Chapter 10

Protoclusters: observations, theory and modeling

Abstract. This chapter attempts to provide new constraints on the scenario for the formation of galaxy clusters, based, in part, on the observational evidence presented in this thesis. The chapter is structured in 3 parts. In part I, we compile the first overview of observational evidence for overdensities of galaxies between $z = 2$ and $z = 6$. The overdensities, estimated from the number densities of star-forming galaxies (Ly$\alpha$ emitting galaxies and Lyman break galaxies) relative to random fields, are $\sim 1 - 7$. If these structures were to collapse under the influence of their own gravity, their masses would be $\sim 10^{14}$ to $10^{15} M_\odot$. Because this is comparable to the masses of clusters of galaxies in the local universe, we define the term ‘protocluster’ as being an object that meets the requirements for forming a bound object on the mass scale of a cluster prior to, or at, the present epoch, but which has not yet collapsed and virialized at the epoch corresponding to its observed redshift. In part II, we use simple theoretical descriptions for the growth of overdensities in a $\Lambda$CDM universe to study the evolution of the sample of candidate protoclusters compiled in part I. Using conservative estimates of the overdensities, we find that the majority of the structures are likely to collapse within a finite time. We identify several structures as meeting the requirements for virialization at $z \approx 0.5$, whereas others are expected to have fully collapsed by the present epoch. We compare the predicted abundance of dark halos as a function of their (linear) overdensities, mass and redshift, to the protocluster data. We find that the protoclusters lie in dark halos with number densities of $10^{-6}$ to $10^{-5}$ Mpc$^{-3}$, and conclude that they are associated with clusters that become virialized between $z \approx 0$ for $M \approx 10^{15} M_\odot$ and $z \approx 1$ for $M \approx 10^{14} M_\odot$. We show that this is in agreement with recent results from N-body simulations. We compare the extrapolated bias of dark halos hosting protoclusters and radio galaxies at $z \sim 3$ to the bias of Lyman break galaxies (LBGs) and distant red galaxies (DRGs) at $z = 3 - 6$. The bias of protoclusters at $z \sim 3 (b \sim 8)$ implies that their present-day descendants lie in dark halos that are $\sim 5 - 10$ times more massive than those hosting the $z = 0$ descendants of luminous LBGs or DRGs, although even the latter populations are associated with group- or moderate cluster-type environments at $z = 0$. In part III, we model the star formation history of cluster red sequence galaxies in order to compare their luminosities extrapolated to $z > 2$ to the protocluster data. We find that the total stellar mass in the cluster red sequence was built up over the redshift range $z \sim 10 - 2$ with star formation rates of several hundreds to a thousand of $M_\odot$ yr$^{-1}$ assuming constant star formation. We show that there is good agreement with the star formation rates as measured for Ly$\alpha$ emitters and LBGs, and the total extrapolated star formation rates in protoclusters. Summarizing, the overdensities, the masses and the star formation rates of protocluster candidates are in general agreement with the properties expected for the progenitors of clusters in the local universe.


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10.1 Introduction

The formation and evolution of structure in the universe is a fundamental research area in modern cosmology. Clusters of galaxies represent the most extreme cosmology from initial conditions in the universe, and are therefore good laboratories for testing evolutionary scenarios for the formation of the large-scale structure, and their properties are closely tied to the cosmological parameters (Bahcall & Fan 1998). While clusters of galaxies have been studied extensively in the relatively nearby universe, their evolutionary history becomes obscure beyond roughly half the Hubble time. Their progenitors are extremely difficult to identify when the density contrast between the forming cluster and the field becomes very subtle, and mass condensations on the scales of clusters are extremely rare at any epoch (Kaiser 1984).

Overdensities of galaxies have been discovered out to $z \approx 6$ (e.g. Pascarelle et al. 1996; Steidel et al. 2005; Keel et al. 1999; Francis et al. 2001; Möller & Fynbo 2001; Venemans et al. 2002, 2004, 2005a,b; Shimasaku et al. 2003; Ouchi et al. 2005; Stiavelli et al. 2005). Some of these structures were found as by-products of wide field surveys using broad or narrow band imaging. Others were traced by a luminous radio galaxy or quasar that pinpointed the overdense regions (e.g. Steidel et al. 1998; Kurk et al. 2000; Pentericci et al. 2000; Venemans et al. 2002, 2004; Miley et al. 2004; Venemans et al. 2005a,b; Overzier et al. 2006a,b; Zheng et al. 2006). Although these structures are all overdense compared to the field, their derived physical properties are generally highly uncertain. In general, the galaxy overdensities are on the order of a few and imply group- or clusterlike masses of $10^{13} - 15$ $M_{\odot}$, projected sizes of several to several tens of (comoving) Mpc, and in some cases measured velocity dispersions of a few hundred km s$^{-1}$ determined from emission line galaxies. Because of the variation, as well as the uncertainties, in their sizes, topologies and masses they have been associated with overdense regions within the large-scale structure, such as filaments, or more special structures that are the progenitors of local clusters and galaxy groups.

The inferred total masses of the suspected protoclusters usually relies on the assumptions that (i) the overdensity measured for one tracer population of galaxies (usually Ly$\alpha$ emitters (LAEs) or Lyman break galaxies (LBGs)) is representative for the total underlying mass overdensity of the structure (its various components being dark matter, gas and other types of galaxies, besides the tracer population) and (ii) that the various other galaxy populations occupy the same volume that is occupied by the tracer population. These assumptions are at present difficult to verify without time-consuming spectroscopic campaigns over large areas of the sky. Can we use any other signatures expected from forming clusters to establish whether the high redshift galaxy overdensities observed are indeed the suspected sites of cluster formation? One of the tell-tale signs of clusters, at least in the relatively nearby universe, is their high luminosity and extended X-ray emission. However, the detectability of distant clusters using X-ray observations is proportional to $(1 + z)^{-4}$, even when it is assumed that there is no evolution in cluster abundance, with redshift. The most distant X-ray clusters that have been found (e.g. Rosati et al. 1998; Mullis et al. 2005) are expected to lie very close to the maximum redshift achievable by current surveys ($z \approx 1.5$). Because the X-ray luminosity of clusters is furthermore proportional to the square of the gas density within the virial radius of the cluster, and virialization is believed to have occurred by $z \approx 1 - 1.5$ (for the most distant and massive clusters known), little X-ray emission is expected from clusters or cluster progenitors beyond $z \approx 2$. X-rays are therefore not useful as a tool for the identification of a (forming) cluster.

There is, however, another property of galaxy clusters that could serve as an indicator of its mass and overdensity at early epochs. The cluster red sequence or color-magnitude relation (CMR) is a preferred region in the magnitude (usually rest frame optical) versus color (usually rest frame $U - B$) diagram of the galaxies
in clusters (see Fig. 10.1). The relation is formed almost exclusively by early-type galaxies. The existence of the red sequence implies that star formation ceased at a sufficiently early epoch to allow the colors to redden passively up to the cluster age at the observed epoch. The early-type galaxies on the red sequence are the most massive and oldest galaxy constituents of clusters, even for clusters at \( z \sim 1 \), where masses of \( \sim 10^{11} M_\odot \) and formation redshifts of \( \sim 2 – 5 \) are inferred (e.g. Ellis et al. 1997; van Dokkum et al. 2000; Stanford et al. 2002, 2005; Blakeslee et al. 2003, 2006; Holden et al. 2005; Postman et al. 2005; Mullis et al. 2005; Mei et al. 2006).

The epoch of cluster formation is presumed to be marked by the violent build-up of the stellar mass contained in this early-type population. Constraints on the star formation history are (i) the color-magnitude relation, (ii) galaxy morphologies, (iii) the metal enrichment of the intracluster medium (ICM), and (iv) the Butcher-Oemler effect (the empirical evidence that distant clusters have a higher fraction of relatively blue (late-type) galaxies than nearby clusters.

**How do the star formation rates observed in protoclusters correspond to predictions based on the formation of these red sequence galaxies?** Are the amplitudes of the galaxy overdensities observed consistent with what structure formation predicts for the progenitors of galaxy clusters? The answers on these important questions may shed new light on the process of structure formation in the universe.

In this chapter we will attempt to address the questions raised above. The structure of this chapter is as follows. In Part I, we review the evidence of galaxy overdensities observed between \( z = 2 \) and 6, and summarize the main properties of the structures used in the subsequent parts of this chapter. In Part II, we review the theory of structure formation, and compare the evolution of overdensities associated with massive dark matter halos to the protocluster data compiled in Part I. We will investigate the abundances, bias and the likely present-day descendant populations of the halos hosting protoclusters with respect to other classes of high redshift objects. In Part III, we construct a simple ‘toy model’ for the star formation history of the red sequence population of galaxy clusters, and compare the evolution of the total luminosity as predicted by our model against the protocluster data.

### 10.2 Part I: A census of protoclusters

At present the investigation of candidate protocluster fields has resulted in the discovery of about 15 structures spanning the redshift range \( z = 2 – 6 \) (corresponding to \( \sim 2.5 \) Gyr of cosmic time). We have compiled an overview of the properties of 12 of the most convincing candidate protocluster objects discovered to date. Below we will briefly describe each target individually, ordered in increasing redshift, and refer to Table 10.1 for a compilation of the key parameters (overdensities and masses) of some of the structures used later in this chapter. The data was compiled from information on radio galaxy protocluster targets gathered by Venemans et al. (2005a), supplemented by data taken from this thesis, and from other targets from literature.
10.2.1 The targets

- **PKS 1138–262 (z = 2.16)** This 7′ × 7′ field contains a significant overdensity of spectroscopically confirmed Lyα emitters around a massive radio galaxy (Kurk et al. 2000; Pentericci et al. 2000; Kurk et al. 2004a,b). Furthermore, the field has been found to be relatively rich in Hα emitting galaxies having different velocity and spatial distributions compared to the Lyα emitters, several spectroscopically confirmed X-ray sources (Pentericci et al. 2002; Croft et al. 2005), as well as candidate 4000Å break objects at the protocluster redshift, indicating that different galaxy populations already exist in clusters several Gyr before virialization. Being among the closest of the candidate protoclusters, this is a particularly important target for linking high redshift protoclusters to clusters at low and intermediate redshifts (z ~ 1) in order to study morphological and kinematical evolution of galaxy clusters and their star formation histories. The host galaxy of the radio source has been found to consist of a massive component surrounded by a large concentration of smaller, disturbed objects, and the entire system in embedded in a 100 kpc halo of emission line gas (see Fig. 1.3 in this thesis).

- **HS1700–FLD (z = 2.30)** A highly significant overdensity at z = 2.300 ± 0.015 was discovered in the course of a large spectroscopic campaign.

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**Table 10.1 — Observational data on protocluster candidates.**

<table>
<thead>
<tr>
<th>Object</th>
<th>z</th>
<th>Sample</th>
<th>Field size (arcmin²)</th>
<th>δz / z</th>
<th>σv (km s⁻¹)</th>
<th>M₀ (M₀)</th>
<th>References¹</th>
</tr>
</thead>
<tbody>
<tr>
<td>PKS 1138–262</td>
<td>2.16</td>
<td>Lyα</td>
<td>7 × 7</td>
<td>3 ± 2</td>
<td>900 ± 240</td>
<td>3-4</td>
<td>1,2,3,4,5</td>
</tr>
<tr>
<td>HS1700–FLD</td>
<td>2.30</td>
<td>BX</td>
<td>8 × 8</td>
<td>6.9 ± 2.1</td>
<td>–</td>
<td>14</td>
<td>6</td>
</tr>
<tr>
<td>MRC 0052–241</td>
<td>2.86</td>
<td>Lyα</td>
<td>7 × 7</td>
<td>2.0 ± 0.5</td>
<td>980 ± 120</td>
<td>3-4</td>
<td>7</td>
</tr>
<tr>
<td>MRC 0943–242</td>
<td>2.92</td>
<td>Lyα</td>
<td>7 × 7</td>
<td>2.2 ± 0.9</td>
<td>715 ± 105</td>
<td>4-5</td>
<td>7</td>
</tr>
<tr>
<td>SA22–FLD</td>
<td>3.09</td>
<td>LBG</td>
<td>9 × 18</td>
<td>3.6 ± 1.1</td>
<td>–</td>
<td>10-14</td>
<td>8</td>
</tr>
<tr>
<td>MRC 0316–257</td>
<td>3.13</td>
<td>Lyα</td>
<td>7 × 7</td>
<td>2.3 ± 0.5</td>
<td>640 ± 195</td>
<td>3-5</td>
<td>7,10</td>
</tr>
<tr>
<td>TN J2009–3040</td>
<td>3.16</td>
<td>Lyα</td>
<td>7 × 7</td>
<td>0.7 ± 0.8</td>
<td>515 ± 90</td>
<td>–</td>
<td>7</td>
</tr>
<tr>
<td>TN J1338–1942</td>
<td>4.11</td>
<td>Lyα</td>
<td>7 × 7 (×2)</td>
<td>3.7 ± 1.0</td>
<td>265 ± 65</td>
<td>6-9</td>
<td>7,11</td>
</tr>
<tr>
<td>SDF</td>
<td>4.86</td>
<td>Lyα</td>
<td>25 × 45</td>
<td>2.0 ± 1.0</td>
<td>–</td>
<td>&gt; 3</td>
<td>15</td>
</tr>
<tr>
<td>TN J0924–2201</td>
<td>5.19</td>
<td>Lyα</td>
<td>7 × 7</td>
<td>1.5 ± 1.6</td>
<td>305 ± 110</td>
<td>4-9</td>
<td>7,16</td>
</tr>
<tr>
<td>SXDF-Object ‘A’</td>
<td>5.70</td>
<td>Lyα</td>
<td>60 × 60</td>
<td>2.3 ± 0.6</td>
<td>∼ 180</td>
<td>1-3</td>
<td>18</td>
</tr>
<tr>
<td>SDSS J0836+0054</td>
<td>5.82</td>
<td>LBG</td>
<td>3.4 × 3.4</td>
<td>1.0 ± 0.5</td>
<td>–</td>
<td>–</td>
<td>19</td>
</tr>
</tbody>
</table>

¹Method of sample selection: (Lyα) narrowband Lyα, (LBG) Lyman break technique, (BX) the ‘BX’ criteria of Adelberger et al. (2005).

²Approximate field size.

³Amplitude of the galaxy overdensity, δz = (Σ – Σ)/Σ, calculated from the surface overdensity (Σ) with respect to the average field density (Σ).

⁴Inferred mass of the overdensity in units of 10¹⁴ M⊙.

by Steidel et al. (2005) to select star-forming galaxies at $z = 2.3 \pm 0.4$. The structure corresponds to the largest, spectroscopically confirmed galaxy overdensity known at $z > 2$, and it is expected to become virialized by $z \sim 0$ on a mass scale of $\sim 10^{15} M_\odot$. Comparison between the best-fit spectral energy distributions of galaxies in the protocluster and the field suggests that the structure is relatively rich in evolved galaxies, as expected from simple theoretical predictions for accelerated structure formation (Steidel et al. 2005).

- **MRC 0052–241 ($z = 2.86$) and MRC 0943–242 ($z = 2.92$)** The 7′ × 7′ fields towards both of these galaxies are rich in Lyman-alpha (Lyα) galaxies in a narrow redshift interval centred on that of the radio sources. The systems each have $\sim 70$ Lyα imaging candidates (of which $> 20$ spectroscopically identified), corresponding to galaxy overdensities of $\delta_c > 2$ and masses of $\sim 3 - 5 \times 10^{14} M_\odot$ (Venemans et al. 2005a).

- **SA22–FLD ($z = 3.09$)** This structure was serendipitously discovered by Steidel et al. (1998), following the presence of a significant ‘spike’ in the spectroscopic redshift distribution of a large sample of $z \sim 3$ Lyman break galaxies in this field (9′ × 18′). The derived number density of these structures implies that they are consistent with being the progenitors of moderately rich galaxy clusters in their early stages of evolution. Subsequent study of this field has shown a similar overdensity in Lyα galaxies, as well as the presence of several large Lyα ‘blobs’ associated with the protocluster (Steidel et al. 2000).

- **MRC 0316–257 ($z = 3.13$)** This structure consists of 31 confirmed Lyα emitters in a 7′ × 7′ field centered on a powerful radio galaxy (Venemans et al. 2005a,b). Its velocity dispersion of $\sim 600 \text{ km s}^{-1}$, roughly halfway between the typical velocity dispersion associated with the local Hubble flow at $z \geq 3$ and that of massive virialized clusters at $z \leq 1$, possibly indicating that the structure is at an intermediate evolutionary stage.

- **TN 2009–3040 ($z = 3.16$)** This target is one of two of the least rich of the overdensities around radio galaxies found by Venemans et al. (2005a).

Although due to its small overdensity it is not considered to be a candidate protocluster, the subclustering of Lyα emitters both in angular and velocity space suggests that they are physically linked to the radio galaxy, maybe in a group. We include it in our analysis below for comparison.

- **TN J1338–1942 ($z = 4.11$)** This is the richest structure among the radio galaxy-selected protocluster targets (Venemans et al. 2005a). Venemans et al. (2002) have estimated a mass of $\sim 10^{15} M_\odot$ based on the overdensity of Lyα emitters within a structure of $\sim 2 \text{ Mpc}$ in radius around the radio galaxy TN J1338–1942. The field has been shown to have a similar excess of Lyman break galaxies selected using HST/ACS over an 3.4′ × 3.4′ area (see chapters 4, 5 & 6 of this thesis), indicating the enhanced star-forming activity and clustering associated with the forming structure and the radio source (Miley et al. 2004; Zirm et al. 2005; Overzier et al. 2006b).

- **SDF ($z = 4.86$)** Strong clustering of Lyα emitters was found in a 20 × 50 Mpc (in co-moving units) elongated region in the Subaru Deep Field (SDF) using a narrowband centred on Lyα at $z = 4.86$ (Shimasaku et al. 2003). The overdensity and large structure size may signal the formation of a galaxy cluster.

- **TN J0924–2201 ($z = 5.19$)** This radio galaxy, the most distant known, has 6 spectroscopically confirmed Lyα emitting companion galaxies, and appears to lie within an overdense region (Venemans et al. 2004). Observations with HST/ACS further indicated an excess of $V_{\text{606}}$-break ($z < 5$) objects to $\sim 99\%$ confidence, suggesting that the radio galaxy lies in a relatively rich environment, possibly a protocluster.

- **SXDF ($z = 5.70$)** Ouchi et al. (2005) discovered two overdense structures of $\sim 1 \text{ Mpc}$ in physical size of Lyα galaxies within the $\sim 1 \text{ deg}^2$ Subaru/XMM-Newton Deep Field (SXDF), corresponding to a similar volume density of moderately rich clusters found in the present-day universe.

- **SDSS J0836+0054 ($z = 5.82$)** The most distant radio-loud quasar known, SDSS J0836+0054...
is associated with a large number of candidate companion objects characterized by very red (1.3 < \textit{i}_{775} \sim 2.0) color (Zheng et al. 2006, see chapter 8 of this thesis). The surface density in this field is approximately six times higher than the number expected from deep ACS fields, although the relatively small number statistics and lack of spectroscopic confirmation make it difficult to quantify the excess.

10.2.2 Derivation of the total mass

The large galaxy overdensities relative to the field observed in each of these structures can be translated into an estimate of the total mass of the structure (cf. Steidel et al. 1998; Venemans et al. 2005a) as follows. The observed galaxy overdensity, $\delta_g$, relates to the mass overdensity, $\delta_m$, through the bias parameter, $b$, that relates galaxies to the underlying dark matter:

$$1 + b \delta_m = C (1 + \delta_g),$$  \hspace{1cm} (10.1)

where

$$C = 1 + f - f(1 + \delta_m)^{1/3}, \quad f = \Omega_m(z)^{4/7} \hspace{1cm} (10.2)$$

is a correction for redshift-space distortion due to peculiar velocities assuming that the object is breaking away from the Hubble expansion (Steidel et al. 1998). Taking the typical bias values of $\sim 2$ - 5 as found for various populations of star-forming galaxies in protoclusters estimated from the clustering of these populations in large field surveys (e.g. Ouchi et al. 2004; Adelberger et al. 2005; Lee et al. 2005), typically yields $\delta_m \sim 0.2 - 2$ for the targets listed in Table 10.1. Assuming that this overdensity is representative for the true mass overdensity of the structure as a whole as measured within a certain structure volume (or surface area), the total mass is given by

$$M \simeq (1 + \delta_m) \rho V,$$  \hspace{1cm} (10.3)

where $\rho$ is the present-day mean density of the Universe.

The mass estimates of the protocluster candidates are typically on the order of $10^{14-15} M_\odot$ (see Table 10.1), similar to that of massive clusters. However, given that for typical cosmological surveys $V \gtrsim 10^{14-15}/\rho$, the genuine progenitors of present-day clusters must be shown to have an overdensity sufficiently large in order for the structure to collapse and virialize by the current epoch. This will be the subject of the following section, where we shall consider the basic theory of structure formation in order to compare its predictions with the observed properties of protocluster candidates summarized above.

10.3 Part II: Theory of structure formation

The structure of this section is as follows. We first present the elements of the spherical collapse model, and use its predictions to investigate whether the properties of our candidate ‘protoclusters’ are consistent with the evolution of the structures into bound objects. Then, we use the framework of structure formation to investigate the evolution of cluster-like halo abundances, and compare its predictions to the available protocluster data.

10.3.1 Linear spherical collapse

In the local Universe, clusters of galaxies are gravitationally bound objects with masses of $\sim 10^{14-16} M_\odot$ (bound objects with mass of a few times $10^{13} M_\odot$ are usually referred to as ‘galaxy groups’). We shall review aspects of the theoretical framework necessary to understand the formation and evolution of galaxy clusters. Clusters are the result of the gravitational collapse of matter. Because most of this mass is in the form of (non-baryonic) dark matter, the theory of collapse can almost in its entirety be developed from the fluid dynamics of collisionless, non-dissipative systems. The current framework for the theory of collapse describes the growth of all structure in the universe as due to perturbations in the density field of an expanding universe with a non-zero cosmological constant. The theory predicts the mass distribution in the present-day universe from the amplitude of the
initial perturbations at recombination as characterized by the matter power spectrum.

Although collapsed structures ranging from dwarf galaxies to massive galaxy clusters are highly non-linear systems, in explaining structure formation we can largely rely on the linear theory of spherical collapse (see, e.g., Peacock 1999). This is because the growth of cosmological structure proceeds approximately linearly up to the point of collapse. After collapse, the structure evolves non-linearly until it is virialized. In the linear collapse model, a uniform (top-hat) spherical overdense region with a density larger than the local critical density will behave like an isolated region that initially expands but then collapses to form a bound object. The turn-around time is the time at which the outermost shell of the system has reached its maximum radius, and decouples from the cosmic expansion resulting from the collapse. In this simplified model, the collapse is complete at twice the turn-around time. In reality, the object does not collapse to a singularity, but stabilizes after a finite time at the radius of virialization, roughly half the radius reached at maximum expansion (see Fig. 10.2). The critical overdensity predicted by the linear collapse model is $\delta_c \approx 1.686$, which is almost independent of cosmology. This critical overdensity can be used as a simple criterion to study the collapse of density perturbations at different epochs and at different mass scales. Once the structure virializes, the true overdensity of the structure will be $\sim 200$, at which the linear model becomes insufficient to describe the dynamical evolution.

### 10.3.2 Comparison between protocluster overdensities and the requirements for spherical collapse

Here we will investigate whether the matter overdensities estimated for protoclusters at $z = 2 - 6$ are sufficient for the structures to collapse, and, if so, at which epoch virialization takes place. As discussed in the previous section, gravitationally bound or collapsed objects of mass $M$ are expected to have formed when their linear matter overdensity, $\delta_L$, exceeds the critical density for collapse of 1.686. In order to make the comparison between $\delta_L$ used in the theory, and the matter overdensity, $\delta_{mr}$, that were estimated based on galaxy overdensities ($\delta_g$) measured towards the protocluster targets discussed in the previous section, we use the approximation given by Mo & White (1996, see Bernardeau (1994) for a derivation) that relates the mass overdensity to the linearly extrapolated overdensity in the early stages of the (spherical) collapse:

$$\delta_L = -1.35(1 + \delta_{mr})^{-2/3} + 0.7875(1 + \delta_{mr})^{-0.58661} + 1.1243(1 + \delta_{mr})^{-1/2} + 1.68647.$$  \hspace{1cm} (10.4)$$

In Fig. 10.3 we present the linearly extrapolated overdensities determined from the protocluster data summarized in Section 10.2 (indicated by the points). For simplicity, the targets have been grouped in approximate redshift where appropriate. The shaded regions indicate the linear overdensities and their 1σ uncertainty when evolved to the present epoch based on the growth of the density perturbations according...
Figure 10.3 — Linear overdensities as a function of redshift based on the observational evidence for protocluster candidates as summarized in Table 10.1. Points with error bars show the linear overdensities corresponding to the measured galaxy overdensities (mostly from Lyα emitters) assuming spherical collapse, and the evolution of the overdensities with redshift is shown by the shaded regions (1σ range). The horizontal bar indicates the critical collapse threshold, $\delta_c = 1.686$, for forming bound objects. The available data suggests that the protocluster candidates have varying properties when evolved to the current epoch: some structures undergo collapse by $z \sim 0.5$, others by $z \approx 0$, while some structures are not dense enough for undergoing collapse even by $z = 0$. See text for details.

Using the approximations to the cosmological growth in a $\Lambda$CDM universe given by Carroll et al. (1992).

In each of the panels of Fig. 10.3 we have indicated the critical linear overdensity for collapse. The simple extrapolation of the measured overdensities to later epochs illustrates a range of interesting aspects of these structures. Some of the structures (e.g., the targets HS1700–FLD at $z = 2.3$, TN J1338–1942 at $z = 4.1$ and SXDF at $z = 5.7$) have sufficiently large overdensities with relatively small uncertainties, that they are expected to have reached the collapse threshold.
well before the present epoch. For these objects, collapse is predicted to occur at \( z \approx 0.5 \). Most of the other structures are consistent with collapse by \( z \approx 0 \) (e.g., PKS 1138–262 at \( z = 2.16 \), SA22–FLD and MRC 0316–257 both at \( z = 3.1 \), SDF at \( z = 4.9 \), TN J0924–2201 at \( z = 5.2 \), and SDSS J0836+0054 at \( z = 5.8 \)). Some of the overdensities observed seem too small for collapse even by \( z = 0 \), for example for 2009–3040 at \( z = 3.1 \), in agreement with the conclusion of Venemans et al. (2005a) that the overdensity is too small to qualify as a protocluster candidate.

Taking the results at face value, we have demonstrated that the current sample of protoclusters represents a class of objects that, generally speaking, meets the requirements for collapse on a mass scale that is comparable to that of galaxy clusters. How do the linear overdensities derived here, and the abundance of protoclusters observed, compare to the predicted halo abundances as a function of redshift and halo mass?

### 10.3.3 The evolution of mass fluctuations

Here we give the necessary ingredients for describing the evolution of mass fluctuations starting from the initial perturbations to the present (see, e.g., Peebles 1980; Peacock 1999; Mo & White 2002; Kaiser 2002; Tozzi 2006). Because the predictions for structure formation rely heavily on the cosmology, observational cosmologists have had to struggle with widely varying model predictions depending on which cosmological model was used. Currently, theory predictions have become considerably more reliable due to the vastly improved accuracy of the fundamental cosmological parameters. In the discussion below we will use the concordance \( \Lambda \text{CDM} \) cosmology, \( \Omega_M = 0.3 \), \( \Omega_\Lambda = 0.7 \), \( h = 0.7 \), \( H_0 = 100h \) km s\(^{-1}\) Mpc.

Because the cosmic density field is approximated to be linear, at any moment we can relate a given mass to the radius of a spherical volume in which that mass is contained

\[
R(M) = \left( \frac{3M}{4\pi\rho_0} \right)^{1/3},
\]

where \( \rho_0 \) is the current average density of the universe. Under the assumption of Gaussian fluctuations in the density field (as predicted by inflation), the entire mass field can be characterized only by its variance

\[
\sigma^2(M) = \frac{4\pi}{(2\pi)^3} \int_0^\infty \frac{dk}{k^3} P_{lin}(k) W^2(kR),
\]

where \( \sigma(M) \) is the rms value of the density fluctuations on mass scale \( M \) when smoothed with a top-hat filter of radius \( R \) having a Fourier transform of \( W(x) = (3/x^3)[\sin x - x \cos x] \), and \( P_{lin}(k) \) is the linear power spectrum of density fluctuations extrapolated to \( z = 0 \). We used a \( \Lambda \text{CDM} \) power spectrum of the form \( P_{lin}(k) \propto k^n T^2(k) \) with \( n = 1 \) and the matter transfer function

\[
T(q) = \frac{\ln(1 + 2.34q)}{2.34q} \times [1 + 3.89q + (16.1q)^2 + (5.46q)^3 + (6.71q)^4]^{-1/4},
\]

\[
q = (k/h)\Gamma,
\]

\[
\Gamma = \exp(\Omega_b + \sqrt{2H_0\Omega_\Lambda/\Omega_M})/\Omega_M H_0 c
\]

from Bardeen et al. (1986); Sugiyama (1995). The power spectrum is normalized by requiring that the present-day rms mass fluctuation in a sphere of radius \( 8 h^{-1} \) Mpc is \( \sigma_8 = 0.9 \).

The above prescriptions for mass fluctuations can be used to predict the number density of present-day bound structures such as galaxy clusters, and their evolution at different epochs. This problem is usually addressed by studying the mass function \( n(M, z)dM \), which describes the number density of bound objects with masses between \( M \) and \( M + dM \) at redshift \( z \), or \( N(M > M, z) \), which is the number density of objects more massive than \( M \). Although the collapse and virialization of overdensities are non-linear in nature, the process of collapse is unlikely to change the total mass contained by the overdensity filtered on a scale \( R \). Therefore, the mass function can be constructed from the number density of regions that have an overdensity \( \delta > \delta_c \).

In the theory pioneered by Press & Schechter (1974), the halo abundance as a function of mass
and redshift can be approximated using the unconditional mass function derived by Sheth & Tormen (1999); Sheth et al. (2001); Sheth & Tormen (2002):

\[
\nu f(\nu) = 2A \left( 1 + \frac{1}{\nu^2p} \right) \left( \frac{\nu^2}{2\pi} \right)^{1/2} \exp \left( -\frac{\nu^2}{2} \right),
\]

(10.10)

where \( \nu' = \sqrt{\nu} \), \( a = 0.707 \), and \( q = 0.3 \) are based on a fit to the mass function of the numerical GIF simulations (Kauffmann et al. 1999), and \( A = 0.322 \) follows from the requirement that the integral of \( f(\nu) \) over all \( \nu \) should give unity. The halo mass \( M \) is related to the rms density fluctuations through the parameter

\[
\nu \equiv \left[ \frac{\delta_c}{D(z)\sigma(M)} \right]^2.
\]

(10.11)

The mass function is accordingly

\[
n(M, z)dM = \frac{\rho}{M} \nu f(\nu) \frac{d\nu}{\nu}.
\]

(10.12)

The typical halo mass that collapses at each redshift, \( M_c(z) \), is defined by \( \nu = 1 \), i.e., \( \sigma(M_c) = \delta_c/D(z) \).

We have used the above formalism to reproduce the evolution of halo abundances as shown by Mo & White (2002). The result is shown in Fig. 10.4, showing the number density of collapsed, dark halos as a function of redshift and (minimum) mass. The figure illustrates that the number density of collapsed halos of mass \( > 10^{15} M_\odot \) are as frequent in the present-day universe, as halos of \( > 10^{12} M_\odot \) at \( z \sim 10 \), and that locally halos of \( > 10^{12} M_\odot \) are 4 orders of magnitude more abundant than those of \( > 10^{15} M_\odot \).

10.3.4 Comparison between protocluster overdensities and halo abundances

In Fig. 10.5 we plot the predicted abundance of the progenitor halos of \( 10^{15} M_\odot \) halos, as a function of their linear overdensity and redshift (the predictions for \( 10^{14} M_\odot \) halos are shown as dotted lines). For your guidance, check that the number density corresponding to the linear collapse threshold of 1.686 (dashed line) at \( z = 0 \) is \( \sim 10^{-6} h^3 \text{ Mpc}^{-3} \), consistent with the abundance of collapsed halos of this mass plotted in Fig. 10.4. The plot further shows that at \( z \sim 2 \), for example, the linear overdensity of such halos was \( \sim 0.8 \). Assuming that the protoclusters are the progenitors of \( \sim 10^{15} M_\odot \) halos (see Table 10.1), how do their overdensities and number densities fit in with these model predictions? Unfortunately, the abundance of the protoclusters is largely unknown given the very few objects discovered to date and the complicated selection effects. However, we can place at least some constraints from the observed spread in (linear) overdensities of \( \delta_c = 0.3 \sim 0.8 \), and the corresponding redshifts of the protoclusters of \( z = 2 \sim 6 \) (points inside the dark shaded region in Fig. 10.5). Furthermore, we indicate the approximate number density of powerful radio galaxies at \( z \sim 3 \) as estimated\(^1\) by Venemans et al. (2002) (indicated by the medium-dark shaded region, allowing for a factor 10 un

\[^1\] Based on the number density of radio galaxies at \( 2.7 < z < 3.4 \) with luminosities exceeding \( 10^{23} \text{ erg s}^{-1} \text{ Hz}^{-1} \text{ sr}^{-1} \) at 2.7 GHz, taking into account a radio source life-time of \( 10^7 \text{ yr} \). See Venemans et al. (2002) for details.
Figure 10.5 — The cumulative number densities of the progenitors of $10^{15} M_\odot$ halos as a function of linear overdensity and redshift (numbered solid lines). Dotted lines are for dark halos of mass $10^{14} M_\odot$. The points inside the dark shaded area indicate the range of overdensities ($\delta_L \sim 0.3 - 0.8$) measured at the corresponding redshifts of the protocluster targets of Table 10.1 and Fig. 10.3. Although there are few constraints on the abundance of protoclusters, we mark the approximate number density of powerful radio galaxies at $z \sim 3$ (medium shaded region), and the number density of protoclusters inferred by Steidel et al. (1998) based on the statistics of LBG redshift spikes (light shaded region). The vertical dashed line indicates the critical collapse threshold of 1.686. If radio galaxies, LBG spikes and the galaxy overdensities observed are associated with the progenitors of $\sim 10^{15} M_\odot$ halos, their number densities of $\log N(>\delta_L) \sim -(5 - 6) h^3$ Mpc$^{-3}$ imply cluster virialization at $z \sim 0.5 - 0$ for the lower value, and no complete collapse by $z = 0$ for the higher value. The hatched area marks the observed number density of clusters at $z \leq 1$ with X-ray luminosities of $\gtrsim 5 \times 10^{43}$ erg s$^{-1}$ (Rosati et al. 2002).

certainty in the number density). We also indicate the number density of $z \sim 3$ protoclusters as inferred by Steidel et al. (1998) based on the statistics of LBG redshift spikes (light shaded region, with a factor 10 uncertainty). If radio galaxies, LBG spikes and the range of structure overdensities observed all trace the same massive halos, their number densities are predicted to be in the range from $N(>\delta_L) = 10^{-6}$ to $10^{-5} h^3$ Mpc$^{-3}$. Following the redshift evolution at these halo abundances using the lines in Fig. 10.5 imply cluster virialization by $z \approx 0.5$ for the overdensities at $z \sim 4 - 6$. Some of the overdensities at $z \sim 3$ seem too small for complete collapse even by $z = 0$.

We also indicated the observed number density of clusters at $z \leq 1$ with X-ray luminosities of $\gtrsim 5 \times 10^{43}$ erg s$^{-1}$ (hatched region in Fig. 10.5). For a massive cluster to collapse by $z \sim 1$, it is predicted to have $\delta_L \approx 0.7$ at $z \sim 4$. This is higher than observed for protoclusters at this redshift: TN J1338–1942 at $z = 4.1$, for example, has $\delta_L \approx 0.5$. 
10.3.5 The bias of protoclusters and their \( z = 0 \) descendants

In the halo mass model given in Section 10.3.3, the bias of the dark halos is given by

\[
b_{\text{DH}} = 1 + \frac{1}{\delta_c} \left[ \nu^2 + b \nu^{3(1-\sigma)} - \frac{\nu^{2\sigma}/\sqrt{a}}{\nu^{2\sigma} + b_a(1-c)/(1-c/2)} \right]^{1/2},
\]

where \( a = 0.707, \ b = 0.5, \ c = 0.6 \) (Sheth et al. 2001). We have estimated the bias of protoclusters, by looking up the \( b_{\text{DH}}(z = 3) \) corresponding to the number densities of luminous radio galaxies and LBG spikes at \( z \sim 3 \) given by Venemans et al. (2002) and Steidel et al. (1998). We find \( b \sim 8 \) (see Fig. 10.6). At this redshift, the bias corresponds to collapsed halos of \( \sim 5 \times 10^{13} M_\odot \). For comparison, we have also indicated in Fig. 10.6 the bias values found for several other classes of objects. At \( z \sim 1 \), luminous radio sources have \( b \sim 2 - 3 \), suggesting that they correspond to massive halos (see chapter 2 of this thesis). At \( z = 3 - 5 \), the clustering of bright \((L \gtrsim L_*)\) LBGs indicate \( b \sim 3 - 5 \) and halo masses of \( \sim 10^{12} M_\odot \) (circles, from Ouchi et al. (2004)). Near-infrared selected galaxies at \( z \sim 3 \) show that the bias may strongly depend on colour (squares, from Daddi et al. (2003)).

For completeness, we have also illustrated the (mild) constraints on the bias of \( i_{775} \) dropouts at \( z \sim 6 \) derived in chapter 8 of this thesis.

Following Ouchi et al. (2004), we can extrapolate the bias measurements for these objects to \( z = 0 \) using (Sheth et al. 2001):

\[
b_{\text{DH}}^0 = 1 + \frac{D(z)}{D(0)} \left[ \nu^2 + b_a \nu^{3(1-\sigma)} - \frac{\nu^{2\sigma}/\sqrt{a}}{\nu^{2\sigma} + b_a(1-c)/(1-c/2)} \right]^{1/2},
\]

and by assuming that the bias of a given class of objects is representative of the bias of the dark halos hosting them, i.e., \( b_a \approx b_{\text{DH}} \). The results are shown in Fig. 10.7, where we plot the number densities of the present-day descendants of the objects in Fig. 10.6 as a function of halo mass. As previously shown by Ouchi et al. (2004), the descendants of all the classes of objects shown lie in the mass range corresponding to groups and clusters of galaxies. Albeit by construction, Fig. 10.7 illustrates that the number density of the \( z = 0 \) descendants of luminous radio galaxies and protoclusters at \( z \sim 3 \) implies masses well in the range for galaxy clusters.

10.3.6 Summary

There are several conclusions that follow from our analysis of the observed properties of protocluster candidates:

1. We calculated the linear overdensities corresponding to the mass overdensities derived from the observations, finding \( \delta \sim 0.3 - 0.8 \).

2. The observed overdensities are in approximate agreement with the amplitudes of massive halo progenitors at \( z = 2 - 6 \).

3. The number densities of massive dark halo progenitors agree approximately with the densities expected for luminous high-redshift radio galaxies (corrected for a radio source life-time of \( \sim 10^7 \) yr), and the estimated number density or protoclusters of Lyman break galaxies.
10.4 Part III: Modeling the history of the cluster red sequence

As detailed in the previous sections, the amplitudes and masses of the galaxy overdensities found suggest that they are likely progenitors of clusters. However, so far we have ignored the baryonic matter component of clusters altogether. Here, we shall construct a very simple ‘toy model’ for the star formation history of clusters. Because it is an established fact that most of the stellar mass contained in the cluster red sequence population was formed at redshifts similar to the redshifts of our protocluster candidates (see Blakeslee et al. 2006, and references therein), we will restrict our analysis to modeling of the red sequence of clusters. The goal of our simulations is to build a library of models that are able to reproduce the observed intrinsic scatters around the color-magnitude relation (CMR) as observed for massive high-redshift clusters, and compare the extrapolated star formation history of those models against the available protocluster data. For these simulations we shall use the observed CMR of three massive clusters chosen from literature. These are Cl 1358+6245 at $z = 0.33$ (van Dokkum et al. 1998), MS 1054–0321 at $z = 0.834$ (Blakeslee et al. 2006), and RDCS 0910+5422 at $z = 1.106$ (Mei et al. 2006). The main observational details for these clusters are summarized in Table 10.2.

10.4.1 The model

The first part of our procedure is similar to the method used by van Dokkum et al. (1998), Blakeslee et al. (2003, 2006) and Mei et al. (2006), who simulated the scatter around the CMR to estimate the mean ages of red sequence galaxies in a number of massive clusters at $z = 0.3 - 1.3$ observed with HST. We have used Bruzual & Charlot (2003, BC03) stellar population models to calculate template colors given a particular star formation history. We only consider the solar metallicity models with a Salpeter initial mass function and high resolution (“bc2003_hr_m62_salp_ssp”). We use the truncated star formation model for the star formation history of cluster galaxies, which assumes that stars form at a constant rate between $t = t_1$ and $t = t_2$, where $t_1$ and $t_2$ are randomly chosen to lie at $t_0 < t_1 < t_2 < t_{\text{end}}$ with $t_0$ the time at recombination and $t_{\text{end}}$ the age of the universe at the redshift of the cluster.

We simulated ‘clusters’ by randomly creating ‘galaxies’ with different formation epochs and different burst durations. Instead of the standard procedure employed by Blakeslee et al. (2003) and Mei et al. (2006) of creating ~ 10,000 model galaxies per simulation (for statistical reasons), we limit the number of model galaxies...
Table 10.2 — Scatters of the color-magnitude relation in 3 massive clusters.

<table>
<thead>
<tr>
<th>Cluster</th>
<th>z</th>
<th>$L_{bol}^{44}$ (erg s$^{-1}$)</th>
<th>$T_X$ (keV)</th>
<th>$f_E$</th>
<th>$N_E^4$</th>
<th>$R_{max}$ (arcmin)</th>
<th>Color</th>
<th>$\sigma_{int}$</th>
<th>Refs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cl1358+6245</td>
<td>0.33</td>
<td>10.65</td>
<td>7.0</td>
<td>0.24</td>
<td>46</td>
<td>$V_{606}$−$I_{814}$</td>
<td>0.022</td>
<td>$\pm$ 0.003</td>
<td>1,2</td>
</tr>
<tr>
<td>MS 1054–0321</td>
<td>0.83</td>
<td>16.43</td>
<td>8.0</td>
<td>0.49</td>
<td>46</td>
<td>$V_{606}$−$z_{850}$</td>
<td>0.080</td>
<td>$\pm$ 0.015</td>
<td>3,4,5,6</td>
</tr>
<tr>
<td>RDCSJ0910+5422</td>
<td>1.11</td>
<td>2.14</td>
<td>7.2</td>
<td>0.37</td>
<td>20</td>
<td>$i_{775}$−$z_{850}$</td>
<td>0.044</td>
<td>$\pm$ 0.010</td>
<td>6,7,8</td>
</tr>
</tbody>
</table>

$a$ Bolometric X-ray luminosity in units of $10^{44}$ erg s$^{-1}$.

$b$ X-ray temperature.

$c$ Elliptical fraction.

$d$ Number of ellipticals on the red sequence.

$e$ Maximum radius for object selection.

$f$ Color used for fitting the color-magnitude relation.

$g$ Observed intrinsic scatter.


Table 10.3 — Color-magnitude relation simulations.

<table>
<thead>
<tr>
<th>Cluster</th>
<th>z</th>
<th>Model</th>
<th>$\sigma$</th>
<th>$\Delta t_{min}$</th>
<th>$\langle \tau \rangle$</th>
<th>$\langle \tau_1 \rangle$</th>
<th>$\langle \Delta \tau \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cl1358+6245</td>
<td>0.33</td>
<td>a</td>
<td>0.022</td>
<td>3.9</td>
<td>6.8</td>
<td>1.6</td>
<td>2.9</td>
</tr>
<tr>
<td>Cl1358+6245</td>
<td>0.33</td>
<td>b</td>
<td>+</td>
<td>2.0</td>
<td>5.7</td>
<td>2.2</td>
<td>4.0</td>
</tr>
<tr>
<td>Cl1358+6245</td>
<td>0.33</td>
<td>c</td>
<td>−</td>
<td>5.4</td>
<td>7.6</td>
<td>1.3</td>
<td>1.9</td>
</tr>
<tr>
<td>MS 1054–03</td>
<td>0.83</td>
<td>a</td>
<td>0.080</td>
<td>1.3</td>
<td>3.9</td>
<td>1.4</td>
<td>2.3</td>
</tr>
<tr>
<td>MS 1054–03</td>
<td>0.83</td>
<td>b</td>
<td>+</td>
<td>0.8</td>
<td>3.7</td>
<td>1.3</td>
<td>3.0</td>
</tr>
<tr>
<td>MS 1054–03</td>
<td>0.83</td>
<td>c</td>
<td>−</td>
<td>2.3</td>
<td>4.3</td>
<td>1.2</td>
<td>2.0</td>
</tr>
<tr>
<td>RDCSJ0910+5422</td>
<td>1.11</td>
<td>a</td>
<td>0.042</td>
<td>1.0</td>
<td>3.3</td>
<td>1.0</td>
<td>2.2</td>
</tr>
<tr>
<td>RDCSJ0910+5422</td>
<td>1.11</td>
<td>b</td>
<td>+</td>
<td>0.1</td>
<td>2.7</td>
<td>1.3</td>
<td>3.0</td>
</tr>
<tr>
<td>RDCSJ0910+5422</td>
<td>1.11</td>
<td>c</td>
<td>−</td>
<td>2.1</td>
<td>3.8</td>
<td>0.9</td>
<td>1.7</td>
</tr>
</tbody>
</table>

$a$ The best-fit model (see text for details).

$b$ The ‘maximally young’ model (see text for details).

$c$ The ‘maximally old’ model (see text for details).

$d$ The minimum model age.

$e$ The mean luminosity-weighted age.

$f$ The mean start time of the star formation.

$g$ The mean duration of the star formation.

to the actual number of red sequence galaxies observed on the CMR. For each randomly simulated cluster we evolve the star formation histories to the redshift ($z_{obs}$) or epoch ($t_{red}$) of observation, and calculate the average color and the scatter of the simulated population to test for the presence of a ‘red sequence’. In case the scatter among the models in a particular simulation run are within the range of the scatters as observed for our three baseline clusters, we accept this ensemble of models to reflect a possible star formation history of the cluster red se-
Figure 10.8 — Points show the scatter in the simulated color-magnitude relation at \( z = 0.33 \). Each simulation consisted of 46 model SEDs with constant formation of variable, random duration (from \( t_1 \) to \( t_2 \)) between the age of the cluster (\( t_{\text{end}} \)) and the age of recombination (\( t_0 \)). In (a), we show the scatter in \( V_{606} - I_{814} \) as a function of the minimum age at \( z = 0.33 \) (\( t_{\text{end}} \)). In (b), we illustrate how the increase in the minimum age of the models implies a decrease in the (mean) duration of the starburst, given the shorter time window allowed for star formation. In (c)–(e), we respectively plot the mean of the luminosity-weighted ages at \( z = 0.33 \), the mean duration of star formation (\( \langle \Delta t \rangle = (t_2 - t_1) \)), and the mean time of onset of the star formation (\( \langle t_1 \rangle \)) versus the scatter found in each of the simulations. Shaded regions indicate the \( \pm 1 \sigma \) range of the observed intrinsic scatter from van Dokkum et al. (1998). Large solid circles indicate the best fit model (middle circle in panel (a)), as well as a maximally young and a maximally old model that still fit the observed intrinsic scatter (left and right circles in panel (a), resp.). These three models are used in Fig. 10.11 to evaluate the star formation history of the red sequence galaxies at \( z > 0.35 \).

sequence. The formation epochs and star formation histories are saved to carry out step 2 of our simulation. In this step, the BC03 models are normalised so that the total stellar mass of the model at \( z \sim 1 \) amounts to the typical mass of red sequence galaxies. Given the set of models that yields an appropriate red sequence when evolved to the observed cluster redshifts, and the proper mass normalisation, the full star formation history of the simulated cluster is now fixed. It is straightforward to compute the total star formation rate or luminosity of the simulated ‘clusters’ at any redshift and in any bandpass. Summarizing, our toy model depends on the following input parameters or constraints:

1. A global model form of star formation history (we assume constant star formation).
2. \( N_e \), the number of morphologically selected elliptical galaxies in the cluster CMR.
3. \( M_e \), the typical mass of cluster elliptical galaxies (~\( 10^{11} M_\odot \)).
4. \( \sigma_{\text{int}} \), the intrinsic scatter at \( z = z_{\text{obs}} \).

Note that the number of progenitor galaxies per red sequence galaxy is not a factor in our simulations. This is justified because we are only interested in modeling the evolution of the total star formation rate, and not the number of galaxies at different magnitudes (i.e., the luminosity function) as a function of redshift. It is, however, implicitly assumed that the stellar mass that comprises a single galaxy at the observed cluster redshift, is coeval and formed with the same star formation history. This assumption is supported by simulations of the formation histories of cluster ellipticals. De Lucia et al. (2006) showed that there is a clear distinction between the epoch at which the stellar mass formed, and the epoch at which that mass was assembled into a single galaxy. The mod-
els also show that for massive cluster ellipticals, the number of equal mass progenitors is quite low ($\sim 2 - 3$). For such equal mass mergers to take place, it is conceivable that the two merging galaxies will roughly have comparable formation histories given that they have comparable mass at the time of the merger. Our model is independent of the exact mechanism that drives the star formation (e.g., induced by merger, monolithic collapse, AGN or superwind feedback, etc.). Although our model uses a very simplistic approach that should not be considered as an alternative for elaborate semi-analytical models or $N$-body simulations of cluster formation, it is sufficient for carrying out a rough order of magnitude comparison with the available data on protocluster candidates.

### 10.4.2 Model Results

For each simulation, we created $N_e$ galaxies with randomly chosen $t_1$, $t_2$ (giving $\Delta \tau = t_2 - t_1$). $\Delta \tau$ was in the range 0.1–10.0 Gyr, with increments of 0.1 Gyr. Our three clusters 1358, 1054 and 0910 have respectively 46, 46 and 20 elliptical galaxies on the CMR. To aid the practicality, we simulated the model populations as a function of the minimum age, $t_{\text{min}}$, of galaxies at the cluster redshift. Figs. 10.8, 10.9 and 10.10 show the scatters calculated from the simulations (points), compared to the observed scatter and its 1$\sigma$ error range (shaded regions). Our best-fit model for 1358 ($z = 0.33$) had a minimum age of 3.9 Gyr ($z > 0.9$), a mean luminosity-weighted age of 6.8 Gyr ($z \sim 2.2$), a mean burst duration of 2.9 Gyr, and a mean star formation starting time of 1.6 Gyr after recombination. For 1054 ($z = 0.83$), the minimum age was 1.3 Gyr ($z > 1.2$) and mean luminosity-weighted age of 3.9 Gyr ($z = 2.4$). For 0910 ($z = 1.11$), the minimum age was 1.0 Gyr ($z > 1.5$) and the mean $t_{\tau} = 3.3$ Gyr ($z \sim 3.1$). The results are summarized in Table 10.3. Our results are in agreement with the results found by van Dokkum et al. (1998), Blakeslee et al. (2006) and Mei et al. (2006) for the same clusters.

Next, we rescaled the $N_e$ SEDs belonging to the best-fit models so that at the epoch corresponding to the redshift of the clusters the total accumulated mass was $M_e = 10^{11} M_\odot$, suitable for cluster ellipticals. A schematic diagram of the star formation histories of the best-fit models are shown in Fig. 10.11. We then
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Figure 10.10 — Simulated scatters in $i_{775} - z_{850}$ of the color-magnitude relation in the cluster RDCS 0910+5422 at $z = 1.106$. The shaded region indicates the ±1σ range of the observed intrinsic scatter from Mei et al. (2006). See the legend of Fig. 10.8 for further details.

de-evolved the star formation histories to earlier times, and calculated the total cluster luminosity at rest frame 1500Å by adding up the luminosities of each of the $N_e$ models. The result is our final model for the star formation history of each cluster, and is shown in Fig. 10.12. Also plotted are two alternative, extreme model outcomes that reproduced the observed intrinsic scatter (dotted lines). In these models, the star formation either shut off later (on average), or started earlier, compared to the best-fit model. We also indicated the effect of dust to the total luminosity (dashed line). The dust contents was assumed to be constant, and decoupled from redshift, taking $E(B - V) = 0.1$ and applying the dust law of Calzetti et al. (2002).

Our model shows (depending on the assumptions) that the massive end of the cluster red sequence may have formed its stars at a relatively constant rate of several hundred $M_\odot$ yr$^{-1}$ over the redshift range $\sim 10 - 2$.

10.4.3 Comparison with protocluster data

One of the main questions that motivated the analysis presented in this chapter is whether the total stellar mass observed at the brightest $\sim 3$ mag end of $z \lesssim 1$ cluster red sequences is consistent with being produced by the Lyman break galaxies (and Lyα emitters) observed in protoclusters at $z = 2 - 6$. Our models, as illustrated in Fig. 10.12, imply that such hypothesis would require an average SFR of several hundred to a thousand $M_\odot$ yr$^{-1}$ over this whole redshift range. An estimate of the measured SFR in protocluster fields due to the observed Lyman break and Lyα populations can be obtained by looking at the typical star formation rate of these objects. Lyα emitters have a typical SFR of a few $M_\odot$ yr$^{-1}$ (Venemans et al. 2005a), which multiplied by the typical number of $\sim 30 - 60$ found in protocluster regions gives about 100-200 $M_\odot$ yr$^{-1}$. This is still an underestimate of the total UV SFR, as field studies indicate that only about 25% of LBGs have a sufficiently large equivalent width in Lyα to be observed and detected as Lyα emitters according to the most common selection criteria. Evidence that this fraction is similar in protoclusters comes from the fact that we found large populations of LBGs in some of these fields (see chapters 5 and 8 of this thesis), indicating that the total UV star formation rate including dust correction could well
be close to a thousand $M_\odot$ yr$^{-1}$. In this estimate we have not included the high star formation rates observed in the radio galaxies themselves of another several hundreds of $M_\odot$ yr$^{-1}$. Hence, the properties of the LBG and Ly$\alpha$ populations in the distant protoclusters are consistent with them being the progenitors of the evolved galaxies on the red sequence at $z \lesssim 1$.

### 10.5 Discussion and conclusions

We summarize our results as follows:

- The observed overdensities associated with the protoclusters candidates listed in Table 10.1 are in rough agreement with those expected for massive halo progenitors at $z = 2 - 6$.

- The number densities of such massive dark halo progenitors are in rough agreement with the number density of luminous high-redshift radio galaxies (assuming a radio source lifetime of $\sim 10^7$ yr), and the estimated number density of LBG overdensities (‘redshift spikes’).

- The overdensities of the protocluster candidates are, on average, sufficiently large for forming a bound object with a mass of $10^{15} M_\odot$ at $z \lesssim 0.5$.

- The estimated number density of protoclus-
ters implies \( b \sim 8 \) at \( z \sim 3 \), roughly twice as high as the bias of luminous galaxies at similar redshifts as measured by, e.g., Daddi et al. (2003) and Ouchi et al. (2004).

- The total SFR of the LBG and Ly\( \alpha \) populations in the distant protoclusters are consistent with them forming the stellar mass in the evolved galaxies on the red sequence at \( z \lesssim 1 \).

The main uncertainties in our conclusions are the following. Firstly, the mass overdensities were calculated from the observed surface overdensities of Ly\( \alpha \) galaxies, and, in some cases, LBGs. These populations are known to be highly biased, and it is unclear if they are representative for the total mass overdensity. It is important that the mass overdensities are confirmed using other tracer populations (e.g. Pentericci et al. 2002; Kurk et al. 2004b; Steidel et al. 2005). Secondly, the galaxy overdensities that we used in our calculations are expected to be conservative estimates. We did not include information on the generally narrow velocity dispersion of the Ly\( \alpha \) emitters, which, if it is narrower than that of similar galaxies in the field, could make the true overdensity in the protocluster region even higher. Thirdly, the exact volume that is occupied by the protocluster candidates has a large associated uncertainty. Fig. 10.5 demonstrates that the requirement for the observed overdensities to collapse within a finite time, becomes less stringent if the protoclusters have masses closer to \( 10^{14} \) \( M_\odot \) than \( 10^{15} \) \( M_\odot \) (dashed lines in Fig. 10.5), implying that the overdensities could be the progenitors of at least moderately rich clusters in the local universe. The largest overdensities observed are expected to have \( \delta_c \approx 1.3 - 1.4 \) at \( z \approx 1 \), implying that they cannot be the progenitors of fully virialized clusters at \( z = 1 \). The X-ray luminous clusters discovered at \( z \sim 1 \) are probably, by selection, extreme clusters that are not representative of clusters in the local universe.

Our results are in excellent agreement with a recent study of protoclusters using \( N \)-body simulations (T. Suwa, private communications). Protoclusters were selected by picking up the particles belonging to clusters at \( z = 0 \) and tracing them back to high redshift. The simulations showed that clusters with masses of \( > 10^{14} \) \( M_\odot \) can be traced back to regions at \( z = 4 - 5 \) of 20–40 Mpc in size that are associated with overdensities of Ly\( \alpha \) emitters and Lyman break galaxies of \( \delta_c \sim 3 \) and mass overdensities in the range 0.2–0.6. For randomly selected regions of the same size, the galaxy and mass overdensities were found to be mostly \( \lesssim 0 \), as expected due to the fact that massive halos are relatively rare. Although some of the overdense regions in the simulations having a similar overdensity as our protocluster candidates do not end up in clusters at \( z = 0 \), the simulations show that most regions with an overdensity on the order of a few at \( z = 5 \) will evolve into clusters more massive than \( 10^{14} \) \( M_\odot \) (\( \gtrsim 50\% \) for \( \delta_c \gtrsim 2 \)).

It is interesting that the descendants of almost all of the classes of high redshift objects shown in Fig. 10.7 (such as radio galaxies, LBGs and DRGs) fall in the mass range corresponding to groups and clusters of galaxies when evolved to \( z = 0 \). This is a natural consequence of the bias of these highly luminous objects. This is further evidence that the descendants of the protoclusters candidates are likely to evolve into even rarer and more massive objects.

Although the protocluster regions contain large numbers of LBGs and Ly\( \alpha \) emitters, their typical observed stellar masses are only a fraction (~1–10%) of those of early type cluster galaxies (see e.g., chapter 5), indicating that they might accumulate more stellar mass through merging and continuous star formation as they evolve from the inhabitants of \( z > 2 \) protocluster regions into, possibly, \( z \lesssim 1 \) red sequence galaxies. It is highly important to also try to detect other high-\( z \) populations that one might expect, such as the ‘distant red galaxies’ found at \( z < z < 4 \) (Franx et al. 2003). Infrared observations with the Spitzer Space Telescope may be used to further constrain how the mass of the progenitors of cluster galaxies relate to the masses of galaxies in virialized clusters. The full sample of protoclusters described in this chapter provides a unique sample for studying clus-
ter evolution at early epochs. Current wide-field optical and infrared surveys that search for clusters at $1 < z < 2$, as well as sensitive Sunyaev-Z'eldovich surveys are expected to further constrain the number densities of clusters as a function of redshift with a large dynamic range in mass. The study of how and when these structures and their galaxies were assembled will yield powerful clues to how the present-day large scale structure of the universe came about.

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References

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