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Summary

Ordinary double points are the simplest type of singularity on an algebraic surface, and can be defined locally by the polynomial \( x^2 - yz \in \mathbb{C}[x, y, z] \). Nodal surfaces are projective surfaces with only ordinary double points as singularities.

Hodge theory and deformation theory of nodal surfaces are similar to those of smooth surfaces in many respects, for example, nodal surfaces also have pure Hodge structures. The similarities and differences are described in detail in Chapters 2 and 3 of the thesis. Our first main result is the infinitesimal Torelli theorem for nodal surfaces in \( \mathbb{P}^3 \), which states that all small non-trivial deformations of nodal surfaces induce non-trivial variations of Hodge structures.

A nodal surface \( F \) is said to have an even set of nodes if there exists a double cover \( f : S \to F \) branched precisely over the nodes. We studied two families of such surfaces in Chapter 4, namely sextic surfaces in \( \mathbb{P}^3 \) with even sets of 56 and 40 nodes respectively.

We found a new geometric construction for a universal family of even 56-nodal surfaces whose double covers \( S \) satisfy \( h^{1,0}(S) = 3 \), by showing that any such surface is in turn the double cover of an even 28-nodal surface \( \Theta/[-1] \) where \( \Theta = S^2C \) is a symmetric theta divisor on the Jacobian of a non-hyperelliptic curve \( C \) of genus 3 and \([-1]\) is induced by the involution on the Jacobian.

The involution inducing the double cover \( f : S \to F \) gives a decomposition of Hodge structures into eigenspaces

\[
H^2(S, \mathbb{Q}) = H^2(S, \mathbb{Q})_+ \oplus H^2(S, \mathbb{Q})_- , \quad \text{with} \quad H^2(S, \mathbb{Q})_+ = H^2(F, \mathbb{Q}).
\]

The double cover \( S \) of an even 40-nodal surface has \( H^2(S, \mathbb{Q})_- \) of Hodge type \((1, 26, 1)\). We seek simple sub-Hodge structures of types \((1, n, 1)\) with \( n > 20 \), since we know very few examples of these. However, by comparing various constructions of families of even 40-nodal surfaces, we showed that \( H^2(S, \mathbb{Q})_- \) always contains a sub-Hodge structure, of type \((1, 20, 1)\), arising from some deformation of a Hilbert scheme of a K3 surface. Thus, this family of examples failed to provide new geometric Hodge structures of interest.

In the final chapter, we extended some constructions from Chapter 2 to more general singularities, using Saito’s theory of mixed Hodge modules. These results provide ways to compute the Hodge decompositions of singular varieties.