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Appendix A

More elaborate derivation of contact probability for fractal globules

To derive the contact probability of a “fractal globule”, we first have to define exactly what we mean by that term. Here, we define our fractal object as one where at each length scale the contacts between the $2^d$ neighboring smaller parts “look” the same: the fraction $f_I$ of the surface in between them divided by the total surface of the individual parts (without taking the contact between the parts into consideration) is equal for all length scales. The last assumption is that the blocks build themselves in a fractal way: $2^d$ consecutive blocks are ordered inside a larger $2 \times 2 \times 2$ block (in the case of 3 dimensions). The effect of this is that the surface of a large blob can be fractal with a dimension higher than $d-1$. We define $S_1$ as the surface of the elementary building block where the globule starts to be fractal.

Using these definitions, we now derive the contact probability, without even needing to know the fractal dimension of the globule, which is constrained, though not necessarily uniquely determined, by the value of $f_I$. Since we are not interested here in these constraints, we will refrain from deriving them. Our derivation here is neither limited to polymers, though with the ordering constraint of the blocks, assuming connected bonds is a rather loose constraint.

Since blocks are connected to each other through their surface that is determined by the surfaces of the smaller blocks it constitutes of, we first derive the surface of a block of $g$ elementary blocks. The first new surface area $S_{2^d}$ is a function of the surface of the elementary blocks and the internal surface fraction $f_I$: 76
\[ S_{2d} = (1 - f_I) S_1 2^d. \] (A.1)

Thus we get for arbitrary \( g = (2^d)^k \):

\[ S_g = (1 - f_I)^k S_1 \] (A.2)

Using that \( k = \log(g) / \log(2^d) \), we find that:

\[ S_g = (1 - f_I)^{\log(g) / \log(2^d)} S_1 = g^{1 + \frac{1}{d} \frac{\log(1 - f_I)}{\log(2)}} S_1. \] (A.3)

For simplicity of the argument we only find the contact probability of monomers with a block of \( g \) monomers, at least \( g 2^{-d} \) monomers apart, where the last condition ensures that monomers are in separate sub-blocks. Thus, the resulting contact probability \( p_c(g) \) is actually a (weighted) average over the interval \([g 2^{-d}, g] \). Since we are not interested in a complete explicit formula, but more in scaling and the dependence on \( f_I \) without caring too much about small corrections, this assumption suffices for us.

The total surface of the sub-blocks is given by \( S_g / (1 - f_I) \), which follows readily from their respective definitions. Then the internal surface of all sub-blocks is given by \( M_g = S_g f_I / (1 - f_I) \). To obtain the contact probability, we find the total number of possible contacts, that can be found between monomers that are within a \( g \) block, more than \( g 2^{-d} \) apart, which is given by \( Q_g = 1/2(1 - 2^{-d})g^2 \). Thus we find for the contact probability:

\[ p_c(g) = \frac{M_g}{Q_g} = \frac{2 f_I S_g}{(1 - f_I)(1 - 2^{-d})g^2} = \frac{2 f_I S_1}{(1 - f_I)(1 - 2^{-d})} g^{1 + \frac{1}{d} \frac{\log(1 - f_I)}{\log(2)}}. \] (A.4)

Thus for the case of a smooth fractal, we have \( f_I = 0.5 \), and we get \( p_c \sim g^{-1 - 1/d} \), which is the same as given in the main text. Since this is the highest possible value of \( f_I \), and we can get anything down to \( f_I = 0 \), we find for the possible values of the exponent: \(-1 > \beta \geq -4/3 \). Note however that if \( f_I \) goes to 0, the prefactor also goes to 0. Thus, getting exactly a \(-1 \) law is impossible with our assumptions, though we can approach it arbitrarily close, with an increasingly small prefactor.
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