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3.3. AGORA: AIRGUN SOURCE SIGNATURE MODEL

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Abstract: Seismic exploration has the potential to make a significant contribution to the soundscape of the North Sea. An airgun works by rapid release of air into water, forming a large bubble, which then pulsates, radiating sound as the bubble successively compresses and rarefies the surrounding water. An airgun array source signature model is described, following Gilmore’s equation of motion, incorporating liquid compressibility, mass diffusion, and thermal effects, and gas pressure laws. Predicted airgun signatures are compared with measurements. The proposed source model, coupled with a propagation model, can be used to generate anthropogenic and natural sound maps for. In this study we focus specifically on sound maps associated with seismic surveys in the Dutch North Sea.
3.3.1. INTRODUCTION

Sound maps can provide a useful insight about the distribution of sound over large regions. Model predictions offer a practical means to fill gaps left where measurements are unavailable [Mennitt et al, 2014]. Sound maps based on model predictions rely on the solution to two key problems: the reliable modelling of sound propagation and of source characteristics. For sound mapping, the solution of large scale broadband propagation problems is required. The modelling of source properties requires an additional effort by understanding its working principles and supporting the source level estimations by measurements. The sound generated by airguns can be estimated by the solution of bubble motion problems by including the effect of liquid compressibility, mass diffusion, thermal effects and momentum. This results in a set of differential equations from various branches of physics. In this section, the calculation of airgun array signatures is investigated. A simulation tool (AGORA) is developed. The calculated source signatures can be used to generate seismic survey sound maps for anthropogenic and natural sources in the Dutch North Sea. The model is an implementation of Ziolkowski’s approach [Ziolkowski, 1970; Ziolkowski et al, 1982] including mass and heat transfer as described by [Laws et al, 1990] and [MacGillivray, 2006]. It solves a differential equation system iteratively. First, the bubble radius, bubble wall velocity, temperature and mass of the bubble are calculated. These quantities are then used for the estimation of the radiated pressure from the airguns. The signature of airgun array can be calculated as a sum of contributions from each single airgun, with bubble interactions treated as a perturbation [Ziolkowski et al, 1982]. Results so obtained are compared with measurements made available by the E&P Sound and Marine Life JIP (henceforth abbreviated “JIP”). After the validation of the source model, the calculated source signatures can be used as an input for seismic sound.

3.3.2. CALCULATION OF PRESSURE FROM SINGLE AIRGUN

The equation of motion can be represented by different equations from various approaches. Gilmore’s equation which is based on the Kirkwood-Bethe approximation is investigated in this section [MacGillivray, 2006].
\[
\frac{du}{dt} = \frac{(1 + \frac{u}{c}) H + \frac{a}{c} (1 - \frac{u}{c}) \frac{dH}{dt} - \frac{3}{2} u^2 (1 - \frac{u}{c})}{a (1 - \frac{u}{c})}
\]

where \(a\) is the bubble radius, \(u = da/dt\) is the bubble wall velocity, \(c\) is the sound speed in the disturbed liquid, which can be calculated from the equation of state as \(c = c_\infty \left(\frac{p_a + B}{p_\infty + B}\right)^{(n-1)/2n}\)

Further, \(c_\infty\) is the sound speed in water, and \(B\) and \(n\) are experimental constants: \(n = 7\) and \(B = 304\) MPa for water. The parameter \(H\) is the specific enthalpy at the bubble wall, which can be calculated similarly by using the equation of state [Ziolkowski,1970] as \(H = \frac{p_a}{\rho_\infty} - \frac{n(p_\infty + B)}{(n-1)p_\infty} \left(\frac{p_a + B}{p_\infty + B}\right)^{(n-1)/n} - 1\), where \(p_\infty\) is the undisturbed hydrostatic pressure \(p_\infty = p_{\text{atm}} + \rho_\infty g z_{\text{gun}}\), \(g\) is the acceleration due to gravity, and \(z_{\text{gun}}\) is the depth of airgun. The pressure \(p_a\) is that at the bubble wall, which can be estimated using the polytropic relation, ignoring the surface tension and the liquid viscosity, i.e., \(p_a = p_{\text{atm}} \left(\frac{a_0}{a}\right)^{3\gamma}\), where \(\gamma\) is the experimentally determined polytropic index. However, the calculation of the pressure at bubble wall by a polytropic relation has several limitations for nonlinear oscillations [Prosperetti,1984; Prosperetti et al, 1988]. Instead of using the polytropic relation, \(p_a\) can be written as \(p_a = \frac{3 m_b R_G T_b}{4 \pi a^2} - 2 \sigma - 4 \mu \frac{u}{a}\) where \(\sigma\) is the surface tension and \(\mu\) is the liquid’s shear viscosity, \(R_G\) is the specific gas constant, \(T_b\) is the temperature of the air in the gas bubble, \(m_b\) is the mass of gas contained in the bubble and \(V_b\) is the volume of the gas bubble. The time derivative of \(H\) can be written

\[
\frac{dH}{dt} = \frac{1}{\rho} \left( R_G \left( T_b \frac{dV_b}{dt} + m_b \frac{dT_b}{dt} \right) - m_b T_b \frac{dV_b}{dt} \right) + \frac{2 \sigma}{a^2} - 4 \mu \left( a \frac{u' - u}{a^2} \right)
\]

where \(\frac{dV_b}{dt}\), \(\frac{dv_b}{dt}\) and \(\frac{dR_G}{dt}\) should be found to include mass and heat transfer [Laws et al,1990; MacGillivray,2006]. The derivative of volume can be simply written as \(\frac{dv_b}{dt} = 4 \pi u a^2\). The efficiency of the airgun can be estimated by an empirical parameter \((\eta)\), which characterises the remaining
air in the gun chamber after the airgun has fired [Li Guo-fa et al, 2010]. The maximum value of the mass in the bubble can be \( \eta m_{\text{gun}} \). The time derivative of mass can

\[
\frac{dm_b}{dt} = \left\{ \begin{array}{l}
\tau \sqrt{\frac{m_{\text{gun}} - m_b}{V_{\text{gun}}}} (p_{\text{gun}} - p_a), m_b \leq \eta m_{\text{gun}} \\
0,
\end{array} \right.
\]

where \( \tau = \tau_0 (V_{\text{gun}})^\beta \) is the port-throttling constant which is related to the airgun-port area (with dimension \([L^2]\)), \( \tau_0 \) is the volume independent port-throttling constant (dimension \([L^{2.35}]\)), \( \beta \) is a dimensionless power law exponent which is empirically determined from the experiments, and \( p_{\text{gun}}, m_{\text{gun}} \) and \( V_{\text{gun}} \) are the pressure, mass and volume of remaining air in the gun chamber. The time derivative of temperature

\[
\frac{dT_b}{dt} = \frac{R_\alpha T_b \frac{dm_b}{dt} - \frac{dQ}{dt} - p_a \frac{dV_b}{dt}}{m_b c_v}
\]

where \( \frac{dQ}{dt} \) is the rate of heat transfer into the air bubble [Laws et al, 1990; MacGillivray, 2006]. This rate can be written as \( \frac{dQ}{dt} = \kappa (4\pi a^2 \Delta T) \) where \( \Delta T = T_b - T_w \) is the temperature difference between the bubble and surrounding water. The parameter \( \kappa \) is also an experimentally determined constant. The buoyancy of the air bubbles changes the hydrostatic pressure because of the rise of the sea surface. This affects the bubble period [MacGillivray, 2006] and can be described by \( \frac{dV_{\text{gun}}}{dt} = \frac{2p_c}{a_0^3} \int_0^t a^3 dt \). The ideal gas law is not valid when the pressure in the gun chamber very high. For this situation, a modified form of temperature can be used [Laws et al, 1990] to describe an effective temperature in the air-gun chamber, \( T_{\text{gun}'} = T_{\text{gun}} \left( 1 + \frac{p_{\text{gun}}}{p_c} \right) \), where \( p_c = 139 \text{ MPa} \) for air. The relevant equations for the bubble motion, mass and heat transfer can be written as a differential equation system, which can be solved by iterative methods [MacGillivray, 2006; Li Guo-Fa et al, 2010] with the initial conditions: \( m_0 = \frac{p_\omega V_{\text{gun}}}{R_\alpha T_{\text{water}}}, T_0 = T_{\text{gun}} \), \( a_0 = \left( \frac{3V_{\text{gun}}}{4\pi} \right)^{1/3} \) and \( u_0 = 0 \). The optimized empirical parameters from [MacGillivray, 2006] are used for the mass and heat transfer.
coefficients ($\beta = 0.52, \eta = 0.8317, \kappa = 22230 \text{ J m}^{-2} \text{s}$ and $\tau_0 = 0.5355 \text{ m}^{2-3\beta} = 0.5355 \text{ m}^{0.44}$.) These parameters should be selected carefully to increase the accuracy of the model results depending on the environment and airgun properties. Then, the radiated pressure can be expressed as a function of enthalpy, bubble wall velocity and bubble radius [Laws et al, 1990] as $s_0(t) = \lim_{R_0 \to \infty} R_0 (p_a - p_\infty) = p_\infty a \left( H + \frac{u^2}{2} \right)$. Where $R_0$ is the distance from the bubble centre to the far field point. The equation gives the pressure in the free field and is called the “notional signature”, without the surface reflection, or “ghost” [Ziolkowski, 1970]. The reflections from sea surface and seabed can be added from propagation theories separately. The sound pressure, including the contribution from the surface ghost, is calculated using image theory as

$$\begin{align*}
p_{\text{airgun}}(r, t) &= \frac{a \rho_\infty \left( H(r, t) + \frac{u^2(r, t)}{2} \right)}{D_1} + \frac{a \rho_\infty \left( H(r, t + t_0) + \frac{u^2(r, t + t_0)}{2} \right)}{D_2}
\end{align*}$$

where $R_S = -1$ is the sea surface reflection coefficient, $D_1$ is the distance between gun and receiver, $D_2$ is the distance between the surface image and receiver as shown by Fig.1, and $t_0$ is time delay, calculated as $t_0 = \frac{D_2 - D_1}{c_\infty}$. The time domain source signature, $s(t; \varphi)$, is calculated as $s(t; \varphi) = \lim_{R_0 \to \infty} R_0 p_{\text{airgun}}(R_0, t; \varphi)$ where $\varphi$ is the dip angle. For a single airgun, the source signature only varies with dip ($\varphi$) angle and does not vary with the azimuth angle ($\psi$). However, the airgun array pattern may also vary with azimuth angle depending on the array geometry. The mean-square sound pressure spectral density level is

$$L_f = 10 \log_{10} \left( \frac{2 \left| P_{\text{airgun}}(r, f) \right|^2}{1 \mu\text{Pa}^2\text{s} \text{Hz}} \right) \text{ dB}$$

and energy source spectral density level is
\[ L_S = 10 \log_{10} \left( \frac{2 | S(f, \phi)|^2}{1 \mu \text{Pa}^2 \text{m}^2 \text{s} \text{Hz}} \right) \text{ dB} \]

where \( p_{\text{airgun}}(r, f) = \int_{-\infty}^{\infty} p_{\text{airgun}}(r, t) \exp(-i2\pi ft) \, df \) is the Fourier transform of the sound pressure and \( S(f, \psi) = \int_{-\infty}^{\infty} s(t, \psi) \exp(-i2\pi ft) \, df \) is the frequency domain source signature.

![Figure 1. The contribution of surface ghost](image)

### 3.3.3. SOURCE SIGNATURE OF AIRGUN ARRAYS

During a seismic survey, multiple airguns are used in an array to amplify the sound. Airguns arrays have horizontal and vertical directivity patterns. The effect of interference with the ghost is to direct the source energy more towards the seabed than in the horizontal direction. Before modelling the directivity, the interaction between the different airguns should also be taking into account. This can be done by adding a time dependent perturbation term to the source signature of each individual airgun [Ziolkowski et al., 1982; MacGillivray, 2006]. Hence, the bubble interactions can be estimated by an effective hydrostatic pressure for the \( m \)th airgun as

\[ p_{m}^{\text{eff}}(t) = p_{\infty} + \sum_{k \neq m} \Delta p_{mk}(t) \]
where $\Delta p_{mk}(t) = \frac{\rho a_k(t')}{D_{mk}} \left( H_k(t') + \frac{u_k(t')^2}{2} \right)$ and $t'$ is the retarded time. This equation affects the enthalpy according to Tail’s equation. Thus, the bubble motion characteristic will be changed. In the next steps of derivation, perturbed airgun signatures are used for the calculation of directivity.

For each individual airgun signature, the time delays between airgun location and centre point of array (described as origin point $x = 0, y = 0, z = 0$) are calculated. The distances between the gun and the ghosts are $D_{m1} = |\hat{r} \cdot \hat{r}_{\text{gun}}| = x_m \cos \psi \cos \theta + y_m \sin \psi \cos \theta + z_m \sin \theta$ and $D_{2m} = |\hat{r} \cdot \hat{r}_{\text{ghost}}| = x_m - N \cos \psi \cos \theta + y_m - N \sin \psi \cos \theta - z_m - N \sin \theta$, where $\hat{r} = (\cos \psi \cos \theta, \sin \psi \cos \theta, \sin \theta)$ is a unit vector, and $\psi$ and $\theta$ are the azimuth and grazing angles ($\varphi = \frac{\pi}{2} - \theta$). The positions of airguns and their surface images are $\hat{r}_{\text{gun}} = (x_m, y_m, z_m)$ and $\hat{r}_{\text{ghost}} = (x_m, y_m, -z_m)$. The time delays are summed to obtain the frequency domain pressure with 3D directivity as [Duren, 1988]

$$P(r, f, \psi, \theta) = \sum_{m=1}^{N} S_{0m}(f) \left[ \frac{\exp(ikwD_{1m})}{D_{1m}} - \frac{\exp(ikwD_{2m})}{D_{2m}} \right]$$

where $N$ is the number of airguns in the airgun array, and $S_{0m}(f)$ is the frequency domain notional source signature of the $m$th airgun. The exponential phase terms represent the time
delays in the horizontal plane. In Figure 2, the vertical variation of energy source spectral density level is shown at different dip angles.

### 3.3.4. COMPARISONS

Some comparisons are done for JIP measurements for Svein Vaage broadband airgun study measurements. One selected as representative of the 30 available airgun shots is used in these comparisons. The positions of receivers and airgun are shown in Figure 3. Three receiver locations are (0 m, 0 m, 30 m), (0 m, 0 m, 100 m) and (10.8 m, 9.8 m, 15 m). The airgun location is nominally (0 m, 0 m, 6 m). In the original measurement set-up, there are many receiver points. However, these three positions are chosen for the comparisons. The model is sensitive to small changes in the source depth. To estimate the location of the source depth during the shot, the nulls in Lloyd mirror pattern are used as 

$$z_s \approx \frac{2 N_{\text{tocks}} d^4}{4} = 6.36 \text{ m}.$$

![Figure 3. The geometry of the measurements](image)

In Figure 4, the comparisons between AGORA and JIP measurements are shown for $V_{\text{gun}} = 1.31 \text{ L (80 in}^3)\text{ and firing pressure 13.8 MPa (2000 psi)}$. 

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Figure 4. Comparisons of sound pressure (left panels) and mean-square sound pressure spectral density level (right panels) ($L_f$) of 1.31 L (80 in$^3$) airgun at different distances (the distances between airgun and receiver location is 16.9 m, 23.6 m and 93.6 m) and elevations. The source depth is 6.36 m and the firing pressure is 13.8 MPa (2000 psi).

On the other hand, the source signatures can be calculated from different long ranges. $R_0$ denotes the distance between dipole centre (0 m, 0 m, 0 m) and measurement location. For these calculations, the measurements at (0 m, 0 m, 100 m) and (10.8 m, 9.8 m, 15 m) coordinates are used. The first measurement is at 100 m beneath of the airgun with 0 degree dip angle. The distance of second measurement at $R_0=20.9$ m with 54.4 degree dip angle. The measurement and calculated results are shown in Figure 5.
Figure 5. The comparisons for the source signature and energy source spectral density level ($L_s$).

The measured and calculated source signatures for $R_0=20.92$ m (dip angle is 54.4 degree) and $R_0=100$ m (dip angle is 0 degree)

The empirical calibration coefficients of mass and heat transfer are not optimized for this dataset. Thus, better agreement may be obtained by optimizing these parameters for JIP dataset. AGORA assumes that the surface is flat. Realistically, the sea surface is not flat, resulting in rough surface scattering. All these uncertainties can lead to differences between the model and measurement results. By using calculated signatures the seismic survey maps are generated for the North Sea in Figure 6. Propagation loss is calculated by a hybrid method based on normal modes and flux theory [described in Chapters 2.2. and 2.3]

Figure 6. Annually averaged seismic survey sound maps of the North Sea for 2007. The receiver depth is 1 m. Frequency range for the broadband map is 30 Hz to 3 kHz. The green lines show the Dutch coastline and the Dutch EEZ outline.
3.3.5. CONCLUSIONS

Calculation of airgun source signatures and their comparisons with E&P Sound and Marine Life JIP measurements are shown in this section. A Matlab script (AGORA) is developed to solve the set of differential equations for the bubble motion and visualize airgun array signatures. Good agreement is obtained between the JIP measurements and the airgun model described in the present section. The choice of exact source and receiver locations, environmental parameters, mass and heat transfer coefficients can affect the accuracy of the calculated results. Thus, these parameters should be selected carefully. After these validation tests, the calculated source energy spectral densities can be used for the calculation of seismic survey sound maps.