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Title: The CM class number one problem for curves
Issue Date: 2016-07-05

STELLINGEN

behorende bij het proefschrift
The CM class number one problem for curves
van Pınar Kılıçer

1. There exist exactly 20 isomorphism classes of cyclic quartic CM fields and 63 isomorphism classes of non-normal quartic CM fields with CM class number one. The fields are listed in Theorems 2.4.5 and 2.3.15.
2. There are exactly 125 CM curves of genus 2, up to isomorphism over $\overline{\mathbb{Q}}$, defined over the reflex field with CM by a maximal order of some non-biquadratic quartic CM field. The curves are those of Bouyer and Streng [LMS J. Comput. Math., 18(1):507–538, 2015].
3. There are exactly 19 absolutely simple CM curves of genus 2 over $\mathbb{Q}$ with CM by a maximal order (Murabayashi – Umegaki).
4. There are exactly 21 absolutely simple CM curves of genus 2 over $\mathbb{Q}$, up to isomorphism over $\overline{\mathbb{Q}}$.
5. There exist exactly 37 isomorphism classes of cyclic sextic CM fields with CM class number one. The fields are listed in Table 3.1.
6. There are exactly 37 absolutely simple CM curves of genus 3 over $\mathbb{Q}$ with CM by a maximal order (Theorem 4.1.1).

Let K be a CM field of degree $2g$ and let F be its the maximal totally real subfield. Let Φ be a primitive CM type of K and let (K^r, Φ^r) be the reflex of (K, Φ). Let t_K denote the number of ramified primes in K/F. Let h_M and d_M denote the class number and the discriminant of a number field M, respectively.

7. If K is a CM class number one quartic CM field, then we have $h_K/h_F = 2^{t_K-1}$ (Proposition 2.3.1).
8. If K is a CM class number one quartic CM field and a rational prime p splits completely in $K^r/\mathbb{Q}$, then $p \geq \sqrt{d_K/d_F^2}/4$ (Lemma 2.3.17).
9. A sextic CM field K containing an imaginary quadratic field k has CM class number one if and only if $h_K/h_F = 2^{t_K-1}$ and $h_k = 1$ (Propositions 3.3.5 and 3.4.5).
10. It does not matter how slowly you go as long as you do not stop. – Confucius