probability of a background hit. But the change in probability is small and hence neglected.

The weighting constants $c_i$ are then determined such that $L_i = 1$ using $c_1 = c_2$ and $c_3 * P(\text{hit}|\text{bkg}) = 0.008\%$. In general, the assumption of $c_1 = c_2$ is only true for both showers producing an equal amount of light. But since a study in which the ratio $c_1/c_2$ has been adjusted according to Monte Carlo true information has shown no improvement in reconstruction performance and obtaining an estimate of the single shower energy distributions is not simple. Therefore, the assumption of $c_1 = c_2$ is used.

Instead of maximizing the likelihood, the negative log likelihood is minimized in order to simplify the computation. Therefore, the position of the second...
shower is given by the minimal value of $-\log L$. The negative logarithmic two shower likelihood is given by

$$-\log L = -\sum_i \left[ \log L_i \right].$$

(29)

**Scan procedure**

In the Scan, “Double Bang” likelihood from Eq.29 is evaluated for each step of one meter in an interval of ±800 m along the Prefit direction, resulting in 1601 scan points. While one shower vertex is set to the Prefit position, the other shower vertex position is set to the scan point. The time of a given scan point is given by the tau lepton propagation time between that point and the Prefit position. For each point, the likelihood is evaluated for all hits present in the event.

The interval range is chosen such that all positions within the detector volume are included. In order to avoid edge effects and limit computational time the likelihood is simply set to the maximum value of $\sum_i -c_3 * P(\text{hit}_i|\text{bkg})$ for scan points outside the detector volume.

The resulting likelihood scans for “Double Bang” events fall into different categories depending on the Prefit performance and event topology. The two main types for “Double Bang” events are likelihood scans with either one or two minima. The two scenarios are discussed in detail in the following.

Once the likelihood is scanned, the second reconstructed position is set to the position of the minimal bin in the likelihood scan while the first vertex position remains at the reconstructed Prefit position. If the likelihood scan has no minimum bin (e.g. is constant) the second vertex position is also set to the Prefit position. This can happen if neither reconstructed vertex is contained and the likelihood is set to the background probability as discussed earlier.

**Single minimum in likelihood**

Scans with one significant minimum are typically caused by events with a very asymmetric energy distribution between the two showers, resulting in one shower which produces most of the hits and the other shower to be basically invisible. This event topology is most likely for events with Bjorken $y$ approaching one or zero (as discussed in Sec. 5.1).

**Bjorken $y \rightarrow 1$:** For such events the tau decay shower has most of the initial neutrino energy. In that case, the neutrino vertex can often be recovered during the Prefit by a hit selection focusing on early hits as discussed in Sec. 5.3.1. A scan of an event with a very asymmetric energy distribution towards the tau decay shower in which the Prefit still selected the neutrino interaction vertex is shown in Fig. 67a. The Prefit position is given at distance zero, where the likelihood shows a little dip. The likelihood has a global minimum at 20 m which corresponds to the simulated tau decay shower maximum position. A scan of an event in which the Prefit failed to reconstruct the neutrino interaction
vertex because the energy distribution is even more asymmetric is shown in Fig. 67b. In this case the zero position is at the tau decay position together with the only significant minimum in the likelihood. At $-20$m a small dip is visible where the neutrino vertex is located.

**Bjorken $y \to 0$:** Such events are only of relevance at the highest energies, since at low energies the tau flight length is typically too short to distinguish “Double Bang” and single shower events. For Bjorken $y$ approaching zero the tau shower can already be difficult to detect at $y$ values of 0.3. The reason being that the neutrino shower hits are earlier than the tau decay shower hits and therefore hit merging has a larger impact. An example of a scan of such an event is shown in Fig. 67c. There is only one significant minimum at zero distance which corresponds to the simulated neutrino interaction vertex.

**Two minima in likelihood**

The scans which result in two minima are events in which the energy distribution between the two showers is reasonably balanced (Bjorken $y$ between 0.3 to 0.9). For such energy distributions both showers produce a significant amount of hits. As a result, both are visible in the likelihood scan as a significant minima. In the case of a successful Prefit, one of the minima is at the Prefit position while the other is either at positive distances (the prefit reconstructed the neutrino hadronic shower position) or at negative distances (the prefit reconstructed the tau decay shower position). The global minimum is typically located at the shower which produced most hits. An example of a scan of such an event is shown in Fig. 67d.
(a) Event with tau flight length 19.8 m, total energy of 205.2 TeV and Bjorken $y$ of 0.87.

(b) Event with tau flight length 17.3 m, total energy of 247.2 TeV and Bjorken $y$ of 0.998.

(c) Event with tau flight length 17.3 m, total energy of 1241.1 TeV and Bjorken $y$ of 0.07.

(d) Event with tau flight length 37.2 m, total energy of 328.6 TeV and Bjorken $y$ of 0.54.

Figure 67: Likelihood scans for different contained “Double Bang” events; zero on the X-axis marks the reconstructed Prefit position; the scan is performed along the Prefit direction; the discrete steps in the likelihood are scan positions outside of the detector volume at which the likelihood is set to be the pure background hypothesis.
The Belle Starr Peak algorithm evaluates the likelihood scans produced by the Scan. The main goals are to identify the significant minima and establish characteristics which help to suppress background events. In order to achieve this, the program utilizes a peak finder algorithm called TSpectrum which is part of the ROOT package [115, 116].

The algorithm was originally developed to determine the peaks and the continuous background for gamma ray spectra. Since the algorithm searches for peaks, the likelihood scans are inverted to turn the minima in the negative log likelihood into maxima. In the following, the background and peak identification of the algorithm and its use in the reconstruction chain are discussed.

### 5.5.1 TSpectrum background estimation

The background estimation is performed by the statistics-sensitive non-linear iterative peak-clipping algorithm (SNIP) developed in 1988 by C.G. Ryan et al. [117]. The algorithm depends on one parameter which is the typical width $\sigma$ of peaks present in the spectrum. Peaks with widths differing from $\sigma$ will generally be included in the continuous background although very significant ones can remain. For the likelihood scans a peak width of $\sigma = 4$ m is used (see Sec. 5.5.3). The background estimation is performed in three consecutive steps.

In the first step the values $y(x)$ of the scan histograms are internally compressed in order to reduce the dynamic range for a robust background estimation. This compression is needed in gamma ray spectra since the $y$-values span six orders of magnitude. Although this is not the case for the likelihood scans, the compression is incorporated in the algorithm package anyway. Nonetheless, the SNIP algorithm is appropriate for estimating the continuous background in the likelihood scans. The compression is performed according to Eq. 30, resulting in the values $C(x)$ and is reverted later.

$$C(x) = \log(\log(y(x) + 1) + 1) .$$

In the second step the actual background estimation takes place. Here, the compressed likelihood scan histograms are iterated with a so-called peak clipping loop (for 20 iterations a sufficiently good background estimate could be achieved). The peak clipping loop estimates the background in each bin $i$ of the scan histograms as given by

$$bkg(bin_i) = \min[C(x), C(x, \sigma)]$$

$$C(x, \sigma) = (C(x + \sigma) + C(x - \sigma))/2 .$$

In the third step, the obtained continuous background is decompressed and subtracted from the scan histogram.

Examples of the SNIP procedure applied to inverted likelihood histograms are shown in Fig. 68.
5.5.2 TSpectrum peak identification

The basic concept of most peak finding algorithms is evaluating the derivative of a given function or histogram. A change in the sign of the derivative from positive to negative identifies a maximum in the function or histogram. In addition, also the second derivative can be considered. Both of these methods suffer from random fluctuations due to low statistics. In order to overcome such problems, TSpectrum preprocesses the scan histograms with a peak enhancer \[118\]. The peak enhancer increases the peak to background ratio of all peaks present in the histogram.

The peaks are enhanced using an algorithm based on an analogy to the tunneling effect in quantum mechanics. Suppose a ball is lying in a given bin on the left flank of a peak. In a classical scenario, the ball will roll down the slope. If one gives the ball a non-zero chance to “tunnel” up the slope, it can move in the opposite direction. In order to utilize this idea to find peaks, one has to set the probabilities such that tunneling to the neighboring bin with higher \(y(x)\) value is more likely than “rolling down” the slope to the bin with lower \(y(x)\) value. When this procedure is iterated often enough and the position of each step is recorded, it results in a histogram with enhanced peaks. Examples of how the preprocessing enhances the peaks can be seen in Fig. 69. The algorithm is implemented as a finite Markov-Chain \[119\] for optimal effectiveness. The processed histograms are then analyzed with a first derivative peak finder algorithm as discussed in \[120\].

5.5.3 TSpectrum for likelihood scans

Applying TSpectrum to a likelihood scan histogram is a combination of the background estimation and peak finding method discussed in Sec. 5.5.1 and Sec. 5.5.2 respectively. The different steps combined have three input parameters: the assumed peak width \(\sigma\) for the background estimation, the maximum number of peaks to recorded (in descending peak height) and a threshold value.
Figure 69: Results of the Markhov-Chain peak enhancement procedure for one likelihood scan; the negative logarithm of the likelihood is shown as a function of distance to the prefit position on the prefit trajectory.

for peak rejection. The peaks are rejected based on their height \( h \) in comparison to the height of the highest found peak \( h_{\text{max}} \) as given by:

\[
h \leq h_{\text{max}} \times T ,
\]

where \( T \) is the threshold value. Discarded peaks do not count towards the maximum number of found peaks. The choice of peak width \( \sigma \) and threshold values can be optimized to match the tau “Double Bang” event signature while suppressing background signatures. Successfully reconstructed “Double Bang” events are expected to have one or two peaks, while background signatures can have any number of peaks. In the following the influence of different threshold and \( \sigma \) values on the results of the peak finder are discussed.

In order to study the influence of different threshold and \( \sigma \) values, events are processed with the peak finder algorithm multiple times. While varying the threshold value a \( \sigma \) value of 4 m is used, while varying the \( \sigma \) value a threshold of 0.3 is used. Furthermore, only events with a reconstructed energy of \( \log_{10}(\text{rec.}E \text{ [GeV]}) \geq 4.5 \) and the first reconstructed vertex contained in the detector volume are considered (see Chap. 6).

The results for varying the threshold value are shown in Fig. 70. Figure 70a shows the number of found peaks for a given threshold value for tau signal events. Figure 70b shows the number of events with only one or two peaks for a given threshold for all channels. As can be seen from Fig. 70a, the number of peaks in an event decreases with an increase in threshold, causing most events to have only one or two peaks at the highest threshold. This causes the increase in events with only one or two peaks as shown in Fig. 70b for all channels. The decrease with threshold value is caused by the definition of the threshold as given in Eq. 33.

The results for varying the \( \sigma \) value are shown in Fig. 71. Figure 71a and Fig. 71b are in analogy with the threshold variation. As shown in Fig. 71a, the number of events with more than eight found peaks decreases with an increase in \( \sigma \), while the number of events with only one or two peaks is almost
constant. This is caused by the fact, that for a small sigma value, the background fluctuates more. As can be seen from Fig. 71b, the number of events with only one or two peaks is approximately constant for all channels.

The final values for the threshold and sigma are fixed such that the number of events where one or two peaks have been found is maximal for tau signal events. As can be seen from Fig. 70b, the tau signal drops for threshold values smaller than 0.3 and remains constant at larger values. Hence, a threshold value of 0.3 has been chosen. As can be seen from Fig. 71b, the tau signal rate is approximately flat for sigma larger than 4 m whereas the rate of all background channels increases slightly. Since for sigma values smaller than 4 m the signal rate drops off, a sigma value of 4 m is chosen.

After determining the sigma and threshold values, the resulting peaks are evaluated for their significance. This is done in order to avoid statistical variations from the peak enhancement step being identified as a peak. Therefore, a
Figure 72: Peak height over bkg scan studies for a used threshold value of 0.3 and sigma value of 4; all selected events have vertices contained in the detector volume; atmospheric muons scaled by $10^{-3}$.

value which determines the significance of peaks compared to the estimated background is used. Such a discriminator could be the difference, C, in height between the peak and the background. The influence of such a cut C on the different channels for different values of C is shown in Fig. 72. The Figure shows for values $C \leq 90$, the number of events with one or two significant peaks increases with increasing C. The reason is that at very low C the likelihood scans are more likely to have more than two significant peaks. For $C \geq 90$ the number of events with one or two significant peaks drops. In this case, even significant peaks are rejected since they no longer yield a sufficiently large value.

Figure 72b shows the rate of events with one or two significant peaks as a function of C for all channels. The rate of the background channels is rapidly decreasing with increasing C while the signal channel only shows small variations up to $C = 90$. Therefore, the value of $C = 90$ was chosen. This value provides for a large yield of signal events while keeping the backgrounds rates low.

Examples for the resulting significant peak positions of the likelihood scans are shown in Fig. 73.

5.5.4 Improvement of reconstructed positions

So far, the reconstructed shower positions are set to the position from the Prefit and that of the global minimum of the likelihood scan. For likelihood scans with two significant peaks, the position of these peaks is a better estimate for the vertex positions. Therefore, the two reconstructed vertex positions are set to the two peak positions in such cases. The effect of readjusting the peak position on the length resolution is shown in Fig. 74. As can be seen, after this readjusting the distribution is more sharply peaked and the tail of badly reconstructed lengths is suppressed. The distribution after adjusting shows two peaks. The
5.5 Belle Starr peak

(a) Likelihood scan of “Double Bang” event with two significant peaks.
(b) Likelihood scan of “Double Bang” event with one significant peak.
(c) TSpectrum processed likelihood scan from Fig. 67c.

Figure 73: Likelihood scan histogram as processed by TSpectrum with parameters $\sigma = 4$ m, threshold = 0.3 and $C = 90$.

larger peak is offset from zero by around $-7$ m. This offset is introduced by the Prefit time offset. Events with two peaks typically do not show the time offset that is corrected for after the Prefit. The time of the positions in the scan are therefore off 5 ns. This offset is later compensated in the last step of the Belle Starr reconstruction.

5.5.5 Vertex position reconstruction performance

Once the vertex positions are determined the position reconstruction performance can be validated. As mentioned before, the reconstructed position is not exactly at the interaction or decay vertex, but closer to the position of the shower maximum. Because most photons are –by definition– emitted at the shower maximum. Therefore, the reconstruction performance is evaluated by comparing the reconstructed positions with the calculated shower maximum position which is computed using MC information. The distance between shower maximum and vertex position is $O(\text{meter})$ and depends on the energy and type of the shower as shown in Fig. 75.

Since for “Double Bang” events two shower vertices are present, the reconstructed positions have to be compared to the correct vertex to obtain the position reconstruction performance. In the following, the vertex reconstructed earlier in time is compared with the simulated neutrino shower maximum and
the one later in time is compared with the simulated tau decay shower maximum. The resulting distributions between the distances of the reconstructed position to the neutrino shower maximum and tau decay shower maximum are shown in Fig. 76. The distribution of distances shown in Fig. 76a has a single peak with a median position resolution of 2.5 m. The distribution of distances shown in Fig. 76b shows two distinct peaks of comparable height separated by around 4 m. These peaks are only present for hadronic decay modes of the tau lepton.

Looking in detail at the differences in the tau hadronic decay modes, the cause for the two peaks in the position resolution can be found in the treatment of the pions in the simulation: While showers from neutral pions are like those of electrons, the showers of charged pions have their maxima offset by several meters from the initial vertex as shown in Fig. 22. This offset is caused by the difference in flight lengths of charged and neutral pions. Since this effect is not taken into account in the shower maximum calculation the shower maximum position is of charged pions is underestimated by about 3.5 m. The tau decays into pions in more than 60% of the hadronic decay modes, therefore the two peaks are of comparable height.

The correlation between the two peak structure and the pion decays becomes clear when the vertex resolution is studied as function of the charged pion energy fraction of the visible energy in the tau decay as shown in Fig. 75. As can be seen, the second peak of the vertex resolution only shows up in a regime where charged pions carry over 80% of the energy.

The different tau decay modes therefore have an influence on the reconstruction. One could think this effect allows to probe the tau decay mode of an event but without knowledge of the tau flight length this effect is indistinguishable from a tau with a larger flight length. Because of this enhancement of the tau flight length, tau decay showers which are dominated by charged pions have
Figure 75: Distance between tau shower maximum and second reconstructed vertex vs charged pion energy fraction of tau decay; Z-axis: rate per year per block.

their distance between the shower maxima significantly enhanced compared to the actual tau flight length, making them easier to distinguish from single shower events.

Figure 76: Vertex resolution studies; the resolution is determined as the distances between reconstructed position and the shower maximum position; shown are tau signal events.
5.6 BELLE STARR RECONSTRUCTION

The optimal reconstruction method is a scan of the two shower likelihood over the full detector volume. The drawback of such a scan is the high demand on computational resources. Therefore, a minimization of the two shower likelihood is performed (full fit). The performance of such a minimization depends on the starting values. The reason being, that the likelihood landscape features many local minima and some of these cannot be overcome by the minimizer. Consequently, starting values close to the true values are needed in order to avoid the minimizer selecting one of the local minima. By performing the full fit after the previous steps, the results from the previous steps can be improved.

The vertex position resolution obtainable by the scan procedure is affected by the Prefit direction resolution of about $2^{\circ}$ median. For instance, a deviation of $3^{\circ}$ from the true direction translates to a position deviation of around 2.5 m assuming a tau flight length of 50 m. This is to be compared to the position resolution of a single shower position fit of around 1 m as shown in Sec. 5.3. Therefore, the full fit is expected to improve the position resolution in particular of the tau decay vertex.

An improved position reconstruction could yield an improvement of the reconstructed direction, depending on the final position resolution performance. A position resolution of around 1 m translates into a direction reconstruction of around $1.5^{\circ}$ at 50 m distance. This would be a significant improvement of the angular resolution of the Prefit which is around $2^{\circ}$ median.

The full fit minimizes the same negative log likelihood as used by the Scan given in Eq. 29 and uses the same hit selection. The used minimizer is the implementation of MINUIT 2 in the ROOT framework [101]. The minimization is performed for two positions and one time (the time of the second vertex is fixed by the tau flight time between the two positions) which yields seven parameters. As start values, the results from the Peak step are used.

Since the fit is still computing intensive, it is only applied on preselected events. The selection is based on quantities such as the reconstructed energy and position, which are discussed in more detail in Chap. 6. The criteria are optimized such that “Double Bang” events are accepted while rejecting single shower and track events. The applied criteria are:

- reconstructed energy: $E_{\text{rec}} \geq 10^{4.5}$ GeV
- reconstructed shower positions: $|Z| \leq 300$ m and $\sqrt{X^2 + Y^2} \leq 500$ m
- reconstructed length: $L \geq 5$ m

5.6.1 Full fit performance

The performance of the full fit position resolution is shown in Fig. 77. As can be seen, both the tau and neutrino vertex resolution are only slightly improved compared to the performance after the peak algorithm shown in Fig. 76. These
small improvements indicate that the trajectory reconstructed by the Prefit is close to the simulated trajectory for most events.

The reconstructed positions of the two showers translate directly to a direction. The achieved position resolution of 2 m median allows to improve on the Prefit direction reconstruction for showers which are a minimal distance apart. This relation is shown in Fig. 78a and Fig. 78b. The achieved angular resolution from the full fit positions and the Prefit is shown as a function of simulated tau flight length. The direction resolution from the full fit is better if the reconstructed tau flight length exceeds around 25 m. At large reconstructed distances, the limited statistics introduce some fluctuations.

Consequently, the direction reconstructed by the Prefit is used for events with reconstructed distance shorter than 25 m and the direction reconstructed by the full fit is used for events with a distance longer than 25 m. This results in a final direction resolution as shown in Fig. 79. As can be seen, the improvement of the direction resolution is small, since most “Double Bang” events have little energy and therefore a reconstructed distance smaller than 25 m.

The final performance is close to that achieved for the single showers as discussed before. The larger tails are caused by events with unfavorable topologies. Such events are rejected by the selection discussed in Chap. 6, yielding an improved angle and position reconstruction for the selected events.
Figure 78: Tau direction reconstruction performance for the full fit and Prefit as function of reconstructed distance between the two vertices for tau signal events; Yellow line shows median; dark blue 68% quantile and light blue 90% quantile.

Figure 79: Tau direction reconstruction performance for the unconstrained fit routine for tau signal events for combining the Prefit and full fit.