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STELLINGEN

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Topics in the arithmetic of del Pezzo and K3 surfaces
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1. Every del Pezzo surface of degree 2 over a finite field is unirational.

For any complex K3 surface X, the Picard lattice of X, denoted by $\operatorname{Pic} X$, embeds into $H^2(X, \mathbb{Z})$; we define the *Picard number of* X to be the rank of $\operatorname{Pic} X$; we define the *transcendental lattice of* X, denoted by $T(X)$, to be the orthogonal complement of $\operatorname{Pic} X$ inside $H^2(X, \mathbb{Z})$.

2. Let X be a complex K3 surface with odd Picard number. Assume that the discriminant of its Picard lattice is not a power of 2. Then the automorphism group of X acts faithfully on $\operatorname{Pic} X$.

Let $\mathbb{P}$ denote the complex weighted projective space whose coordinates are x, y, z, w, with weights $1, 1, 1, 3$, respectively. Let $t \in \mathbb{C}$ be a complex number; we define the surface X_t to be the $K3$ surface in $\mathbb{P}$ given by the equation

$$X_t \colon w^2 = x^6 + y^6 + z^6 + tx^2y^2z^2.$$

3. The surface X_t has Picard number that is equal to either 19 or 20.

   If it equals 19, then the Picard lattice of X_t is an even lattice of rank 19, determinant $2^5\,3^3$, signature $(1, 18)$, and its discriminant group is isomorphic to $\mathbb{Z}/6\mathbb{Z} \times (\mathbb{Z}/12\mathbb{Z})^2$.

4. Assume that the Picard number of X_t equals 19. Then the transcendental lattice $T(X_t)$ is isometric to a sublattice of $U(3) \oplus A_2(4)$ of rank 3, signature $(2, 1)$, determinant $2^5\,3^3$, and its discriminant group is isomorphic to $\mathbb{Z}/6\mathbb{Z} \times (\mathbb{Z}/12\mathbb{Z})^2$.

5. Assume that t^3 is an element of the set $\left\{0, -\left(\frac{33}{2}\right)^3, -5^3\right\}$. Then X_t has Picard number 20.

6. There exists an elliptic curve E_t such that X_t is isogenous to the Kummer surface associated to $E_t \times E_t$.

7. Let L_1 and L_2 be the two lattices having Gram matrices equal to the matrices

$$\begin{pmatrix} 60 & 0 & 0 \\ 0 & -2 & 1 \\ 0 & 1 & -2 \end{pmatrix} \text{ and } \begin{pmatrix} 132 & 0 & 0 \\ 0 & -2 & 1 \\ 0 & 1 & -2 \end{pmatrix},$$

respectively. Then each of L_1 and L_2 has, up to isometries, exactly one integral overlattice of index 3; these are denoted by S_1 and S_2, respectively. The lattices S_1 and S_2 have Gram matrices equal to the matrices

$$\begin{pmatrix} 6 & 1 & -1 \\ 1 & -2 & 1 \\ -1 & 1 & -2 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 14 & 1 & -1 \\ 1 & -2 & 1 \\ -1 & 1 & -2 \end{pmatrix},$$

respectively. The lattices S_1 and S_2 complete a list, say Σ, due to work of Bogomolov and Tschinkel [1], and Nikulin [2], of six lattices such that the following statement holds: let X be a K3 surface defined over a number field, let $X_{\mathbb{C}}$ denote the base change of X to $\mathbb{C}$, and assume that $X_{\mathbb{C}}$ has Picard number equal to three; if the Picard lattice of $X_{\mathbb{C}}$ is not isometric to any of the lattices in Σ, then rational points on X are potentially dense.

8. Let X be a complex K3 surface and assume that its Picard lattice is isometric to the lattice having Gram matrix equal to

$$\begin{pmatrix} -2 & 0 \\ 0 & 4 \end{pmatrix}.$$

Then X admits exactly two automorphisms.

9. La filosofia è scritta in questo grandissimo libro che continuamente ci sta aperto innanzi a gli occhi (io dico l'universo), ma non si può intendere se prima non s'impara a intender la lingua, e conoscer i caratteri, ne' quali è scritto. Egli è scritto in lingua matematica, e i caratteri son triangoli, cerchi, ed altre figure geometriche, senza i quali mezi è impossibile a intenderne umanamente parola; senza questi è un aggirarsi vanamente per un oscuro laberinto. [G. Galilei, *Il Saggiatore,* 1623.]

[1] F. A. Bogomolov, Yu. Tschinkel, *Density of rational points on elliptic K3 surfaces*, 2000.
[2] V. V. Nikulin, *K3 surfaces with a finite group of automorphisms and a Picard group of rank three*, 1984.