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We define a variety to be the analytification of an irreducible reduced separated scheme over \( \mathbb{C} \). For any variety \( X \), we defined, in Section 5.2 (p. 97), the complexes \( \tilde{\Omega}^p_X \) of sheaves of differential \( p \)-forms.

1. Let \( X \) be a projective variety of dimension \( n \) with singular locus of codimension \( d \) and \( \pi : \tilde{X} \to X \) a log-resolution of \( X \) with normal crossing exceptional divisor \( E \). Then, the weight-\( k \) part of the mixed Hodge structure on \( H^k(X, \mathbb{Q}) \) is given by

\[
\text{Gr}_W^k H^k(X, \mathbb{C}) = \bigoplus_{p+q=k} H^{p,q}(X) \quad \text{where} \quad H^{p,q}(X) := \mathbb{H}^q(X, \tilde{\Omega}^p_X)
\]

and there are isomorphisms

\[
\tilde{\Omega}^p_X = \begin{cases} 
R\pi_* \Omega^p_X (\log E), & p \leq d, \\
R\pi_* \Omega^p_X (\log E)(-E), & p \geq n - d,
\end{cases}
\]

where \( \Omega^p_X (\log E) \) is the sheaf of differential \( p \)-forms with logarithmic pole along \( E \).

A nodal surface \( F \) is a projective variety of dimension 2 with only ordinary double points as singularities. We say that \( F \) is even if there exists a double cover \( f : S \to F \) branched exactly on its set of nodes.

2. The complex \( \tilde{\Omega}^p_F \) is concentrated in degree 0 and coincides with the sheaf defined in [Steenbrink, 1977].

3. Let \( F \subset \mathbb{P}^3 \) be a nodal surface of degree \( d \geq 4 \). Then \( F \) satisfies the infinitesimal Torelli property, that is, the infinitesimal period map

\[
dP^k : H^1(F, \tilde{T}_F) \to \text{Hom}(H^{1,1}(F), H^{0,2}(F)),
\]

where \( \tilde{T}_F = \text{Hom}(\tilde{\Omega}^1_F, \mathcal{O}_F) \), is injective.
4. For each non-hyperelliptic curve $C$ of genus 3, the Jacobian $J(C)$ is principally polarized by a symmetric divisor $\Theta$ and we can choose a curve $B \in |2K_{\Theta}|$ on $\Theta$. Each general such pair $(C, B)$ determines a unique even 56-nodal sextic surface. Conversely, a general even 56-nodal sextic surface $F \subset \mathbb{P}^3$ whose double cover $S$ satisfies $h^{1,0}(S) = 3$ lies in the 12-dimensional family parametrized by the set of all such pairs $(C, B)$.

5. The family of even 40-nodal sextic surfaces is 28-dimensional. A general such surface $F \subset \mathbb{P}^3$ is tangent to a unique Kummer surface $K$ (an even 16-nodal quartic surface) along a curve $C$. The curve $C$ has arithmetic genus 15 and contains all the nodes of $F$ and $K$.

6. For any even 40-nodal surface $F$ with double cover $S$, the covering involution on $S$ induces a decomposition of $H^2(S, \mathbb{Q})$ into eigenspaces, one of which is of Hodge type $(1, 26, 1)$. This eigenspace contains a sub-Hodge structure of type $(1, 20, 1)$, which is in turn isomorphic to a deformation of a sub-Hodge structure of $H^2(Z[2], \mathbb{Q})$ where $Z[2]$ is the Hilbert scheme of two points on a K3 surface $Z$.

7. The set $\Sigma$ of nodes on an even 40-nodal or 56-nodal sextic surface is independent in degree 6, that is, for any subset $\Sigma' \subset \Sigma$, there exists a sextic surface $Y$ such that $Y \cap \Sigma = \Sigma'$.

8. Let $(A, \Theta)$ be a principally polarized abelian surface, $C$ a curve in the linear system $|2\Theta|$, and $\pi: S \to A$ a double cover of $A$ branched along $C$. Then $S$ satisfies the infinitesimal Torelli property.

9. There is no variety $X$ with $h^{2,0}(X) = 1$ whose cohomology group $H^2(X, \mathbb{Q})$ is known to contain a simple sub-Hodge structure of type $(1, r, 1)$ with $r > 20$.

10. Let $(X, \mathcal{O}_X)$ be a ringed space over a field $k$ and $D^b(X)$ the bounded derived category of $\mathcal{O}_X$-modules. Cones of “geometric” morphisms in $D^b(X)$, that is, those induced by Grothendieck’s six functors, are functorial.

11. Provability is a weaker notion than truth.

— D.R. Hofstadter, Gödel, Escher, Bach: An Eternal Golden Braid.