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Multifield inflation and the adiabatic condition

Single field inflationary models might seem very appealing because of their theoretical minimality and their ability to fit the cosmological data. There are, however, good reasons to think that this may not be a very plausible description of the energy content of the Universe at the energy scale of inflation. Indeed, all of the completions of the Standard Model which accomplish, for example, unification of the gauge coupling constants, are populated by several new degrees of freedom at higher energies. The presence of many fields during inflation may imply a variety of observable effects departing from those predicted in standard single-field slow-roll inflation, including features in the power spectrum of primordial inhomogeneities [44–56], large primordial non-gaussianities [43, 57–62] and isocurvature perturbations [63–71]. A detection of any of these signatures would therefore represent an extremely significant step towards elucidating the fundamental nature of physics taking place during the very early universe. Likewise, the non-detection of any special feature also poses interesting challenges, in particular how to explain the symmetries protecting the smooth background in a more fundamental theory of inflation.

If inflation is well described by a single degree of freedom it does not mean that the Universe *has* one degree of freedom at those energy scales. The decoupling of different degrees of freedom is a well known phenomenon which is of course not only restricted to inflation but applies to generic physical situations (for a review see e.g. [72]). It could be further argued that this is a necessary condition for the construction of any physical theory, otherwise Planck scale physics would be necessary for describing low-energy phenomena. How different degrees of freedom decouple at different energy scales is the subject of Effective Field Theories (EFT). EFT may then be the clue to reconcile the predictions of single field inflation, well in accordance with the data, with the unlikeliness of having just one degree of freedom at the energy scale of inflation. For example, a straightforward solution to this apparent paradox is to assume a hierarchy between the energy scale of inflation H (driven by a field with mass $m \ll H$), and all the additional degrees of freedom, such that their mass $M \gg H$[1]. After these heavy degrees of freedom are integrated out, one expects a low energy EFT in which UV-physics are parametrized by non trivial operators suppressed by factors of order H^2/M^2. The resulting low energy EFT is therefore expected to offer negligible departures from a truncated version of the same theory -single field inflation- wherein heavy fields are simply disregarded from the very beginning.

While this is a perfectly consistent scenario, there are certain situations in which the heavy degrees of freedom may leave big imprints in the low energy EFT. In the particular case of inflation, large couplings between light and heavy fields have been shown to substantially modify the properties of the low energy curvature perturbations [55, 73–81]. Would nature be so kind as to be in a such a state, the observation of its effects would offer a unique opportunity for the characterization of fields much heavier than the Hubble scale. This exciting possibility demands a thorough understanding both of the dynamics of decoupling and of data analysis.

In this chapter we will show, through a detailed study of multifield inflation, that the decoupling of different energy scales does allow for observing the effects of an additional heavy field within an effective low energy theory. After explicitly constructing a low energy EFT in the inflationary time-dependent background, we will show the restrictions that apply on the strength and time variation of the couplings of the low energy -single field- operators such that they can be interpreted in terms of the UV -multi field- theory.

The content of this chapter is based on two papers:

- *"Heavy fields, reduced speeds of sound and decoupling during inflation"*, A. Achucarro, V. Atal, S. Cespedes, J. O. Gong, G. A. Palma and S. P. Patil, Phys. Rev. D **86** (2012) 121301 [arXiv:1205.0710 [hep-th]].
- *"On the importance of heavy fields during inflation"*, S. Cespedes, V. Atal and G. A. Palma, JCAP **1205** (2012) 008 [arXiv:1201.4848 [hep-th]].

[1] Whether this is easy or difficult to achieve in specific theories is another fundamental question.

2.1 Multifield inflation

We start by defining the basics of multifield inflation. We follow the formalism developed in [55, 67, 68]. Our starting point is the action for a set of multiple scalar fields ϕ^a ($a = 1, ..., N$, with N the total number of fields) minimally coupled to gravity

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}g^{\mu\nu}\gamma_{ab}\partial_\mu\phi^a\partial_\nu\phi^b - V(\phi)\right] , \tag{2.1}$$

where R is the Ricci scalar of the spacetime metric $g_{\mu\nu}$, γ_{ab} is the sigma model metric of the space spanned by ϕ^a and V is the scalar potential. The equation of motion for ϕ^a can be written as

$$\Box\phi^a + \Gamma^a_{bc}\partial_\mu\phi^b\partial^\mu\phi^c - V^a = 0 , \tag{2.2}$$

where $\Box = \nabla_\mu\nabla^\mu$, $V^a \equiv \gamma^{ab}V_b$ and $V_b = \partial_b V$. The Christoffel symbols Γ^a_{bc} are associated with the field space metric γ^{ab}, are are given by

$$\Gamma^a_{bc} = \gamma^{ad}(\partial_b\gamma_{dc} + \partial_c\gamma_{bd} - \partial_d\gamma_{bc})/2 . \tag{2.3}$$

We can furthermore construct a Ricci tensor and Ricci scalar associated to γ_{ab}, such that $\mathbb{R} = \gamma^{ab}\mathbb{R}^c_{acb}$ and $\mathbb{R}^c_{acb}$ is the Riemann tensor given by:

$$\mathbb{R}^c_{acb} = \partial_c\Gamma^a_{bd} - \partial_d\Gamma^a_{bc} + \Gamma^a_{ce}\Gamma^e_{db} - \Gamma^a_{de}\Gamma^e_{cb} . \tag{2.4}$$

We now study the cosmological solutions of this system.

2.1.1 Homogeneous and isotropic backgrounds

Let us first study the equations for the homogeneous and isotropic cosmological background, characterized by a scalar field solution $\phi^a = \phi^a_0(t)$ only dependent on time. For this we consider a flat FLRW metric of the form

$$ds^2 = -dt^2 + a(t)^2\delta_{ij}dx_i dx_j , \tag{2.5}$$

where $a(t)$ is the scale factor describing the expansion of flat spatial foliations. Then, the equations of motion determining the evolution of the system of fields are given by

$$D_t\dot{\phi}^a_0 + 3H\dot{\phi}^a_0 + V^a = 0 , \tag{2.6}$$

$$3H^2 = \dot{\phi}^2_0/2 + V , \tag{2.7}$$

where $H = \dot{a}/a$ is the Hubble expansion and we have introduced the covariant time derivative

$$D_t X^a = \dot{X}^a + \Gamma^a_{bc}\dot{\phi}^b_0 X^c \ . \tag{2.8}$$

We are further using the following definition for the total kinetic energy

$$\dot{\phi}^2_0 \equiv \gamma_{ab}\dot{\phi}^a_0\dot{\phi}^b_0 \ , \tag{2.9}$$

from which we can write the following relation

$$\dot{H} = -\dot{\phi}^2_0/2 \ . \tag{2.10}$$

The equations of motion written in the field basis, as in (2.6), are not particularly useful in gaining intuition on how the system is evolving. First of all, we do not know how the kinetic energy is distributed among the different fields. As the curvature perturbations are associated with time translations of the background trajectory (see e.g. [81]), we cannot easily identify to which linear combination of the fields perturbations $\delta\phi^a$ they correspond. Additionally, if the mass matrix is non-diagonal in the field basis, we cannot know whether a hierarchy of masses, which would greatly simplify the system, exists or not.

In order to cure the first problem, we can project the equations of motion into the *kinematic* basis, which is defined as the projection of the equations of motion into the direction tangential and normal to the trajectory[2]. For this, we define orthogonal unit vectors T^a and N^a in such a way that, at a given time t, $T^a(t)$ is tangent to the path, and $N^a(t)$ is normal to it. In this thesis we will consider two-field models, such that there is only one vector normal to the trajectory (see [83] for a generalization to more fields). In this particular case, this set of vectors can be defined as:

$$T^a = \dot{\phi}^a_0/\dot{\phi}_0 \ , \tag{2.11}$$

$$N^a = \gamma^{ab}N_b \qquad \text{with} \qquad N_a = \sqrt{\det\gamma}\,\epsilon_{ab}T^b, \tag{2.12}$$

where ϵ_{ab} is the Levi-Civita symbol with $\epsilon_{11} = \epsilon_{22} = 0$ and $\epsilon_{12} = -\epsilon_{21} = 1$. These definitions ensure that $T_aT^a = 1$, $N_aN^a = 1$ and $T_aN^a = 0$. The decomposition is shown in figure 2.1. Projecting the background equation of motion (2.6) along T^a yields

$$\ddot{\phi}_0 + 3H\dot{\phi}_0 + V_T = 0, \tag{2.13}$$

[2]An alternative decomposition which better accommodates the notion of light and heavy fields is the *mass* basis. In this basis the second covariant derivative of the potential is diagonal through all the trajectory [82]. There is not however a single basis vector associated to the curvature perturbations.

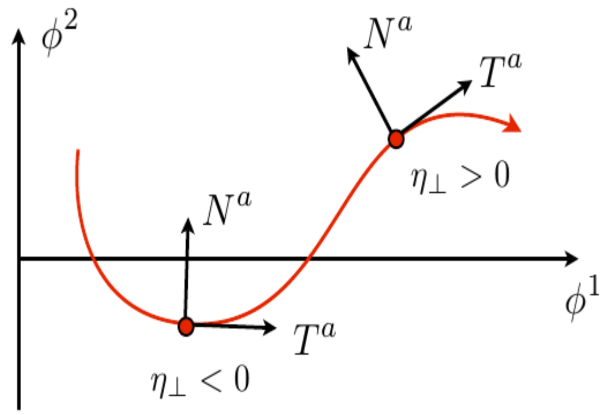

FIGURE 2.1: The decomposition of the trajectory in the kinematic basis, defined by the vectors N_a and T_a, normal and tangential to the trajectory respectively.

where $V_T \equiv T^a V_a$. On the other hand, projecting along N^a, one obtains

$$D_t T^a = -\frac{V_N}{\dot{\phi}_0} N^a, \tag{2.14}$$

where $V_N \equiv N^a \partial_a V$. Just as in single-field inflation, we may define the slow-roll parameters accounting for the time variation of various background quantities:

$$\epsilon \equiv -\dot{H}/H^2 \quad , \quad \eta^a \equiv -\frac{1}{H\dot{\phi}_0}\frac{D\dot{\phi}_0^a}{dt} \ . \tag{2.15}$$

Notice that η^a is a two dimensional vector field telling us how fast $\dot{\phi}_0^a$ is changing in time. We may decompose η^a along the normal and tangent directions by introducing two independent parameters $\eta_{||}$ and $\eta_\perp$ as

$$\eta^a = \eta_{||} T^a + \eta_\perp N^a \ . \tag{2.16}$$

Then, one finds that

$$\eta_{||} = -\frac{\ddot{\phi}_0}{H\dot{\phi}_0} \ , \tag{2.17}$$

$$\eta_\perp = -\frac{V_N}{H\dot{\phi}_0} \ . \tag{2.18}$$

Notice that $\eta_{||}$ may be recognized as the usual η slow-roll parameter in single field inflation (η_ϕ in eq. (1.19)). On the other hand $\eta_\perp$ tells us how fast T^a rotates in time, and therefore it parametrizes the rate of turning of the trajectory followed by the scalar field dynamics. This may be seen more clearly by using (2.14) together with (2.18) to deduce the following relations

$$\frac{DT^a}{dt} = -H\eta_\perp N^a \ , \tag{2.19}$$

$$\frac{DN^a}{dt} = +H\eta_\perp T^a. \tag{2.20}$$

Thus, if $\eta_\perp = 0$, the vectors T^a and N^a remain constant along the path. On the other hand, if $\eta_\perp > 0$, the path turns to the left, whereas if $\eta_\perp < 0$ the turn is towards the right. The value of $\eta_\perp$ is therefore telling us how quickly the angle determining the orientation of T^a is varying in time. By calling this angle θ we may therefore make the identification

$$\dot{\theta} \equiv H\eta_\perp \ . \tag{2.21}$$

With the help of this definition, one deduces that the radius of curvature κ characterizing the turning trajectory, is given by

$$\kappa^{-1} \equiv |\dot{\theta}|/\dot{\phi}_0. \tag{2.22}$$

As in conventional single-field inflation, the background dynamics may be understood in terms of the values of the dimensionless parameters ϵ, $\eta_{||}$ and $\eta_\perp$. For instance, slow-roll inflation will happen as long as:

$$\epsilon \ll 1 \quad , \quad |\eta_{||}| \ll 1. \tag{2.23}$$

These two conditions ensure that both H and $\dot{\phi}_0$ evolve slowly. On the other hand, a large value of $\eta_\perp$ does not necessarily imply a violation of the slow-roll regime (2.23). As we will see later, the regime of large $\eta_\perp$ will offer the most interesting phenomenology.

2.1.2 Perturbations

We now consider the dynamics of scalar perturbations $\delta\phi^a(t,\boldsymbol{x}) = \phi^a(t,\boldsymbol{x}) - \phi_0^a(t)$. It is convenient to work with the gauge invariant quantities v^T and v^N given by:

$$v^T = aT_a\delta\phi^a + a\frac{\dot{\phi}}{H}\psi \ , \tag{2.24}$$

$$v^N = aN_a\delta\phi^a \ , \tag{2.25}$$

where ψ is the scalar perturbation of the spatial part of the metric (proportional to δ_{ij}) in flat gauge. It is useful to consider a second set of fields (u^X,u^Y) in addition to

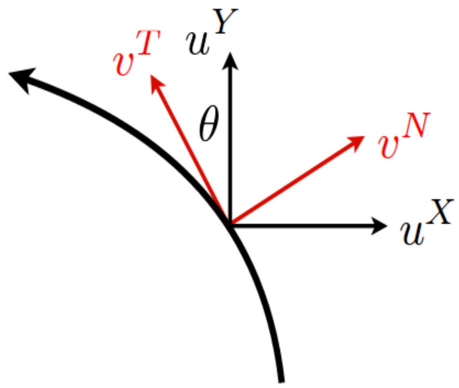

FIGURE 2.2: The u fields represent fluctuations with respect to a fixed local frame, whereas the v-fields repreent fluctuations with respect to the path (parallel and normal).

(v^T, v^N). Let us consider the following time dependent rotation in field space

$$\begin{pmatrix} u^X \\ u^Y \end{pmatrix} \equiv R(\tau) \begin{pmatrix} v^N \\ v^T \end{pmatrix} , \tag{2.26}$$

where the time dependent rotation matrix $R(\tau)$ is defined as

$$R(\tau) = \begin{pmatrix} \cos\theta(\tau) & -\sin\theta(\tau) \\ \sin\theta(\tau) & \cos\theta(\tau) \end{pmatrix} , \quad \theta(\tau) = \theta_0 + \int_{-\infty}^{\tau} d\tau a H \eta_\perp , \tag{2.27}$$

where θ_0 is the value of $\theta(\tau)$ at $\tau \to -\infty$. Thus, we assume that the trajectory is straight in the far past, which will help us later in properly quantizing the system. The rotation angle $\theta(\tau)$ precisely accounts for the total angle covered by all the turns during the inflationary history up to time τ, and coincides with the definition introduced in eq. (2.21). Figure (2.2) illustrates the relation between the v-fields introduced earlier and the canonical u-fields. To continue, the equations of motion for the canonically normalized fields are

$$\frac{d^2 u^I}{d\tau^2} - \nabla^2 u^I + \left[R(\tau)\,\Omega R^t(\tau)\right]^I_J u^J = 0, \quad I = X, Y, \tag{2.28}$$

where R^t represents the transpose of R. In addition, Ω is the mass matrix for the v-fields, whose entries are

$$\Omega_{TT} = -a^2 H^2 \left(2 + 2\epsilon - 3\eta_\parallel + \eta_\parallel \xi_\parallel - 4\epsilon\eta_\parallel + 2\epsilon^2 - \eta_\perp^2\right) , \tag{2.29}$$

$$\Omega_{NN} = -a^2 H^2 (2 + \epsilon) + a^2 \left(V_{NN} + H^2 \epsilon \mathbb{R}\right) , \tag{2.30}$$

$$\Omega_{TN} = -a^2 H^2 \eta_\perp \left(3 + \epsilon - 2\eta_\parallel - \xi_\perp\right) , \tag{2.31}$$

where $\xi_{\|} = -\dot{\eta}_{\|}/(H\eta_{\|})$ and $\xi_{\perp} = -\dot{\eta}_{\perp}/(H\eta_{\perp})$. Additionally, V_{NN} is the tree level mass, defined as the second derivative of the potential projected along the perpendicular direction $V_{NN} = N^a N^b \nabla_a \nabla_b V$. To finish, expanding the original action (2.1) to quadratic order in terms of the u-fields, one finds:

$$S = \frac{1}{2}\int d\tau d^3x \left[\sum_I \left(\frac{du^I}{d\tau}\right)^2 - (\nabla u^I)^2 - \left[R(\tau)\,\Omega R^t(\tau)\right] u^I u^J\right] . \tag{2.32}$$

Thus, we see that the fields $u^I = (u^X, u^Y)$ correspond to the canonically normalized fields in the usual sense. Given that these fields are canonically normalized, it is now straightforward to impose Bunch-Davies conditions on the initial state of the perturbations.

2.1.3 Curvature and isocurvature modes

Another useful field parametrization for the perturbations is in terms of curvature and isocurvature fields $\mathcal{R}$ and $\mathcal{S}$ [66]. In terms of the v-fields, these are defined, to linear order, as

$$\mathcal{R} \equiv \frac{H}{a\dot{\phi}} v^T \qquad \text{and} \qquad \mathcal{S} \equiv \frac{H}{a\dot{\phi}} v^N . \tag{2.33}$$

Instead of working with $\mathcal{S}$, it is in fact more convenient to define [3]

$$\mathcal{F} \equiv \frac{\dot{\phi}}{H}\mathcal{S} . \tag{2.34}$$

The quadratic order action for these perturbations is

$$S_2 = \frac{1}{2}\int dt d^3x a^3 \left[\frac{\dot{\phi}_0^2}{H^2}\dot{\mathcal{R}}^2 - \frac{\dot{\phi}_0^2}{H^2}\frac{(\nabla\mathcal{R})^2}{a^2} + \dot{\mathcal{F}}^2 - \frac{(\nabla\mathcal{F})^2}{a^2} - M_{\text{eff}}^2\mathcal{F}^2 - 4\dot{\theta}\frac{\dot{\phi}_0}{H}\dot{\mathcal{R}}\mathcal{F}\right] . \tag{2.35}$$

Here M_{eff} is the effective mass of $\mathcal{F}$ given by

$$M_{\text{eff}}^2 = m^2 - \dot{\theta}^2 , \tag{2.36}$$

where $m^2 \equiv V_{NN} + \epsilon H^2 \mathbb{R}$. Notice that $\dot{\theta}$ couples both fields. In the case in which we the isocurvature mode $\mathcal{F}$ is heavy, this coupling reduces its effective mass, suggesting a breakdown of the hierarchy that would permit a single field effective description as $\dot{\theta}^2 \sim m^2$. As we are about to see, this expectation is somewhat premature. The linear

[3] A definition of $\mathcal{R}$ and $\mathcal{F}$ valid to all orders in perturbation theory is given in [81].

equations of motion in Fourier space are

$$\ddot{\mathcal{R}} + (3 + 2\epsilon - 2\eta_{||})H\dot{\mathcal{R}} + \frac{k^2}{a^2}\mathcal{R} = 2\dot{\theta}\frac{H}{\dot{\phi}_0}\left[\dot{\mathcal{F}} + \left(3 - \eta_{||} - \epsilon + \frac{\ddot{\theta}}{H\dot{\theta}}\right)H\mathcal{F}\right] , \tag{2.37}$$

$$\ddot{\mathcal{F}} + 3H\dot{\mathcal{F}} + \frac{k^2}{a^2}\mathcal{F} + M^2_{\text{eff}}\mathcal{F} = -2\dot{\theta}\frac{\dot{\phi}_0}{H}\dot{\mathcal{R}} . \tag{2.38}$$

Note that $\mathcal{R}$ = constant and $\mathcal{F} = 0$ are non-trivial solutions to these equations for arbitrary $\dot{\theta}$. When $\mathcal{F}$ is heavy, $\mathcal{F} \to 0$ shortly after horizon exit, and $\mathcal{R} \to$ constant.

2.1.4 Power spectrum

From the observational point of view, the main quantities of interest coming from inflation are its predicted n-point correlation functions characterizing fluctuations. These quantities provide all the relevant information about the expected distribution of primordial inhomogeneities that seeded the observed CMB anisotropies. It is of particular interest to compute two-point correlation functions, corresponding to the variance of inhomogeneities' distribution. To deduce such quantities we have to consider the quantization of the system, and this may be achieved by expanding the canonical pair u^X and u^Y in terms of creation and annihilation operators $a^\dagger_\alpha(k)$ and $a_\alpha(k)$ respectively, as

$$u^I(\mathbf{x}, \tau) = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \sum_\alpha \left[e^{i\mathbf{k}.\mathbf{x}} u^I_\alpha(\mathbf{k}, \tau) a_\alpha(\mathbf{k}) + e^{-i\mathbf{k}.\mathbf{x}} u^{I*}_\alpha(\mathbf{k}, \tau) a^\dagger_\alpha(\mathbf{k}) \right] , \tag{2.39}$$

where $\alpha = 1, 2$ labels the two modes to be encountered by solving the second order differential equations for the fields $u^I_\alpha(k, \tau)$. In order to satisfy the conventional field commutation relations, the mode solutions need to satisfy the additional constraints consistent with the equations of motion

$$\sum_\alpha \left(u^I_\alpha \frac{u^{J*}_\alpha}{d\tau} - u^{I*}_\alpha \frac{u^J_\alpha}{d\tau} \right) = i\delta^{IJ} . \tag{2.40}$$

By examining the action (2.32) one sees that in the short wavelength limit $k^2/a^2 \gg \Omega$, where Ω symbolizes both eigenvalues of the matrix Ω, the equation of motion for the u-fields reduce to

$$\frac{d^2u^I}{d\tau^2} - \nabla^2 u^I = 0, \quad I = X, Y, \tag{2.41}$$

In the UV limit, the mode equations are uncoupled, and they satisfy the simple harmonic oscillator equation. We can then choose the Bunch-Davies initial conditions for each one the fields, as

$$u^X_\alpha(k, \tau) = \frac{e^{-ik\tau}}{\sqrt{2k}}\delta^1_\alpha , \quad u^Y_\alpha(k, \tau) = \frac{e^{-ik\tau}}{\sqrt{2k}}\delta^2_\alpha , \tag{2.42}$$

valid in the limit $k\tau \gg 1$. Notice that here we have chosen to associate the initial state $\alpha = 1$ with the field direction X and $\alpha = 2$ with the field direction Y. This identification is in fact completely arbitrary and does not affect the computation of two-point correlation functions. In other words, we could modify the initial state (2.42) by considering an arbitrary (time independent) rotation on the right hand side, without changing the prediction of observables. Then, given the set of solutions u_α^X and u_α^Y, one finds that the two-point correlation function associated to curvature modes $\mathcal{R}$ is given by:

$$\mathcal{P}_{\mathcal{R}}(k,\tau) = \frac{k^3}{2\pi^2}\sum_\alpha \mathcal{R}_\alpha(k,\tau)\mathcal{R}^*_\alpha(k,\tau) \tag{2.43}$$

where $\mathcal{R}_\alpha$ is related to the pair u_α^X and u_α^Y by the field redefinitions described in the previous sections. When (2.43) is evaluated at the end of inflation, for wavelengths k much bigger than the horizon ($k/a \ll H$), it corresponds to the power spectrum of curvature modes. One may also define the two-point correlation function $P_{\mathcal{S}}(k,\tau)$ and the cross-correlation function $P_{\mathcal{RS}}(k,\tau)$ in analogous ways

$$\mathcal{P}_{\mathcal{S}}(k,\tau) = \frac{k^3}{2\pi^2}\sum_\alpha \mathcal{S}_\alpha(k,\tau)\mathcal{S}^*_\alpha(k,\tau) \tag{2.44}$$

$$\mathcal{P}_{\mathcal{RS}}(k,\tau) = \frac{k^3}{2\pi^2}\sum_\alpha \mathcal{R}_\alpha(k,\tau)\mathcal{S}^*_\alpha(k,\tau) + h.c. \tag{2.45}$$

In the next section we discuss very briefly the observational status regarding the presence of isocurvature perturbations in the CMB, and its implications.

2.1.5 The fate of isocurvature perturbations

The presence of isocurvature modes leads to very specific effects on the angular temperature power spectra. For example, an isocurvature component will oscillate with a different phase with respect to the adiabatic components of the primordial plasma [84], changing the location of the peaks in the temperature two-point function (see figure 1.2). Because the position of the acoustic peaks are measured very accurately, CMB data can put very stringent bounds on their presence. These constraints are usually stated in terms of the primordial isocurvature fraction, defined as

$$\beta_{iso}(k) = \frac{\mathcal{P}_{\mathcal{S}}(k)}{\mathcal{P}_{\mathcal{R}}(k) + \mathcal{P}_{\mathcal{S}}(k)} \ . \tag{2.46}$$

The Planck constraints for β_{iso} are given specifically for models in which the isocurvature component is attributed to one of the different elements of the photon-baryon plasma (CDM, neutrino density or neutrino velocity) and different correlations are assumed between the curvature and isocurvature components. For the simplest cases with a scale independent β_{iso}, a nominal bound can be taken to be $\beta_{iso} < 0.03$ [30], from which we can conclude that observations of the CMB highly disfavor the presence of isocurvature perturbations. The simplest way to satisfy this bound is by assuming a very heavy isocurvature field[4]. The power spectrum for a generic massive scalar field χ in a de Sitter expansion is given by [20]:

$$\mathcal{P}_\chi(k) \simeq \left(\frac{H}{2\pi}\right)^2 \left(\frac{H}{M}\right)^2 \left(\frac{k}{aH}\right)^3 . \tag{2.47}$$

where M is the mass of the field χ. The power spectrum for such a field goes as $(k/aH)^3$ for superhorizon modes $(k/aH) \to 0$. An isocurvature field described by such power spectrum rapidly decays after horizon crossing, and hence its presence will be consistent with the Planck isocurvature constraints.

This is not, however, a necessary condition for not measuring isocurvature perturbations, as the bound coming from Planck really means that *at the time of decoupling* the perturbations are measured to be adiabatic. From the time of inflation to the decoupling of the CMB there is ample time and a diversity of physical processes, and there are many ways in which isocurvature perturbations during inflation may decay so that we only happen to measure adiabatic fluctuations at the time of decoupling. This is how active multifield dynamics can be consistent with the lack of observable isocurvature perturbations in the CMB. In figure 2.3 we show, schematically, the possible decays of the isocurvature fluctuations. Which dynamical process is responsible for the decay of the isocurvature field depends heavily on the mass and the evolution of the mass of this field. Ignoring particle production and effects coming from very rapid time evolution of the background (which will call our attention later), we can distinguish three situations:

- The isocurvature field is heavy throughout all the trajectory.
- The isocurvature field is light throughout all the trajectory.
- The isocurvature field has a mass which varies along the trajectory

If the isocurvature field is heavy throughout all the trajectory, the isocurvature fluctuations will decay after horizon crossing, just as we showed in eq. (2.47). This ensures that heavy scalars do not influence the dynamics of the perturbations at superhorizon scales, in particular, that they do not spoil the conservation of curvature perturbations at long wavelengths. This is case 1 in figure 2.3. If the isocurvature field is light and interacts with the curvature field, the curvature perturbations will continue to evolve after horizon crossing. Now, the isocurvature perturbations might still decay if their

[4] As usual, heavy and light field are defined with respect to the Hubble parameter. A heavy field is a field with mass $M \gg H$, while a light field has a mass $M \ll H$.

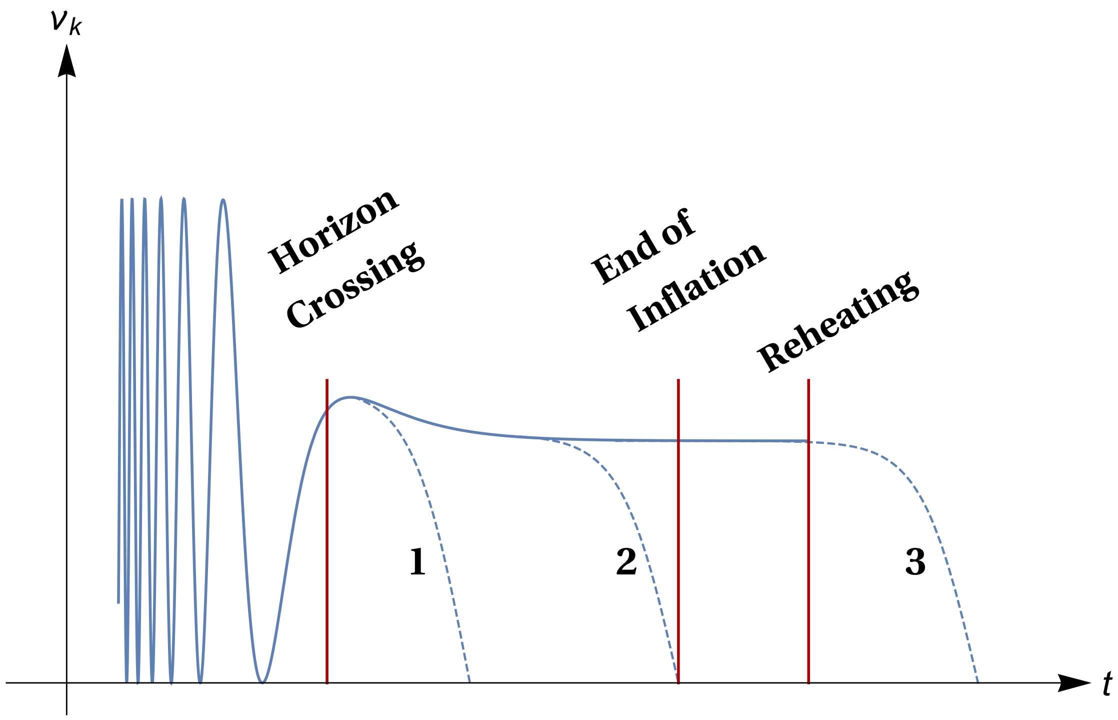

FIGURE 2.3: Schematic view, in arbitrary units, of the possible decays of an isocurvature mode v_k. An isocurvature mode may decay after it exits the horizon (if its very massive - case 1), between horizon exit and the end of inflation (if it becomes massive during this part of the background evolution - case 2), or after thermalization in the plasma era (if it has strong interactions with radiation - case 3). While very different in nature, all these three mechanisms will be consistent with no measurable isocurvature fluctuations in the CMB spectra.

mass becomes large after the mode has exited the horizon (see e.g. [85–88]). Curvature perturbations will now freeze *after* horizon crossing. This is case 2 in figure 2.3, and we will show in the last chapter of this thesis (Chapter 5) a relevant example in which this mechanism operates. The last possibility is to rely on the thermalization process after reheating: if the isocurvature field has not yet decayed during inflation, it can thermalize with the radiation bath and reach adiabaticity [89]. This is case 3 in figure 2.3. The exact thermalization process depends on the initial condition and the couplings of the different fluids, and is thus sensitive to the physics of reheating.

If heavy isocurvature modes rapidly decay, does this mean that heavy fields are unobservable in the CMB? As we have already stated in the introduction, fortunately not. The key to answering this question comes from realizing that a heavy field may have a light propagating mode. This is a natural consequence of considering the coupled nature of the equation of motion for the heavy and light field. As we will see explicitly in the next section, whenever the coupling between both fields is large, a heavy isocurvature field will also contribute to the low energy EFT. Considering that it is impossible to probe the energy scale of inflation directly, this scenario may be a unique way to access very high energy states observationally.

2.2 The effective field theory of turning trajectories

In this section we build the effective theory for the curvature perturbations in the case in which there is a strong turn in the inflationary trajectory supported by a heavy field (see figure 2.4 for an illustration). In this case unsuppressed interactions —kinematically coupling curvature perturbations with heavy fields— are unavoidably turned on. As a consequence, if the turning rate is large compared to the rate of expansion H, the impact of heavy physics on the low energy dynamics becomes substantially amplified, introducing large non-trivial departures from a naively truncated version of the theory.

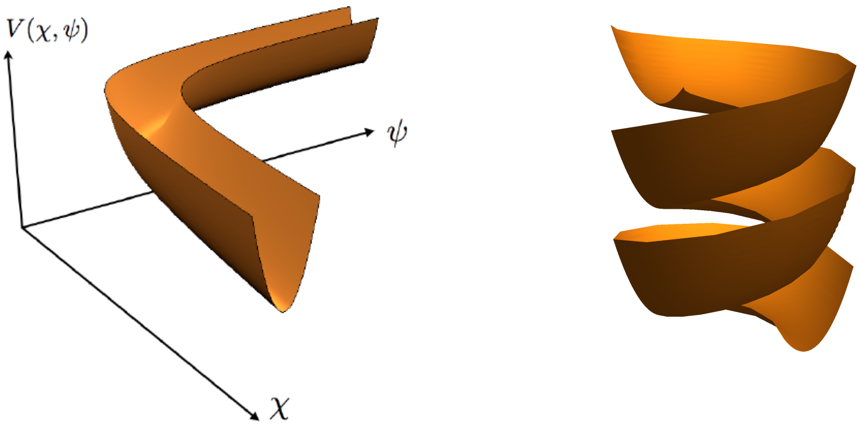

FIGURE 2.4: Two examples of strong turns in a two-field potential. They correspond to sharp (left panel) and soft (right panel) turns. In both cases the effect of the normal heavy field might be sizeable in the low energy effective theory.

It is possible to deduce an effective theory for the curvature mode $\mathcal{R}$ by integrating out the heavy field $\mathcal{F}$ when $M_{\text{eff}} \gg H^2$, provided that certain additional conditions are met. To see this, let us first briefly analyze the expected evolution of the fields $\mathcal{R}$ and $\mathcal{F}$ when the trajectory is turning at a constant rate ($\dot{\theta}=$ constant). To begin with, because we are dealing with a coupled system of equations for $\mathcal{R}$ and $\mathcal{F}$, in general we expect the general solutions for $\mathcal{R}$ and $\mathcal{F}$ to be of the form [76]

$$\mathcal{R} \sim \mathcal{R}_- e^{i\omega_- t} + \mathcal{R}_+ e^{i\omega_+ t} \,, \tag{2.48}$$

$$\mathcal{F} \sim \mathcal{F}_- e^{i\omega_- t} + \mathcal{R}_+ e^{i\omega_+ t} \,, \tag{2.49}$$

where ω_+ and ω_- denote the two frequencies at which the modes oscillate. The values of ω_- and ω_+ will depend on the mode's wave number k in the following way: In the regime $k/a \gg M_{\rm eff}$, both fields are massless and therefore oscillate with frequencies of order $\sim k/a$. As the wavelength enters the intermediate regime $M_{\rm eff} \gg k/a \gg H$ the degeneracy of the modes breaks down and the frequencies become of order (this estimations will be refined in the following section)

$$\omega_- \sim k/a \ , \qquad \omega_+ \sim M_{\rm eff} \ . \tag{2.50}$$

Subsequently, when the modes enter the regime $k/a < H$ the contributions coming from ω_+ will quickly decay and the contributions coming from ω_- will freeze (since they are massless). Notice that the amplitudes $\mathcal{R}_+$ and $\mathcal{F}_-$ necessarily arise from the couplings mixing curvature and isocurvature perturbations, and therefore they vanish in the case $\eta_\perp = \dot\theta/H = 0$. Additionally, on general grounds, the amplitudes $\mathcal{F}_+$ and $\mathcal{R}_+$ are expected to be parametrically suppressed by $k/M_{\rm eff}$ in the regime $M_{\rm eff} \gg k/a$, and therefore we may disregard high frequency contributions to (2.48) and (2.49). Then, in the regime $M_{\rm eff} \gg k/a$, time derivatives for $\mathcal{F}$ can be safely ignored in the equation of motion (2.38) and we may write (since $H \ll M_{\rm eff}$ we may also disregarded the friction term $3H\dot{\mathcal{F}}$):

$$\frac{k^2}{a^2}\mathcal{F} + M_{\rm eff}^2 \mathcal{F} = 2\dot\phi_0 \eta_\perp \dot{\mathcal{R}} \ . \tag{2.51}$$

This leads to an algebraic relation between $\mathcal{F}$ and $\mathcal{R}$ given by:

$$\mathcal{F}_{\mathcal{R}} = \frac{2\dot\phi_0 \eta_\perp \dot{\mathcal{R}}}{k^2/a^2 + M_{\rm eff}^2} \ , \tag{2.52}$$

which precisely tells us the dependence of low frequency contributions $\mathcal{F}_-$ in terms of $\mathcal{R}_-$ defined in eqs. (2.48) and (2.49). To continue, we notice that (2.51) is equivalent to disregarding the term $\dot{\mathcal{F}}^2$ of the kinetic term in the action (2.35). Keeping this in mind, we can replace (2.52) back into the action and obtain an effective action for the curvature perturbation given by

$$S_{\rm eff} = \frac{1}{2}\int dt d^3x \, a^3 \frac{\dot\phi_0^2}{H^2}\left[\frac{\dot{\mathcal{R}}^2}{c_s^2(k)} - \frac{k^2\mathcal{R}^2}{a^2}\right] \ , \tag{2.53}$$

where c_s is the speed of sound of adiabatic perturbations, given by:

$$c_s^{-2} = 1 + \frac{4H^2\eta_\perp}{k^2/a^2 + M_{\rm eff}^2} \ . \tag{2.54}$$

In deriving this expression we have assumed that $\dot\theta$ remained constant. In the more general case where $\dot{\theta}$ is time dependent we expect transients that could take the system away from the simple behavior shown in eqs. (2.48) and (2.49), and the effective field theory could become invalid. The validity of the effective theory will depend on whether the kinetic terms for $\mathcal{F}$ in eq. (2.38) can be ignored, and this implies the following condition on $\mathcal{F}_{\mathcal{R}}$ of eq. (2.52):

$$|\ddot{\mathcal{F}}_{\mathcal{R}}| \ll M_{\text{eff}}^2 |\mathcal{F}| \tag{2.55}$$

Now, recall that unless there are large time variations of background quantities, the frequency of $\mathcal{R}$ is of order $\omega_- \sim k/a$. Thus, any violation of condition (2.55) will be due to the evolution of background quantities, which will be posteriorly transmitted to $\mathcal{R}$. This will allows us to ignore higher derivatives of $\dot{\mathcal{R}}$ in (2.55) and simply rewrite it in terms of background quantities as:

$$\Big|\frac{d^2}{dt^2}\left(\frac{2\dot{\phi}_0\eta_\perp}{k^2/a^2 + M_{\text{eff}}^2}\right)\Big| \ll M_{\text{eff}}^2 \Big|\frac{2\dot{\phi}_0\eta_\perp}{k^2/a^2 + M_{\text{eff}}^2}\Big| \tag{2.56}$$

This relation expresses the adiabaticity condition that each mode k needs to satisfy in order for the effective field theory to stay reliable. To further simplify this relation, we may take into consideration the following points: (1) When $k^2/a^2 \gg M_{\text{eff}}^2$ the two modes decouple (recall eq. (2.41)) and the turn has no influence on the evolution of curvature modes. On the other hand, in the regime $k^2/a^2 \lesssim M_{\text{eff}}^2$ contributions coming from the time variation of k^2/a^2 are always suppressed compared to M_{eff}^2 due to the fact that we are assuming $H^2 \ll M_{\text{eff}}^2$. (2) We focus on situations in which the time derivatives of quantities such as $\dot{\phi}_0$ and H, which describe the evolution of the background along the trajectory, are suppressed in comparison to $\eta_\perp = \dot{\theta}/H$. This corresponds to neglecting changes in the slow-roll parameters ϵ and $\eta_\parallel$ [5]. (3) Because of this, the rate of change of M_{eff}^2 will necessarily be at most of the same order than $\dot{\theta}$. Then, by neglecting time derivatives coming from $\dot{\phi}_0$, H, k^2/a^2 and M_{eff}^2 and focussing on the order of magnitude of the various quantities appearing in (2.56) we can write instead a simpler expression given by:

$$\Big|\frac{d^2}{dt^2}\dot{\theta}\Big| \ll M_{\text{eff}}^2\Big|\dot{\theta}\Big| \tag{2.57}$$

Actually, a simpler alternative expression may be obtained by conveniently reducing the number of time derivatives, and disregarding effects coming from the change in sign of $\dot{\theta}$

$$\Big|\frac{d}{dt}\ln\dot{\theta}\Big| \ll M_{\text{eff}} \ . \tag{2.58}$$

[5]In general, a turn in field space will also induce changes in these parameters [90, 91]. Whether the time variation of $\eta_\perp$ or ϵ and $\eta_\parallel$ is dominant is a model dependent issue.

This adiabaticity condition simply states that the rate of change of the turn's angular velocity must stay suppressed with respect to the masses of heavy modes, which otherwise would become excited. Notice that, we may also choose to express this relation in terms of the variation of the speed of sound, which is a more natural quantity from the point of view of the effective field theory.

$$\left| \frac{d}{dt} \ln(c_s^{-2} - 1) \right| \ll M_{\rm eff} \ . \tag{2.59}$$

We have tested the adiabaticity condition in several situations in [92], where turns induced by the potential and turns induced by the metric were studied. The validity of the adiabatic condition was demonstrated to describe very accurately the threshold between the single field and the two-field regimes. There are however some open questions: if c_s is time independent, the adiabatic condition seems to be automatically satisfied. Does this mean that we can consistently describe, in the low energy regime, a system with an arbitrarily reduced speed of sound? Furthermore, the adiabatic condition given by equation (2.59) was derived rather heuristically. Indeed, there should be a more precise condition stated in terms of the time variation of the frequency of the heavy mode. In this sense, the estimation of the eigenfrequencies in (2.50) should be done more carefully. In the next sections we tackle these question by studying more precisely the UV cut-off of the theory in the case that $\dot{\theta}$ is constant, and by precisely determining the eigenfrequencies of the two-field system.

2.3 Discussion: EFT with $c_s \ll 1$.

The observation that heavy fields can influence the evolution of adiabatic modes during inflation [73] has far reaching phenomenological implications [55, 76, 81, 92] that requires a refinement of our understanding of how high and low energy degrees of freedom decouple and how one splits "heavy" and "light" modes on a time-dependent background. As we have showed in the previous section, provided that there is only one flat direction in the inflaton potential, heavy fields (in this discussion, field excitations orthogonal to the background trajectory) can be integrated out, resulting in a low energy effective field theory (EFT) for adiabatic modes exhibiting a reduced speed of sound c_s given by (2.54). In the $k \to 0$ limit, the speed of sound is given by

$$c_s^{-2} = 1 + \frac{4\dot{\theta}^2}{M_{\rm eff}^2} \,, \tag{2.60}$$

where $\dot{\theta}$ is the turning rate of the background trajectory in multi-field space, and $M_{\rm eff}$ is the effective mass of heavy fields, assumed to be much larger than the expansion rate H.Given that $M_{\rm eff}$ is the mass of the fields we integrate out, one might doubt the validity of the EFT in the regime where the speed of sound is suppressed [77, 78, 80, 93], as this requires $\dot\theta^2 \gg M_{\rm eff}^2$. In this section we elaborate on this issue by studying the dynamics of light and heavy degrees of freedom when $c_s^2 \ll 1$. To this end, we draw a distinction between isocurvature and curvature field excitations, and the true heavy and light excitations. We show that the light (curvature) mode $\mathcal{R}$ indeed stays coupled to the heavy (isocurvature) modes when strong turns take place ($\dot\theta^2 \gg M_{\rm eff}^2$), however, decoupling between the physical low and high energy degrees of freedom persists in such a way that the deduced EFT remains valid. This is confirmed by a simple setup in which H decreases adiabatically, allowing for a sufficiently long period of inflation. In this construction, *high energy degrees of freedom* are never excited, and yet *heavy fields* do play a role in lowering the speed of sound of adiabatic modes.

We are interested in (2.37) and (2.38) in the particular case where $\dot\theta$ is constant and much greater than $M_{\rm eff}$. We first consider the short wavelength limit where we can disregard Hubble friction terms and take $\dot\phi_0/H$ as a constant. In this regime, the physical wavenumber $p \equiv k/a$ may be taken to be constant, and (2.37) and (2.38) simplify to

$$\begin{aligned} \ddot{\mathcal{R}}_c + p^2\mathcal{R}_c &= +\,2\dot\theta\dot{\mathcal{F}}\,, \\ \ddot{\mathcal{F}} + p^2\mathcal{F} + M_{\rm eff}^2\mathcal{F} &= -\,2\dot\theta\dot{\mathcal{R}}_c\,, \end{aligned} \tag{2.61}$$

where we have defined $\mathcal{R}_c = \big(\dot\phi_0/H\big)\mathcal{R}$. The solutions to these equations are found to be [76]

$$\begin{aligned} \mathcal{R}_c &= \mathcal{R}_+ e^{i\omega_+ t} + \mathcal{R}_- e^{i\omega_- t}\,, \\ \mathcal{F} &= \mathcal{F}_+ e^{i\omega_+ t} + \mathcal{F}_- e^{i\omega_- t}\,. \end{aligned} \tag{2.62}$$

The two frequencies ω_- and ω_+ are precisely given by

$$\omega_\pm^2 = \frac{M_{\rm eff}^2}{2c_s^2} + p^2 \pm \frac{M_{\rm eff}^2}{2c_s^2}\sqrt{1 + \frac{4p^2(1-c_s^2)}{M_{\rm eff}^2 c_s^{-2}}}\,, \tag{2.63}$$

with c_s given by (2.60). The pairs $(\mathcal{R}_-, \mathcal{F}_-)$ and $(\mathcal{R}_+, \mathcal{F}_+)$ represent the amplitudes of both low and high frequency modes respectively, and satisfy

$$\mathcal{F}_- = \frac{-2i\dot\theta\omega_-}{M_{\rm eff}^2 + p^2 - \omega_-^2}\mathcal{R}_-\,, \quad \mathcal{R}_+ = \frac{-2i\dot\theta\omega_+}{\omega_+^2 - p^2}\mathcal{F}_+\,. \tag{2.64}$$

Thus the fields in each pair oscillate coherently. Of course, we may only neglect the friction terms if both frequencies satisfy $H \ll \omega_\pm$. This implies $H \ll pc_s$, which is what is meant by short wavelength regime. Integrating out the heavy mode consists in ensuring that the high frequency degrees of freedom do not participate in the dynamics of the adiabatic modes. This can only be done in a sensible way if there is a hierarchy of the form $\omega_-^2 \ll \omega_+^2$, which given (2.63) necessarily requires

$$p^2 \ll M_{\text{eff}}^2 c_s^{-2} \,. \tag{2.65}$$

This defines the regime of validity of the EFT, in which one has

$$\omega_+^2 \simeq M_{\text{eff}}^2 c_s^{-2} = m^2 + 3\dot{\theta}^2 \ , \tag{2.66}$$

$$\omega_-^2 \simeq p^2 c_s^2 + (1 - c_s^2)^2 p^4 / (M_{\text{eff}}^2 c_s^{-2}), \tag{2.67}$$

and one can clearly distinguish between low and high energy degrees of freedom. The adiabatic condition can then be more precisely stated as

$$\frac{\dot{\omega_+}}{\omega_+^2} \ll 1 \tag{2.68}$$

with ω_+ given by (2.66). Notice that the dispersion relation for the light mode may change depending on M_{eff} and c_s as:

$$\omega_-^2 \simeq p^2 c_s^2 \qquad \text{for} \quad p^2 \ll M_{\text{eff}}^2/(1 - c_s^2)^2 \ , \tag{2.69}$$

$$\omega_-^2 \simeq (1 - c_s^2)^2 p^4 / (M_{\text{eff}}^2 c_s^{-2}) \quad \text{for} \quad M_{\text{eff}}^2/(1 - c_s^2)^2 \ll p^2 \ll M_{\text{eff}}^2 c_s^{-2} \ . \tag{2.70}$$

This last possibility, in which $\omega^2 \sim p^4$ is only possible if $c_s^2 \ll 1$. Then, we see that condition (2.65) may be rewritten as

$$\omega_-^2 \ll M_{\text{eff}}^2 c_s^{-2} \ , \tag{2.71}$$

which allows to recognize the cut-off scale Λ_{UV} (which we use interchangeably with ω_+) given by

$$\Lambda_{\text{UV}}^2 = \omega_+^2 \simeq M_{\text{eff}}^2 c_s^{-2} \ . \tag{2.72}$$

One can also re-express (2.65) using (2.60) and (2.36) as

$$p^2 \ll 4m^2/(3c_s^2 + 1) \ . \tag{2.73}$$

From this, we see that contrary to the naive expectation based on M_{eff}, the range of comoving momenta for low energy modes *actually increases as the speed of sound decreases*. Furthermore, upon quantization [76] one finds $|\mathcal{R}_-|^2 \sim c_s^2/(2\omega_-)$ and $|\mathcal{F}_+|^2 \sim 1/(2\omega_+)$, implying that high frequency modes are relatively suppressed in amplitude. Thus, we can safely consider only low frequency modes, in which case $\mathcal{F}$ is completely determined by $\mathcal{R}_c$ as $\mathcal{F} = -2\dot{\theta}\dot{\mathcal{R}}_c / \big(M_{\text{eff}}^2 + p^2 - \omega_-^2\big)$. Notice that $\omega_-^2 \ll M_{\text{eff}}^2 + p^2$, so ω_-^2 may be disregarded here.

We now outline four crucial points that underpin our general conclusions:

1. The mixing between fields $\mathcal{R}$ and $\mathcal{F}$, and modes with frequencies ω_- and ω_+ is *inevitable* when the background trajectory bends. If one attempts a rotation in field space in order to uniquely associate fields with frequency modes, the rotation matrix would depend on the scale p, implying a non-local redefinition of the fields.

2. Even in the absence of excited high frequency modes, the heavy field $\mathcal{F}$ is forced to oscillate in pace with the light field $\mathcal{R}$ at a frequency ω_-, so $\mathcal{F}$ continues to participate in the low energy dynamics of the curvature perturbations.

3. When $\dot{\theta}^2 \gg M_{\text{eff}}^2$, the high and low energy frequencies become $\omega_+^2 \simeq M_{\text{eff}}^2 c_s^{-2} \sim 4\dot{\theta}^2$ and $\omega_-^2 \simeq p^2(M_{\text{eff}}^2 + p^2)/(4\dot{\theta}^2)$. Thus the gap between low and high energy degrees of freedom is amplified, and one can consistently ignore high energy degrees of freedom in the low energy EFT.

4. In the low energy regime, the field $\mathcal{F}$ exchanges kinetic energy with $\mathcal{R}$ resulting in a reduction in the speed of sound c_s of $\mathcal{R}$, the magnitude of which depends on the strength of the kinetic coupling $\dot{\theta}$. This process is adiabatic and consistent with the usual notion of decoupling in the low energy regime (2.65), as implied by (2.68).

At the core of these four observations is the simple fact that in time-dependent backgrounds, the eigenmodes and eigenvalues of the mass matrix along the trajectory do not necessarily coincide with the curvature and isocurvature fluctuations and their characteristic frequencies. With this in mind, it is possible to state more clearly the refined sense in which decoupling is operative: *while the fields $\mathcal{R}$ and $\mathcal{F}$ inevitably remain coupled, high and low energy degrees of freedom effectively decouple.*

We now briefly address the evolution of modes in the ultraviolet (UV) regime $p^2 \gtrsim M_{\text{eff}}^2 c_s^{-2}$. Here both modes have similar amplitudes and frequencies, and so in principle could interact via relevant couplings beyond linear order (which are proportional to $\dot{\theta}$). Because these interactions must allow for the non-trivial solutions $\mathcal{R}$ = constant and $\mathcal{F} = 0$ (a consequence of the background time re-parametrization invariance), their action is very constrained [81]. Moreover, in the regime $p^2 \gg M_{\text{eff}}^2 c_s^{-2}$ the coupling $\dot{\theta}$ becomes negligible when compared to p, and one necessarily recovers a very weakly coupled set of modes, whose $p \to \infty$ limit completely decouples $\mathcal{R}$ from $\mathcal{F}$. This can already be seen in (2.52), where contributions to the effective action for the adiabatic mode at large momenta from having integrated out $\mathcal{F}$, are extremely suppressed for $k^2/a^2 \gg M_{\text{eff}}^2$, leading to high frequency contributions to (2.53) with $c_s = 1$.

Given that we have established the UV cut-off of the theory from the perspective of the full two-field theory, we can compare this scale to the strong coupling scale as given by the single field EFT. Indeed, the EFT will come with an intrinsic strong coupling scale $\Lambda_{\rm s.c.}$ at which perturbative unitarity in the scattering matrix is lost. If we want the predictions for the power spectrum and bispectrum coming from the EFT to be reliable, it is important that the strong coupling scale $\Lambda_{\rm s.c.}$ is much above the Hubble scale ($\Lambda_{\rm s.c.} \gg H$). Furthermore, re-establishing unitarity demands that new degrees of freedom appears at an energy scale $E \leqslant \Lambda_{\rm s.c.}$. As we have previously showed, the new degrees of freedom are characterized by the scale $\Lambda_{\rm UV}$ (or equivalently ω_+), and then it will also be important to ensure that this scale satisfies $\Lambda_{\rm UV} \leqslant \Lambda_{\rm \, s.c.}$.

2.3.1 Perturbativity for constant c_s

Demanding that the curvature perturbations are weakly coupled at the energy scale of inflation implies bounds on the strength of the non linear interactions of the low energy EFT. The operators of the EFT and their hierarchy vary according to whether they are time dependent or not. The perturbative regime will then be sensitive to this time dependence, so we might divide the discussion in whether the operators -in our case, functions of the speed of sound- are time dependent or not.

For the case in which the speed of sound is constant (we will later refer to the time dependent case), the strong coupling scale can be calculated from the action (1.48) with $\dot{c_s} = \ddot{H} = M_3^4 = 0$. The strong coupling scale is precisely defined as the scale at which quantum corrections to correlation functions become comparable to the tree level contribution. It is given by [34, 79]

$$\Lambda^4_{s.c.} = 4\pi M^2_{\rm pl}|\dot{H}|c_s^5(1-c_s^2)^{-1} \ . \tag{2.74}$$

Imposing that the observable modes were in the weakly coupled regime at the time they exited the horizon, $\Lambda_{\rm s.c.} > H$, implies the following bound on c_s

$$c_s > 0.01 \ . \tag{2.75}$$

This is consistent with the bound coming from the absence of observed non-gaussianities, $c_s > 0.024$ [40]. The bound in (2.75) comes however with an important drawback: it is possible that the theory becomes strongly coupled at an energy which is below the energy at which the heavier degrees of freedom becomes excited, $\Lambda_{\rm UV}$, given by

$$\Lambda_{\rm UV} \sim M^2 c_s^{-2} \ . \tag{2.76}$$

This clashes with the general idea that it should be possible to define an EFT below the UV cut-off of the theory. The key to circumvent this problem is by noticing that the speed of sound we have derived has an energy dependence

$$c_s^{-2}(k) = 1 + \frac{4\dot{\theta}^2}{\left(k^2/a^2 + M_{\rm eff}^2\right)} \,. \tag{2.77}$$

As we showed explicitly in eqs. (2.69) and (2.70), at sufficiently high energies the dispersion relation changes from $\omega(p) \propto p$ to $\omega(p) \propto p^2$. The strong coupling bound (2.74) is derived assuming a constant dispersion relation, and we may expect deviations when assuming a momentum dependent dispersion relation. This was done in [79, 94], and the new strong coupling scale was found to be:

$$\Lambda_{s.c.} = \left(8\pi c_s^2\right)^{2/5} \left(\frac{2\epsilon H^2 M_{\rm pl}^2}{\Lambda_{\rm UV}^4}\right)^{2/5} \Lambda_{\rm UV} \,, \tag{2.78}$$

which implies, as expected, $\Lambda_{s.c.} > \Lambda_{\rm UV}$. Furthermore, an extended version of the EFT action (1.48) can be constructed such that the *new physics regime* (in which the dispersion relation becomes $\propto p^2$) is incorporated [94].

2.3.2 Example

We now analyse a model of slow-roll inflation that executes a constant turn in field space, implying an almost constant and heavily suppressed speed of sound for the adiabatic mode. We will show that very simple two-field models of inflation exhibit all of these features. These are models that have a spiral structure in field space, just as in the right panel of figure 2.4. Further phenomenological implications of these models will be discussed in Chapter 4. To begin, let us consider fields $\phi^1 = \theta$, $\phi^2 = \rho$ with a metric $\gamma_{\theta\theta} = \rho^2$, $\gamma_{\rho\rho} = 1$, $\gamma_{\rho\theta} = \gamma_{\theta\rho} = 0$ (thus $\Gamma^\theta_{\rho\theta} = \Gamma^\theta_{\theta\rho} = 1/\rho$ and $\Gamma^\rho_{\theta\theta} = -\rho$), and potential

$$V(\theta,\rho) = V_0 - \alpha\theta + \frac{1}{2}m^2(\rho-\rho_0)^2 \,. \tag{2.79}$$

This model would have a shift symmetry along the θ direction were it not broken by a non-vanishing α. This model is a simplified version of one studied in [74], where the focus instead was on the regime $M_{\rm eff} \sim m \sim H$ (see also [95] where the limit $M_{\rm eff}^2 \gg H^2 \gg \dot{\theta}^2$ is analysed). The background equations of motion are

$$\begin{aligned} \ddot{\theta} + 3H\dot{\theta} + 2\dot{\theta}\frac{\dot{\rho}}{\rho} &= \frac{\alpha}{\rho^2} \,, \\ \ddot{\rho} + 3H\dot{\rho} + \rho\left(m^2 - \dot{\theta}^2\right) &= m^2\rho_0 \,. \end{aligned} \tag{2.80}$$

The slow-roll attractor is such that $\dot{\rho}$, $\ddot{\rho}$ and $\ddot{\theta}$ are negligible. This means that H, ρ and $\dot{\theta}$ remain nearly constant and satisfy the following algebraic equations near $\theta = 0$

$$\begin{aligned} 3H\dot{\theta} =& \frac{\alpha}{\rho^2} \,, \\ \dot{\theta}^2 =& m^2 \left(1 - \frac{\rho_0}{\rho}\right) \,, \\ 3H^2 =& \frac{1}{2}\rho^2\dot{\theta}^2 + V_0 + \frac{1}{2}m^2(\rho - \rho_0)^2 \,. \end{aligned} \tag{2.81}$$

These equations describe circular motion with a radius of curvature ρ and angular velocity $\dot{\theta}$. Here $M_{\rm eff}^2 = m^2 - \dot{\theta}^2$, implying the strict bound $m^2 > \dot{\theta}^2$. Thus the only way to obtain a suppressed speed of sound is if $\dot{\theta}^2 \simeq m^2$. Our aim is to find the parameter ranges such that the background attractor satisfies $\epsilon \ll 1$, $c_s^2 \ll 1$ and $H^2 \ll M_{\rm eff}^2$ simultaneously. This is given by

$$1 \gg \frac{\rho_0}{4}\left(\frac{m\sqrt{3V_0}}{\alpha}\right)^{1/2} \gg \frac{V_0}{6m^2} \gg \frac{\alpha}{4\sqrt{3V_0}m} \,. \tag{2.82}$$

If these hierarchies are satisfied, the solutions to (2.81) are well approximated by

$$\rho^2 = \frac{\alpha}{\sqrt{3V_0}m} \,, \quad \dot{\theta} = m - \frac{m\rho_0}{2}\left(\frac{m\sqrt{3V_0}}{\alpha}\right)^{1/2} \,, \tag{2.83}$$

and $H^2 = V_0/3$, up to fractional corrections of order ϵ, c_s^2 and $H^2/M_{\rm eff}^2$. We note that the first inequality in (2.82) implies $\rho \gg \rho_0$, and so the trajectory is displaced off the adiabatic minimum at ρ_0. However, the contribution to the total potential energy implied by this displacement is negligible compared to V_0. After n cycles around $\rho = 0$ one has $\Delta\theta = 2\pi n$, and the value of V_0 has to be adjusted to $V_0 \to V_0 - 2\pi n\alpha$. This modifies the expressions in (2.83) accordingly, and allows us to easily compute the adiabatic variation of certain quantities, such as $s \equiv \dot{c}_s/(c_sH) = -\epsilon/4$, and $\eta_{||} = -\epsilon/2$, where $\epsilon = \sqrt{3}\alpha m^2/(2V_0^{3/2})$. These values imply a spectral index $n_{\mathcal{R}}$ for the power spectrum $\mathcal{P}_{\mathcal{R}} = H^2/(8\pi^2\epsilon c_s)$ given by $n_{\mathcal{R}} - 1 = -4\epsilon + 2\eta_{||} - s = -19\epsilon/4$. It is now possible to find reasonable values of the parameters in such a way that observational bounds are satisfied. Using (2.83) we can relate the values of V_0, α, m and ρ_0 to the measured values $\mathcal{P}_{\mathcal{R}}$ and $n_{\mathcal{R}}$, and to hypothetical values for c_s and $\beta \equiv H/M_{\rm eff}$ as

$$\begin{aligned} V_0 =& \frac{96}{19}\pi^2(1 - n_{\mathcal{R}})\mathcal{P}_{\mathcal{R}}c_s \,, \\ m^2 =& \frac{8}{19}\pi^2(1 - n_{\mathcal{R}})\mathcal{P}_{\mathcal{R}}c_s^{-1}\beta^{-2} \,, \\ \alpha =& 6\left(\frac{16}{19}\right)^2\pi^2(1 - n_{\mathcal{R}})^2\mathcal{P}_{\mathcal{R}}c_s^2\beta \,, \\ \rho_0 =& 16c_s^3\beta\sqrt{\frac{2}{19}(1 - n_{\mathcal{R}})} \,. \end{aligned} \tag{2.84}$$

Following WMAP7, we take $\mathcal{P}_{\mathcal{R}} = 2.42 \times 10^{-9}$ and $n_{\mathcal{R}} = 0.98$ [96]. Then, as an application of relations (2.84), we look for parameters such that

$$c_s^2 \simeq 0.06\,, \quad M_{\rm eff}^2 \simeq 250 H^2\,, \tag{2.85}$$

(which imply $H^2 \simeq 1.4 \times 10^{-10}$), according to which $V_0 \simeq 5.9 \times 10^{-10}$, $\alpha \simeq 1.5 \times 10^{-13}$, $m \simeq 4.5 \times 10^{-4}$ and $\rho_0 \simeq 6.8 \times 10^{-3}$, from which we note that m, ρ_0 and $\alpha^{1/4}$ are naturally all of the same order. We have checked numerically that the background equations of motion are indeed well approximated by (2.83), up to fractional corrections of order c_s^2. More importantly, we obtain the same nearly scale invariant power spectrum $\mathcal{P}_{\mathcal{R}}$ using both the full two-field theory described by (2.37) and (2.38), and the single field EFT described by the action (2.53). The evolution of curvature perturbations in the EFT compared to the full two-field theory for the long wavelength modes is almost indistinguishable given the effectiveness with which (2.65) is satisfied, with a marginal difference $\Delta\mathcal{P}_{\mathcal{R}}/\mathcal{P}_{\mathcal{R}} \simeq 0.008$. This is of order $(1 - c_s^2)H^2/M_{\rm eff}^2$, which is consistent with we made in [92]. Despite the suppressed speed of sound in this model, a fairly large tensor-to-scalar ratio of $r = 16\epsilon c_s \simeq 0.020$ is predicted.

As expected, for $c_s^2 \ll 1$ a sizable value of $f_{\rm NL}^{(\rm eq)}$ is implied. The cubic interactions leading to this were deduced in Ref. [81] which for constant turns is given by

$$f_{\rm NL}^{(\rm eq)} = \frac{125}{108}\frac{\epsilon}{c_s^2} + \frac{5}{81}\frac{c_s^2}{2}\left(1 - \frac{1}{c_s^2}\right)^2 + \frac{35}{108}\left(1 - \frac{1}{c_s^2}\right)\,. \tag{2.86}$$

This result is valid for any single-field system with constant c_s obtained by having integrated out a heavy field. Recalling that the spectral index n_T of tensor modes is $n_T = -2\epsilon$, for $c_s \ll 1$ we find a consistency relation between three potentially observable parameters, given by $f_{\rm NL}^{(\rm eq)} = -20.74\, n_T^2/r^2$. In the specific case of the values in (2.85), we have $f_{\rm NL}^{(\rm eq)} \simeq -4.0$. This value is relatively large, so future observations could constrain this type of scenario. Finally, one can ask if the EFT corresponding to (2.85) remains weakly coupled throughout. As we have already mentioned, for small values of c_s, the dispersion relation has a dominant quartic piece which implies a strong coupling scale (2.78). For the values (2.85), we find $\Lambda_{\rm s.c.}/\Lambda_{\rm UV} \simeq 2$, implying that the EFT obtained by integrating a heavy field remains weakly coupled all the way up to its cut-off scale $\Lambda_{\rm s.c.}$.

2.3.3 Perturbativity for rapidly varying c_s

So far, the analysis has been made assuming that the speed of sound is constant in time. In the case in which the speed of sound is varying with time there is a new scale, the characteristic time of variation of c_s, that might affect the discussion above. While a constant $c_s \neq 1$ implies the appearance of a third order interaction, a time varying c_s creates an infinite number of interactions (in which the coupling of the higher order operators are given by higher order derivatives of c_s). This demands recalculating the strong coupling scale as given in eq. (2.74).

For the case in which the slow-roll parameter ϵ has a sharp change during inflation it is indeed possible to easily organize all the higher order interactions (assuming they all come from ϵ and its derivatives), and give an order of magnitude for the strong coupling scale. This was done by Cannone, Bartolo and Matarrese in [97] (and also by Adshead and Hu using different estimators [98]). We might expect these results to hold also for the speed of sound case. In [97] the change in ϵ was parametrized as a step in the Hubble parameter as

$$\dot{H}(t) = \dot{H}_0(t)\left[1 + \epsilon_{\text{step}} F\left(\frac{t - t_b}{b}\right)\right] \tag{2.87}$$

where ϵ_{step} is the magnitude of the step and F is a function centred at t_b, where the step happened, and b parametrizes its sharpness. Defining $\beta = 1/bH$, it was showed that the perturbative expansion is valid if we impose

$$\beta \ll 160\ . \tag{2.88}$$

We will refer to this condition as the feature unitarity bound. While we will make use of this bound, let us note that a quantitative analysis might in principle shows some deviations from this result. First of all, the previous result was obtained taking into account only the time dependence of the Hubble parameter. The nth-order lagrangian can then be calculated by Taylor expanding the Hubble parameter up to that order. Then it is possible to group all the terms at nth-order in a single vertex (for example π^n) by successive integration by parts, greatly simplifying the calculation.

There are however many more terms that are allowed by the symmetries of the system. In particular, there is a tower of M_n^4 coefficients multiplying nth-order operators (just as M_3^4 was needed for computing $\mathcal{L}_3$) which are in principle time dependent and different from zero. In the absence of a UV theory that gives us a recipe for consistently calculating M_n^4, any estimate on how they determine the perturbative regime must be made with caution. Additionally, the intuition in terms of scattering amplitudes is borrowed from the standard QFT techniques which assume time-independent vertex coefficients. Intuitively, this will be applicable to time-dependent coefficients if they obey an adiabatic condition of the form $|\dot{\lambda}/\lambda T| \ll 1$, where T is the timescale of the scattering process. Within this regime, higher order interactions should be suppressed. Although this might relax the strong coupling bound coming from the scattering amplitudes, it is not clear how time dependence would affect the other strong coupling scales [98].

Let us note that a very sharp change in the speed of sound might also violate the two-field adiabatic condition, $\dot{\omega}_+ \ll \omega_+^2$. We can express this bound in terms of u and s as:

$$\begin{aligned}\frac{\dot{\omega}_+}{\omega_+^2} &= \frac{c_s^2}{M_{\rm eff}^2}\frac{d}{dt}\left(\frac{M_{\rm eff}}{c_s}\right) \\ &= s\left(\frac{H}{M_{\rm eff}}\right)\left(\frac{-3c_s^4}{1+3c_s^2}\right) \ll 1 \ . \end{aligned} \tag{2.89}$$

The parameter $H/M_{\rm eff}$ may take a wide range of values, from a minimal of $H/M_{\rm eff} \sim 10^{-2}$ to $H/M_{\rm eff} \sim 10^{-16}$ in the extreme cases of inflation happening just below the Planck scale or at the TeV scale (see e.g. [99, 100]). The feature unitarity bound given by equation (2.88) can also be expressed in terms of s and c_s. For this, we need to choose a functional form for the speed of sound. We will use the following ansatz

$$c_s^2(t) = \bar{c_s}^2(t)\left[1+\sigma F\left(\frac{t-t_b}{b}\right)\right] \ , \tag{2.90}$$

where $\bar{c_s}$ is the unperturbed value of c_s, and σ and F are respectively the amplitude and the shape of the feature. We choose an exponential for F in e-folds, such that

$$F = e^{-\beta^2\left(N-N_f\right)^2} \ . \tag{2.91}$$

Under this parametrization, we can write a relation between s and c_s in the following form (for definitiveness with signs we choose $\sigma < 0$)

$$|s| = \beta\left(\frac{c_s^2}{\bar{c_s}^2}-1\right)\sqrt{-\ln\left(\frac{c_s^2/\bar{c_s}^2-1}{\sigma}\right)} \ . \tag{2.92}$$

Then, the bound (2.88) can be written as

$$\beta = |s|\left[-\ln\left(\frac{c_s^2/\bar{c_s}^2-1}{\sigma}\right)\right]^{-1/2}\left(\frac{c_s^2}{\bar{c_s}^2}-1\right)^{-1} \ll 160. \tag{2.93}$$

We compare the bounds (2.89) (with $H/M_{\rm eff} \sim 10^{-2}$) and (2.93) (with $\bar{c_s} = 1$) in figure 2.5. In orange we show the region excluded either by dynamical excitation of the heavy field (when $\dot{\omega}_+ \sim \omega_+^2$) or by loss of unitarity given by the constant speed of sound bound (2.78). We also show the speed of sound trajectories that satisfy the feature unitarity bound (2.93), given the gaussian ansatz for the shape of the speed of sound feature (we do not expect different qualitative results when considering different shapes, e.g. a *tanh* step in the speed of sound). We see that all the trajectories that satisfy the feature

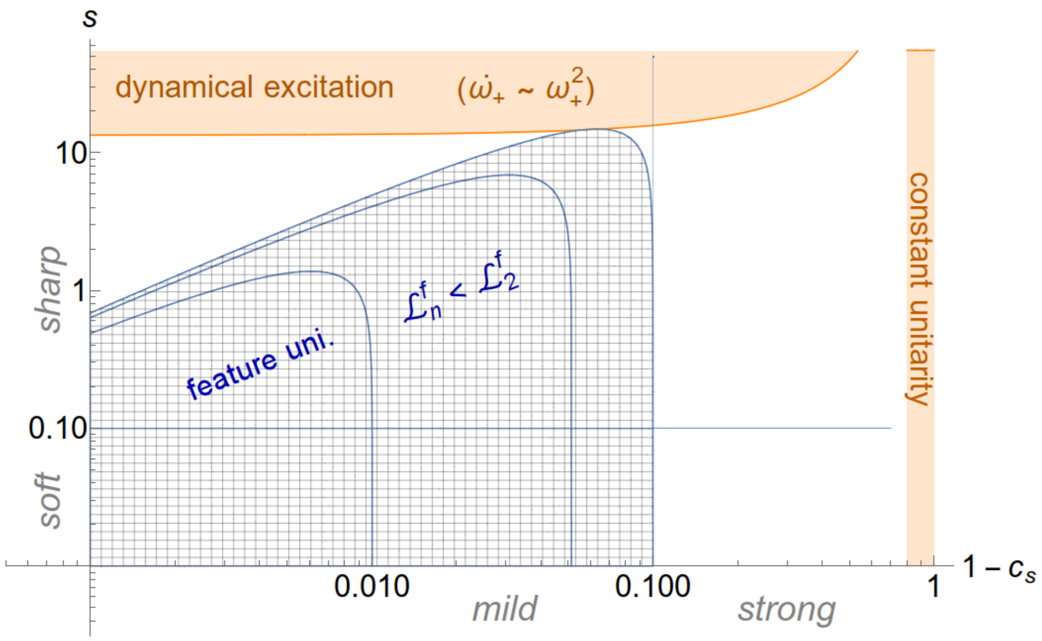

FIGURE 2.5: Different regimes for reduction in the speed of sound. Regions in orange are forbidden either by violation of the unitarity bound for constant c_s eq. (2.78) (in the plot this region is enlarged for visualization) or adiabaticity ($\dot{\omega}_+ \sim \omega_+^2$), as given in eq. (2.89). The meshed gray region is allowed by imposing the feature unitarity bound for the gaussian transient reduction, as given in eq. (2.93). We arbitrarily consider the soft/sharp limit to be at $s = 0.1$ and the mild/strong limit to be at $1 - c_s = 0.1$.

unitarity bound also satisfy the dynamical excitation bound. The zone in which feature unitarity is lost but the heavy field is not excited demands to be studied in more detail.

Independently of its origin, the speed of sound of the adiabatic mode during inflation may be phenomenologically divided into whether it has a slow or a fast time evolution. In figure 2.5 we have also plotted these different regimes in terms of $1 - c_s$ and s. While s determines whether the time variation of the speed of sound is *soft* or *sharp*, $1 - c_s$ determines whether the reductions in the speed of sound is *mild* or *strong*[6]. These different regions will result in n-point correlation functions with different characteristics. On the one hand, if the speed of sound evolves slowly, we can use the standard slow-roll techniques. This is the soft regime of figure 2.5. In this case the predictions for both the mild and strong regime can be analytically calculated. On the other hand, a fast evolution of the speed of sound will demand the use of different techniques for calculating the spectra of the curvature perturbations. In this case the mildness of the feature will prove important for using perturbative techniques, so we will restrict our study to the sharp and mild regime. We will devote the two following chapters to studying the observational constraints for these two different phenomenological regions, the sharp and mild (Chapter 3) and the soft (Chapter 4) regime.

[6]Let us note that here strong does not refer to the strong coupling regime, but rather to reductions in c_s that satisfy $1 - c_s < 0.1$.