It has been recently reported that a superconducting triplet spin valve (TSV) based on the half-metallic CrO$_2$ can show “colossal” variations of the critical temperature, up to more than 1 K. This can be achieved when the magnetic noncollinearity between the mixer (F$_1$) and the drainage ferromagnetic layer (F) is maximized. In this work we investigate further such TSV devices looking at two aspects: first, we present the dependence of the TSV effect on the thickness of the mixer layer; second, we look at the perpendicular upper critical field $H_{c2\perp}$ as a function of the temperature. The thickness dependence, which is nonmonotonic as expected, represents a further proof that the effect is due to the generation of equal-spin triplet Cooper pairs, while what we observe for the $H_{c2\perp}$ versus $T$ curves is an interesting and peculiar behavior: there is a clear deviation from the universal linear dependence and the average slope is suppressed much more than what can be described with the formalism used for conventional proximized structures.

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I. INTRODUCTION

Ferromagnetism and superconductivity are mutually exclusive states because of the inherent competition between the exchange energy, which aligns the spins parallel, and the superconducting pairing, which aligns them antiparallel. However, supercurrents can be injected from a superconductor (S) into a ferromagnet (F) and survive for surprisingly long distances. This is possible only if conventional singlet Cooper pairs are converted into more unconventional triplet ones, with the spins aligned parallel and therefore not affected by the exchange interaction. In this case the limiting factors for the propagation in the F layer are temperature and spin-diffusion length. In a standard ferromagnet, such as Co, the spin-diffusion length is about 60 nm, while in a half-metallic ferromagnet, with full spin polarization such as CrO$_2$, triplet Cooper pairs can survive for distances up to several hundred nm [1,2]. In a half-metal (HM), indeed, at the Fermi level only one spin band is available and so at low temperatures no spin-flip scattering event, which would break the Cooper pairs, is allowed. These distances are remarkably larger than the characteristic short-range penetration lengths of the singlet component: about 1–2 nm for standard ferromagnets, atomic lengths for half-metals. In the case of half-metals not only are the injected superconducting correlations long range but they are also fully spin polarized, which makes CrO$_2$-based devices appealing systems to develop superconducting spintronics applications [3].

The mechanism which converts the singlet into an equal-spin triplet is realized at the S/F interface in two steps: a spin mixing process followed by spin rotation [4–6]. The spin rotation converts the $m_s = 0$ triplet component ($\uparrow \downarrow + \downarrow \uparrow$), generated by the spin mixing, into the parallel-spin triplet $m_s = 1$ component ($\uparrow \uparrow$). This conversion is possible if some sort of magnetic inhomogeneity is provided at the S/F interface. In the past decade many experimental works [1,2,7–9] unequivocally showed evidence of a long-range proximity effect in S/F/HM Josephson junctions. In most of these experiments the magnetic inhomogeneity was engineered by introducing an extra thin ferromagnetic layer F$_1$ between S and F on both sides. If the magnetization directions of F and F$_1$ are misaligned there is triplet generation. The crucial role played by the magnetic inhomogeneity was shown in a clear way, validating the theoretical model described above. Furthermore, a theoretical formalism was recently developed to specifically describe the proximity effect in the case of half-metals [10]. However, there are still several open questions about the mechanism and full control of the triplet generation has not been achieved yet.

In the last few years there has been a growing interest in superconducting triplet spin valve (TSV) devices [11], as a useful tool to provide better insight into the triplet generation. A TSV is a multilayer structure S/F$_1$/N/F, which can be thought of as half of a Josephson Junction with only one interface for singlet to triplet conversion, and therefore easier to control. The thin normal metal layer N is simply used to decouple the magnetizations F$_1$ and F. When the magnetization of F$_1$ (called the mixer layer) and F (the drainage layer) are noncollinear, there is generation of long-range triplets. The leakage of Cooper pairs into F results in a decrease of the superconducting order parameter of the S layer, and a drop of $T_c$, provided that the S layer is not thicker than a few times the superconducting coherence length $\xi_S$. The maximum efficiency of triplet generation, so the biggest suppression, is reached when the angle between the two magnetization directions is 90 deg.

HMs form a special class of ferromagnets, with clear relevance for triplet proximity effects. In our recent work [12], by using TSVs based on the half-metallic CrO$_2$ we could show a huge $T_c$ suppression, well over 1 K. Such a large effect is more than an order of magnitude bigger than for TSVs based on standard ferromagnets [13–15]. An explanation for this was recently given by Mironov and Buzdin [16], who showed that the special boundary conditions for a S/F/HM system (as opposed to an S/F$_1$/F$_2$ system) lead to the appearance of an extra triplet component in the S and F layers, with a much stronger effect on $T_c$ of the spin valve as a consequence.

In the experiments presented in Ref. [12] we have to keep in mind that, to induce the magnetic noncollinearity between F$_1$ and F, the magnetic field was rotated from in-plane to out-of-plane. For this reason we had to ascertain that the measured $\delta T_c$ was not due to the difference between the parallel and perpendicular upper critical field $H_{c2\perp}$. In particular, in the
out-of-plane configuration, several spurious effects such as vortices and stray field of the magnetic layers can influence the superconducting properties. In Ref. [12] this problem was circumvented by comparing the behavior of the TSV to equivalent structures where either the F$_1$ or the F layer was removed, so where the triplet generation was not expected. However, measuring $H_{c2}$ as a function of temperature would actually provide a more fundamental description of the TSV effect. The study of $H_{c2}$ not only quantifies the efficiency of triplet generation but also probes the change in behavior of the superconductor due to the triplet proximity effect.

In this paper we study CrO$_2$-based TSVs similar to the ones studied in Ref. [12], with MoGe as S layer and Ni as mixer layer, looking at two aspects not explored so far. First we study the dependence of the TSV efficiency on the Ni thickness. From the theoretical framework we know that there should be an optimum thickness for the mixer layer. Indeed, since the equal-spin triplet component is generated starting from the $m_2 = 0$ component created in the mixer layer, it follows that this layer has to be thick enough to give a sufficient amplitude of the $m_2 = 0$ component, but thinner than the decay length (a few nm). Here we confirm that there is a nonmonotonic dependence with the optimum thickness value around 1.5 nm. The result is consistent with previous studies of a Ni mixer layer in Josephson junctions based on Co [8], and also consistent with the theoretical results of Ref. [16]. It is a further proof, even if indirect, that the observed TSV effect is due to triplet generation.

The second point we address is the effect of the triplet generation on the phase diagram of the superconductor $H_{c2}(T)$. For the reasons explained above, $H_{c2}(T)$ of the stack seems to be a more fundamental quantity to study the TSVs, particularly in our device geometry. With the field applied out-of-plane (the TSV is “on”) we observe an unexpected behavior: the $H_{c2}(T)$ phase diagram strongly deviates from the universal linear behavior observed for both isolated superconductors and structures with no triplet generation. Next to that, there is a strong suppression of the critical field, namely, a decrease of the average slope, which cannot be described with the standard formalism for proximized S/F hybrids. The link between triplet generation and $H_{c2}(T)$ dependence is confirmed by comparing TSVs with different Ni thickness, so with different efficiency in triplet generation. We show how this reflects in a very different critical field behavior, the study of which can be used to have insightful information about the triplet generation.

In Sec. II we describe sample preparation and measurement details while in Sec. III we present the results on the $T$ variation as a function of the spin mixer thickness (A) and we discuss the features due to spin accumulation; in Sec. IV we show the results for the temperature dependence of the perpendicular upper critical field for the TSVs (B) compared with the isolated superconductor and the S/F hybrids in absence of triplet generation (A). To conclude, in Sec. V we highlight the main results of our study.

II. SAMPLE PREPARATION AND EXPERIMENTS

Our TSV device is a stack of four layers, namely, Mo$_{70}$Ge$_{30}$ (25)/Ni($d_{Si}$)/Cu(5)/CrO$_2$(100) [Fig. 1(b)], where the numbers express the thickness in nanometers. For simplicity $\mu$m width was patterned on a film of CrO$_2$. The bridge direction is parallel to the magnetic easy axis of CrO$_2$, which is along the c-(001) direction of the TiO$_2$ surface. Right before the deposition of the trilayer, CrO$_2$ was Ar etched in order to remove the native Cr$_2$O$_3$ oxide from the surface. The importance of this step, fundamental to achieve a good interface transparency, has been extensively discussed in Ref. [12]. The thickness of the mixer layer was varied, with values $d_{Si} = 0, 1.0, 1.5, 2.2$, and $3.0$ nm. We performed electrical measurements in a physical properties measurement system. The angle $\theta$ between the surface plane and the applied magnetic field $H_a$ could be varied. When $H_a$ is rotated out of the plane, the magnetization vectors of both the Ni and the CrO$_2$ layer are gradually pulled out-of-plane. However, the magnetization loops (presented in Ref. [12]) show a different saturation field for the two ferromagnetic layers, so that at a given field below the saturation there is an angle between the two vectors, meaning the Ni layer has a stronger perpendicular component. This difference accounts for the magnetic noncollinearity. The $R(T)$ curves (Sec. III) were obtained with the magnetic field both in-plane ($\theta = 0^\circ$) and out-of-plane ($\theta = 90^\circ$). The measurement configuration and the definition of $\theta$ are represented in the sketch of Fig. 1(a). When $H_a$ is in-plane it is always parallel to the current direction. The $R(H)$ measurements (Sec. IV) were only performed in the out-of-plane configuration. Further details about the measurement setup can be found in Ref. [12].

FIG. 1. (a) Coordinate system used in angle dependent magnetotransport measurements, showing the direction of the current $j$, the applied field $H_a$, and the angle $\theta$ between them. (b) Sketch which shows the four layers forming the TSV, with their respective thickness. (c) Optical micrograph of a typical TSV where a MoGe/Ni/Cu trilayer bridge of 10-$\mu$m width was patterned on a film of CrO$_2$. The thicknesses of the different layers are the same as in (b). $d_{Si}$ was varied, with values $d_{Si} = 0, 1.0, 1.5, 2.2$, and $3.0$ nm.

Mo$_{70}$Ge$_{30}$ is called MoGe. Via sputtering and liftoff the trilayer MoGe/Ni/Cu was patterned in a four-terminal structure with a bridge 10-$\mu$m wide on top of a CrO$_2$ film grown by chemical vapor deposition on a crystalline TiO$_2$ (100) substrate. More details about the film growth can be found in Ref. [12] (Supplemental Material). An optical image of a typical device is shown in Fig. 1(c). The bridge direction is parallel to the magnetic easy axis of CrO$_2$, which is along the c-(001) direction of the TiO$_2$ surface. Right before the deposition of the trilayer, CrO$_2$ was Ar etched in order to remove the native Cr$_2$O$_3$ oxide from the surface. The importance of this step, fundamental to achieve a good interface transparency, has been extensively discussed in Ref. [12]. The thickness of the mixer layer was varied, with values $d_{Si} = 0, 1.0, 1.5, 2.2$, and $3.0$ nm. We performed electrical measurements in a physical properties measurement system. The angle $\theta$ between the surface plane and the applied magnetic field $H_a$ could be varied. When $H_a$ is rotated out of the plane, the magnetization vectors of both the Ni and the CrO$_2$ layer are gradually pulled out-of-plane. However, the magnetization loops (presented in Ref. [12]) show a different saturation field for the two ferromagnetic layers, so that at a given field below the saturation there is an angle between the two vectors, meaning the Ni layer has a stronger perpendicular component. This difference accounts for the magnetic noncollinearity. The $R(T)$ curves (Sec. III) were obtained with the magnetic field both in-plane ($\theta = 0^\circ$) and out-of-plane ($\theta = 90^\circ$). The measurement configuration and the definition of $\theta$ are represented in the sketch of Fig. 1(a). When $H_a$ is in-plane it is always parallel to the current direction. The $R(H)$ measurements (Sec. IV) were only performed in the out-of-plane configuration. Further details about the measurement setup can be found in Ref. [12].
the resistive transition of the TSVs as a function of the temperature, with the magnetic field (0.5 T) applied in-plane ($\theta = 0^\circ$) and out-of-plane ($\theta = 90^\circ$), $T_c$ was defined by using the operational parameter $T_{50\%}$, namely, the temperature at 50% of the resistive transition. The maximum $T_{50\%}$ variation, a measurement of the TSV efficiency, is given by $\delta T_{50\%} = T_{50\%}(0^\circ) - T_{50\%}(90^\circ)$, for TSV25 devices in a field of 0.5 T, as a function of the thickness of the mixer layer $d_{Ni}$ and (right panel) dependence of the TSV efficiency as a function of the interface transparency, measured by the reciprocal of the normal resistance $1/R_N$, for different series of TSV25s, each with a different $d_{Ni}$ value. Dashed and dash-dotted lines show the $\delta T_{50\%}\text{max} = \delta T_{50\%}$ for the isolated superconductor MoGe(25) and the trilayer MoGe(25)/Ni(1.5)/Cu(5), respectively. The full symbols represent the points used for the plot in the left panel.

At the onset of the transition there is a small but distinct peak, with resistance exceeding the normal value $R_N$. By rotating the field from in-plane to out-of-plane, for TSV25 the peak gets slightly reduced and broadened (together with the transition width). For TSV50, instead, the peak is completely suppressed. This feature is likely due to spin accumulation, as proposed in Ref. [12], and will be more extensively discussed in Sec. III B.

As expected, TSV25 shows a lower $T_c$ and, in line with the theory of proximity effect in a TSV, a larger $T_c$ variation. $\delta T_c$ is about 1.6 K for TSV25 and 0.65 K for TSV50. $T_{50\%}(90^\circ)$ for TSV25 has been extrapolated because at 4.2 K, the minimum temperature we could reach in that particular measurement, the resistance was still slightly above the threshold of 50% of $R_N$, even if very close. We have to stress that these values result from the triplet leakage added to spurious effects such as vortex resistance and stray fields of the F$_1$ and F layers. However, as explained in Ref. [12], these effects at 0.5 T do not exceed a few hundred mK. In the left panel of Fig. 2(b) we plot $\delta T_c$ as a function of $d_{Ni}$ for TSV25s: as expected the TSV efficiency shows a strongly nonmonotonic behavior with a peak centered around 1.5 nm, which confirms that to be the optimum thickness. For both $d_{Ni} = 0$ and 3 nm, for which no triplet generation is expected, the measured variation $\delta T_c \simeq 350$ mK provides us the magnitude of the triplet-independent effects. The interface transparency between the CrO$_2$ film and the trilayer above has been proven to be a crucial parameter for the injection of the Cooper pairs into the CrO$_2$ and therefore for the TSV efficiency [12]. The interface transparency is proportional to the reciprocal of the normal resistance $1/R_N$: if the transparency is low, the barrier resistance, in series with the low resistance of CrO$_2$, is high, and so $R_N$ is also high. For the TSVs of the left panel of Fig. 2(b), the values of $R_N$ are comparable but not fully equal. For this reason, in order to isolate the effect of the variation of the Ni thickness dependence we should compare structures with similar interface transparency, an experimental parameter not controllable with high precision. Nonetheless the differences among the TSVs do not change the qualitative behavior presented. This becomes clear in the right panel of Fig. 2(b), where the $\delta T_c$ of different series of TSVs is plotted as a function of the interface transparency. Each curve is a series with a different Ni thickness, while the dashed and the dash-dotted horizontal lines show the $T_c$ variation for a single MoGe(25) layer and a MoGe(25)/Ni(1.5)/Cu(5) trilayer, respectively. The full symbols represent the points used for the plot in the left panel [series Ni(1.0) and Ni (2.2) are not presented for clarity but are consistent with the general behavior]. For low transparencies (low $1/R_N$ values) the leakage is suppressed and all the curves approximately converge to the value of the trilayer. Increasing the transparency results in a growing TSV efficiency and the closer the Ni thickness is to the optimum value the stronger the effect. From the plot it is clear that the TSVs with $d_{Ni} = 1.5$ nm are the most efficient, which is in line with the theoretical result that the optimum lies around $d_{Ni} = d_F = \xi_F$ [16].

**B. Spin accumulation**

In Sec. III A we pointed out that most of the $R(T)$ curves show a peak at the onset of the superconducting transition. This peak is also present at zero field. In the earlier work [12], zero-field peaks were usually not present but that work
FIG. 3. (a) Transition curves \( R(T) \) for a TSV MoGe(25)/Ni(1.5)/Cu(5)/CrO\(_2\)(100), at zero field (dot-dashed line) and with a field of 0.5 T applied in plane (dashed line) or out of plane (full line). (b) \( R(T) \) curves for a TSV with the same characteristics as in (a), but with a much lower interface transparency. The dot-dashed line is for the measurement at zero field; the dashed line is for the measurement in-field (\( \mu_0 H = 0.5 \) T, in-plane). (c) Spin accumulation peaks at the onset of the \( R \) vs \( T \) transition in an out-of-plane field of 0.5 T for TSVs with different Ni layer thickness: 1.0 nm (dashed line), 1.5 nm (full line), and 2.2 nm (dot-dashed line). The inset shows the transitions at zero field (dot-dashed line) and in-field out-of-plane (full line) for the trilayer MoGe(25)/Cu(5)/CrO\(_2\)(100), with no nickel.

Concerned devices with \( d_3 = 50 \) and 25 nm but lower interface transparency. The peak is typically broadened when a field is applied. Furthermore, when we go from an in-plane to an out-of-plane configuration, it gets broader and slightly suppressed. This evolution can be appreciated for the TSV with \( d_3 = 1.0 \) nm in Fig. 3(a), where we show the transition curves at zero field and 0.5 T, both in-plane and out-of-plane. The normal state resistance for this structure is about 2.4 \( \Omega \).

We associate the peak with spin accumulation at the SF’N/F interface. Indeed, right above \( T_c \) the current path is mainly confined in the CrO\(_2\) layer, due to its low resistivity. The SF’N/F interface, thus, plays a role in the total measured resistance. At the onset of the superconducting state, triplet Cooper pairs start to be created and, depending on the magnetic configuration of the different layers, at the interface with CrO\(_2\) they can either be injected or broken and reflected as polarized Cooper pairs or polarized quasiparticles. The reflection results in spin accumulation and thus an excess resistance. The dependence of the peak height on the angle of the applied magnetic field makes us discard the charge imbalance as a possible explanation of the observed effect.

For devices in which the interface transparency is lower, the peak is not observed. As an example, in Fig. 3(b) we show the transitions, at zero field (dot-dashed line) and with a field (0.5 T) applied in-plane (dashed line), for a TSV with the same characteristics as in Fig. 3(a) (\( d_{\text{Ni}} = 1.0 \) nm), but with a significantly lower interface transparency, as signaled by the higher \( R_N \) value (about 36 \( \Omega \)). This is, on the one hand, because of the barrier hindering the proximity with CrO\(_2\) and therefore the polarization and, on the other hand, because the higher \( R_N \) value makes a contribution coming from the spin accumulation less relevant.

If we compare the transitions for different \( d_{\text{Ni}} \) values [Fig. 3(c), dashed, full, and dash-dotted line for \( d_{\text{Ni}} = 1.0, 1.5, \) and 2.2 nm, respectively], we see that the relative peak height is increasing with decreasing the Ni thickness, with a maximum height/\( R_N \) ratio obtained for the trilayer without nickel (inset). The same trend is observed for the peak at zero field. This is consistent with the spin accumulation mechanism described above: by increasing \( d_{\text{Ni}} \) we increase the amount of pair breaking inside the nickel layer so that less and less Cooper pairs reach the interface with CrO\(_2\), decreasing the amount of accumulation.

All the TSVs show a slight reduction of the peak height by rotating the field out-of-plane. This can be explained by the fact that when the field is rotated out-of-plane the conditions for the leakage into the CrO\(_2\) layer are optimized and the amount of reflected polarized Cooper pairs (or quasiparticles) is then reduced. In general, to have an exact description of the mechanism responsible for the spin accumulation is not trivial. Indeed, beside \( F_1 \) and \( F \), the magnetization of the surface of CrO\(_2\) plays an important role. If residues of the native oxide layer CrO\(_2\) are still present after the Ar-cleaning procedure, there are extra magnetic moments, \( F' \), which can influence the process. The relative orientation of the magnetizations \( F_1 \), \( F' \) (if present), and \( F \) determines how many singlet Cooper pairs are converted into triplets and how many triplets leak into CrO\(_2\) or are reflected, giving rise to spin accumulation. In a trilayer with no nickel layer, triplets can be generated by the noncollinearity between \( F' \) and \( F \), or simply by the misaligned magnetic moments of the domains of CrO\(_2\), as proposed by previous works [1,2]. In particular, similar peaks were observed for devices MoGe/CrO\(_2\), where the CrO\(_2\) film was grown on sapphire [17] and for Co nanowires contacted by superconducting W bars [18]. This explains why a peak can be observed also for a trilayer MoGe/Cu/CrO\(_2\), both at zero field and in-field [see inset of Fig. 3(c)]. The properties of \( F' \), as well as the interface transparency, are parameters not perfectly controllable. This is a cause of different types of behavior observed for different samples, with respect to peak height or peak evolution from zero field to in-field measurements. In order to achieve a more clear picture about the spin accumulation mechanism further experiments are needed.

IV. PERPENDICULAR UPPER CRITICAL FIELDS \( H_{c2\perp} \)

Since the changes in \( T_c \) as a function of applied field are large, it becomes more reasonable to describe the effects in terms of the critical field \( H_{c2\perp} \) of the proximized system, and construct a \( H_{c2\perp} \) phase diagram. In our TSVs, beside the
standard (singlet) proximity between the S layer and the F1/N/F sandwich, there is a triplet channel for the leakage of Cooper pairs which depends on the amount of (in)homogeneity and may therefore depend on H in a nontrivial manner. In that sense the $H_{c2}$ phase diagram will be anomalous because of an underlying change in $T_c$, which is not present in standard proximity systems.

### A. $H_{c2}$ of the S layer and of S/F hybrids

We first characterize our system in absence of triplet generation. We measured the resistance versus magnetic field dependence $R(H)$ at different temperatures for (i) a MoGe(25) single film to characterize the isolated superconductor, (ii) a trilayer MoGe(25)/Ni(1.5)/Cu(5), and (iii) a trilayer MoGe(25)/Cu(5)/CrO$_2$(100) [Figs. 4(a)–4(c), respectively; the numbers in parentheses are the thicknesses in nm]. Cases (ii) and (iii) are studied to look at the pair-breaking effect due to either the Ni or the CrO$_2$ layer, independently. The Cu layer, used in the TSV only to decouple the ferromagnets, is much thinner than the coherence length $\xi_N$ (of the order of 1 $\mu$m) so it does not play a role in suppressing the order parameter. For all the measurements presented in this section the field was applied perpendicular to the plane of the structure, in order to have a reference for the TSV effect, which is maximized when the field is applied in this geometry.

In Fig. 4 we can immediately notice a large difference in the normal resistance $R_N$ of the three structures: 840 $\Omega$ for the isolated MoGe [Fig. 4(a)], 240 $\Omega$ for the trilayer MoGe/Ni/Cu [Fig. 4(b)], and 4.5 $\Omega$ for the trilayer MoGe/Cu/CrO$_2$ [Fig. 4(c)]. Above $T_c$, the MoGe has high resistivity ($\rho_{MoGe} = 200 \mu\Omega\cdot\text{cm}$) and therefore is shorted by the Cu layer ($\rho_{Cu} = 2 \mu\Omega\cdot\text{cm}$) in (ii) and by the CrO$_2$ ($\rho_{CrO_2} = 6 \mu\Omega\cdot\text{cm}$) in (iii). The low value of $R_N$ for MoGe/Cu/CrO$_2$ is due to the fact that the CrO$_2$ is not patterned. In addition it is an indication of a good interface transparency [12]. The peak at the onset of the superconductivity, observed for $R(T)$ curves and discussed in Sec. III B, is clearly visible also for the $R(H)$ curves of the trilayer MoGe/Cu/CrO$_2$. The peak becomes broader when lowering the temperature.

For the isolated MoGe and the trilayer MoGe/Ni/Cu the value of $H_{c2}$ is extrapolated as shown by the black dashed lines in Fig. 4(a), as the field value at which the fitting line of the linear part of the transition intersects the $R_N$ value. This is a standard construction for cases when the S layer is weakly pinning and freely flowing vortices lead to a flux flow resistance $\rho_f = H/H_{c2} \rho_N$ [20].

For the trilayer MoGe/Cu/CrO$_2$ [see Fig. 4(c)], the shape of the transition is very different because of the feature described in Sec. III B. In addition it is important to point out that the total resistance we measure is the result of the parallel between MoGe which is highly resistive and the other layers. The contribution to the resistance due to vortex motion is $R_f = R_N/B_{c2}$. In the normal state, the resistance of the superconducting layer $R_N$ is about 840 $\Omega$ and it is shorted by the low resistive CrO$_2$ layer ($R_{CrO_2} \simeq 2 \Omega$). Therefore for the stack MoGe/Cu/CrO$_2$ (and the TSVs) only the lowest fraction (0.2%) of vortex resistance contributes to the total resistance in the observed transitions. In this region the flow of vortices is not coherent and the dependence of the resistance on the magnetic field is not linear. For this reason the method shown in Fig. 4(a) is no longer justified to determine $H_{c2}$, also because too little is known about vortex dynamics in proximized systems. The only reliable way we have in this case to define $H_{c2}$ is to consider the field at which, coming from the normal state, the resistance starts to increase as shown in Fig. 4(c) ($H_{c2p}$ value). The operational threshold is 0.3 times the peak height above the normal value $R_N$, namely, $R_N + 0.3(R_{max} - R_N)$, where $R_{max}$ is the maximum resistance value of the peak. As explained above, the peak originates from the breaking of Cooper pairs and starts to set in at the onset of superconductivity, even if not necessarily of the proximity effect. Therefore this value, which we call $H_{c2p}$, is an upper limit for the real upper critical field, which could be a bit...
lower. For comparison we also look at the dependence of the field value defined as in Fig. 4(a), because only below this value the resistance drops to zero. This value, which we call $H_{c2}$ [see Fig. 4(c)], represents a lower limit for the real $H_{c2,\perp}$. Also, representing the onset of flux flow, it is definitely related to $H_{c2,\perp}$.

Very small oscillations, possibly due to vortex dynamics, are visible at high fields in the tail of the transition curve for the lowest temperatures in Figs. 4(a) and 4(b). The oscillations become much more evident for the trilayer MoGe/Cu/CrO$_2$ [Fig. 4(c)], in particular at 4.2 and 4.4 K at around $1.2\,T$. Here the total resistance of the multilayer is low, so the relative contribution of the oscillations, which seems to be of the order of a few ohms, is more relevant. The sharpness of the oscillation at 4.2 K is an artifact due to the limited amount of points acquired for that specific measurement. For the curves where big oscillations are present we fitted the background points acquired for that specific measurement. For the curves at 4.2 K, because only below this field value defined as in Fig. 4(a), because only below this field value the resistance drops to zero. This value, which we call $H_{c2}$ [see Fig. 4(c)], represents a lower limit for the real $H_{c2,\perp}$.

For comparison we also look at the dependence of the field value defined as in Fig. 4(a), because only below this value the resistance drops to zero. This value, which we call $H_{c2}$ [see Fig. 4(c)], represents a lower limit for the real $H_{c2,\perp}$. Also, representing the onset of flux flow, it is definitely related to $H_{c2,\perp}$.

Figure 5 shows the phase diagrams $H_{c2,\perp}$ versus $T$ obtained from Figs. 4(a) (circles), 4(b) (squares), and 4(c) (full and empty triangles, for $H_{c2,\perp}$ and $H_{c2,\perp}$, respectively).

To quantitatively analyze proximized systems, the formalism presented by Fominov et al. in Ref. [21] is typically used. The formalism, based on the quasiclassical Usadel equations which describe the gap- and pair-correlation functions, was originally developed to describe proximity effects in S/N systems, in particular the behavior of $T_c$ as a function of such parameters as layer thicknesses, interface transparency, and diffusion constants. It was extended to S/F hybrids with a weak ferromagnet in the dirty limit (the mean free path $l$ is the shortest length after the Fermi wavelength), starting from the linearized Usadel equations. In many cases the so-called single-mode approximation (SMA) can be used and the final equations reduce to

$$\ln(t_0) = \Psi\left(\frac{1}{2}\right) - \Psi\left(\frac{1}{2} - \frac{\Omega^2}{2\Omega_0}\right),$$

$$\Omega \tan\left(\frac{\gamma d_s}{\xi_S}\right) = \frac{\gamma}{\gamma_0}$$

where $t_0 = T_c/T_{cS}$ is the critical temperature of the proximized system $T_c$, normalized by the critical temperature of the isolated superconductor $T_{cS}$. $d_s$ is the thickness of the superconductor, $\xi^2 = (2/\pi \xi_{S0})$ is the coherence length and $\Psi$ the digamma function, and $\Omega$ is the pair-breaking parameter responsible for the suppression of $T_c$. In Eq. (2), $\Omega$ depends on $\gamma$, which measures the strength of the proximity effect, and on $\gamma_0$, which describes the effect of the boundary transparency [22]. These equations allow us to describe the $T_c$ variation as a function of the layer thickness (either of S or F), at zero field. Due to the additivity of the pair breaking [23], it is trivial to extend the formalism to include the role of the magnetic field, as shown in Ref. [24]. In this case in Eqs. (1) and (2) $\Omega^2$ is substituted by

$$\Omega^2(t) = \Omega_0^2 + h_{c2,\perp}(t)$$

where $t = T/T_{cS}$ and $h_{c2,\perp}(t) = (2\pi/\phi_0)H_{c2,\perp}^2\xi_S^2$. $\Omega_0^2$ is the pair-breaking parameter at zero field. By combining Eqs. (1)–(3) and solving them numerically it is possible to obtain and fit the $H_{c2,\perp}$ versus $T$ dependence. Close to $T_c$ (smaller critical fields), the dependence is linear.

The isolated superconductor can be described by the equations above in the limit $\Omega_0 \rightarrow 0$. The $T_c$ of MoGe is approximately $6.9\,K$, close to the bulk value [25]. By fitting the curve we obtain $\xi_S \approx 2.7\,nm$, which gives a coherence length $\xi_{S0} \approx 4.2\,nm$, consistent with what is reported in literature for MoGe [25]. The same result for $\xi_{S0}$ can be obtained from Ginzburg-Landau by the linear relation which describes the perpendicular $H_{c2}$ dependence of a superconducting film near $T_c$:

$$\mu_0 H_{c2,\perp} = \frac{\phi_0}{2\pi \xi_S^2} \left(1 - \frac{T}{T_c}\right)$$

where $\phi_0$ is the magnetic quantum flux.

In our trilayer MoGe/Ni/Cu, Ni is a relatively strong ferromagnet (exchange energy $E_{ex} \approx 200\,meV$). This raises the question about the validity of Usadel equations (dirty limit). The two main conditions for the validity are (i) $E_{ex} \ll E_F$, with $E_F$ the Fermi energy, and (ii) $\xi_S \gg l$. With $E_F$ the diffusion coefficient of F and $l$ the mean free path. If we take a Fermi velocity $v_F = 10^6\,m/s$ and we calculate $D_F$ from $v_F$ and the experimental resistivity $\rho_{Ni} = 7\,\mu\Omega\,cm$, the estimated values are $E_F \approx 2.8\,eV$, $\xi_S \approx 1.5\,nm$, and $l \approx 5.4\,nm$. While (i) is satisfied, (ii) is not. However, the Ni layer of our system is really thin and the value of $l$, calculated from the resistivity measured for much thicker films, could be overestimated. If we assume that the Usadel equations are valid and we try to fit the trilayer data with Eqs. (1)–(3), we obtain $\xi_{S0} \approx 4.5\,nm$, a value close to the one of the isolated MoGe, but the $T_c$ suppression from 7.0 to 6.3 K cannot be reproduced. The minimum $T_c$ we can obtain (in the case of very small $\gamma_0$) is $6.5\,K$, a signal that the SMA (often used in the limit of thin superconductors) cannot properly describe our
data. It is possible that the multimode method or the method of fundamental solution [21] could provide more accurate results but more likely a new formalism specifically developed for strong ferromagnets is needed. The analysis above cannot be applied to the trilayer MoGe/Cu/CrO$_2$ where the proximity is with a thick half-metallic ferromagnet. In this case as expected $T_c$ is suppressed even more, to a value of about 5.4 K.

Even if we cannot have quantitative information about the proximity from the analysis above, we can qualitatively see that the effect of the pair breaking is a simple shift of the $H_{c2}(T)$ curve towards lower $T_c$ values. The slope, dependent only on $\xi_S$, which is an intrinsic propriety of the superconductor not affected by the pair breaking, should remain unchanged. This is what we observe for the trilayer MoGe/Ni/Cu and for MoGe/Cu/CrO$_2$, if we consider $H_{c2}$. Instead, $H_{c2}(T)$ shows a lower slope. This observation seems to suggest that $H_{c2}$ is the proper definition of $H_{c2l}$. However, since a certain amount of triplet generation cannot be completely excluded in this case (as mentioned in Sec. III B), the behavior of $H_{c2}$ could simply describe the dependence of the superconductor before the CrO$_2$ gets proximized.

**B. $H_{c2l}$ of triplet spin valve structures**

As we can see from Fig. 6(b) the full TSV shows a behavior very different from the trilayers. In Fig. 6(b) we show the phase diagram for the TSV MoGe(25)/Ni(1.5)/Cu(5)/CrO$_2$(100) compared with the trilayer MoGe(25)/Cu(5)/CrO$_2$(100), for both $H_{c2}$ (dashed lines, full squares) and $H_{c2l}$ (dot-dashed lines, empty squares), defined as described in Sec. IV A [Fig. 4(c)]. A selection of the $R(H)$ transitions for the TSV, from which the values of $H_{c2}$ and $H_{c2l}$ are obtained, is shown in Fig. 6(a). The feature observed, namely, the peak at the onset of the transition and the oscillations visible at around 0.8 and 1.2 T, have been extensively discussed in Secs. III B and IV A.

The critical temperature of the TSV, about 5.7 K, is slightly higher than the trilayer MoGe/Cu/CrO$_2$ where there is no (or little) triplet generation. This is not consistent with what was expected and with what was observed in Ref. [12] and it is likely due to differences between the interface transparencies of the samples we compare. If we look at $H_{c2l}$, the relation is linear but with a significantly lower slope, 0.65 times the slope of the trilayer. This suggests that what we observe for a TSV is not simply a pair breaking due to S/F proximity, but a stronger effect due to the leakage of triplets in the ferromagnet. In this case the multilayer behaves more like an S/N than an S/F proximized system.

If we look at $H_{c2}$, the slope is suppressed even more. Here, we see another interesting and unexpected feature: the dependence is linear very close to $T_c$ but at around $T = 0.85T_c$ it strongly deviates to higher values. Interestingly enough, this unusual behavior is pronounced for TSV-Ni(1.5) and TSV-Ni(1.0) and it gradually disappears by increasing or decreasing the Ni thickness. This seems to suggest a relation between nonlinearity and triplet generation. In Sec. IV A we addressed the problem of large oscillations in the transition curves at low temperatures. The lower the temperature (higher critical field) the more oscillation modes are included in the transition. For this reason we have to point out that the values extrapolated at lower temperatures are less precise. However, the range of variation is not such to modify the qualitative behavior of our observation. Moreover at low fields no oscillations are present so that the definition of the slope of the curve is not affected by that. A similar nonlinear behavior has been reported in Ref. [26] for Nb/Cu bilayers. The authors propose several, although not conclusive, explanations for the observation, but in general the effect is attributed to the proximity effect and the boundary conditions. Even if we do not have enough elements to claim that the physical mechanism for the similar behavior is the same, such effects observed in an S/N bilayer seem to be consistent with what we measure in a S/F-type structure with a long-range proximity effect.

If we compare the slope of the phase diagram for TSVs with different Ni thickness, we expect a stronger effect the closer we are to the optimum value. This is confirmed by the experimental results shown in Fig. 7, where we plot the absolute value of the slope obtained by the fit of the $H_{c2l}(T)$ curves as a function of the Ni thickness. Dashed and dot-dashed lines show the value for the isolated MoGe(25) and the trilayer MoGe(25)/Ni(1.5)/Cu(5), respectively.

As for the $T_c$ variation [Fig. 2(b)], we observe a nonmonotonic behavior, with a maximum effect between 1.0 and 1.5 nm. A similar value has been obtained for the $\delta T_c$ dependence. This suggests that there is a direct correlation between slope of the $H_{c2l}(T)$ curve and the TSV efficiency, namely, the triplet generation.
The determination of the actual $H_{c2}$ value is complicated by the presence of the spin accumulation peak, however we could define an upper and lower limit, $H_{c2u}$ and $H_{c2l}$, which allowed us to compare the TSVs with the systems with no triplet generation. The behavior is very different from the standard S/F proximized systems: the slope is strongly suppressed, similarly to what happens in S/N systems, and for $H_{c2}(T)$ we observe a clear deviation from the universal linear dependence. Both effects are prominent for Ni thicknesses around 1.5 nm and vanish for thinner or thicker Ni layers, which indicates a link with the efficiency of the triplet generation. This behavior cannot be described by simple pair breaking, as described by the formalism developed so far for S/F hybrids. A new theoretical formalism along the lines of Ref. [16] is needed to describe the behavior of the critical field in presence of triplet superconductivity.

V. CONCLUSION

To summarize, we showed that for superconducting triplet spin valves based on the half-metallic CrO$_2$ the efficiency of the singlet to triplet conversion, and thus the magnitude of the spin valve effect, depends on the thickness of the spin mixer layer. In our case the spin mixer is Ni and the optimum value is found to be around 1.5 nm. This dependence has already been shown for other systems such as Josephson junctions based on Co with a similar outcome. However, it has been shown before not in a TSV and it is a further, although indirect, proof of the triplet nature of the observed effects. In addition we addressed the phenomenon of spin accumulation, which we believe can explain the emergence of the peak observed at the onset of the superconductivity for both $R(T)$ and $R(H)$ transitions. Furthermore we studied the dependence of the perpendicular upper critical field as a function of the temperature for a TSV. The behavior is very different from the standard S/F proximized systems: the slope is strongly suppressed, similarly to what happens in S/N systems, and for $H_{c2}(T)$ we observe a clear deviation from the universal linear dependence. Both effects are prominent for Ni thicknesses around 1.5 nm and vanish for thinner or thicker Ni layers, which indicates a link with the efficiency of the triplet generation. This behavior cannot be described by simple pair breaking, as described by the formalism developed so far for S/F hybrids. A new theoretical formalism along the lines of Ref. [16] is needed to describe the behavior of the critical field in presence of triplet superconductivity.

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