STELLINGEN

In the theorems below, the following notation is used. Let k be an algebraically closed field of characteristic 0 and L an algebraic function field of transcendence degree 1 over k. Denote by g_L the genus of L. Further, denote by M_L the set of normalized discrete valuations on L that are trivial on k and define the absolute values $|\cdot|_\nu := e^{-\nu(\cdot)}|$ ($\nu \in M_L$) and define the ring of T-integers $\mathcal{O}_T = \{x \in L : |x|_\nu \leqslant 1 \text{ for } \nu \notin T\}$. For $x \in O_T$ define $|x|_T := \prod_{\nu \in T} |x|_\nu$. For $x_1, \ldots, x_n \in L$, put $H_T(x_1, \ldots, x_n) := \prod_{\nu \in T} \max_{1 \leqslant i \leqslant n} |x_i|_\nu$. Let $K = k(t)$ be the field of rational functions in the variable t and S a finite subset of M_K containing the valuation ν_∞ with $\nu_\infty(t) = -1$.

1. Let $n \geqslant 3$. Assume $x_1, \ldots, x_n \in L$ and $\sum_{i=1}^{n} x_i = 0$ but that no non-empty proper subsum vanishes. Then
$$H_T(x_1, \ldots, x_n) \leqslant e^{\binom{n-1}{2} \max(2g_L - 2 + \#T, 0)} \Big(\prod_{i=1}^{n} |x_i|_T \Big) \Big(\prod_{\nu \notin T} \max_i (|x_i|_\nu) \Big)^{n-1}.$$
In particular, if $x_1, \ldots, x_n$ are k-linearly independent, then we can replace $\max(2g_L - 2 + \#T, 0)$ by $2g_L - 2 + \#T$.
(Corollary 2.2.11)

2. Let $n > 2$. If $l_1, \ldots, l_n$ are positive integers satisfying $\frac{1}{l_1} + \cdots + \frac{1}{l_n} \leqslant \frac{1}{\binom{n-1}{2}}$, then the equation $x_1^{l_1} + \cdots + x_n^{l_n} = 0$ does not have a solution $x_1, \ldots, x_n \in k[t]$ such that $x_1, \ldots, x_n$ are non-constant and pairwise coprime.

3. Let L be a finite normal extension of K and T the set of normalized valuations of L lying above those in S, $\{l_1, \ldots, l_n\}$ a set of linear forms in two variables with coefficients in L which is invariant under the action of $\mathrm{Gal}(L/K)$, $\mathbb{A}$ an admissible tuple (Definition 4.3.1) and λ_1, λ_2 the successive minima of $\mathcal{C} = \prod_{\nu \in S} \mathcal{C}_\nu$ (see Section 3.1 of this thesis), where
$$\mathcal{C}_\nu = \{\mathbf{x} \in K_\nu^2 : |l_i(\mathbf{x})|_\omega \leqslant A_{i\omega} \text{ for } i = 1, \ldots, n,\ \omega \in T,\ \omega | \nu\}.$$
Then
$$\lambda_1 \lambda_2 \geqslant \left(\prod_{\omega \in T} \max_{1 \leqslant i < j \leqslant n} \frac{|\det(l_i, l_j)|_\omega}{A_{i\omega} A_{j\omega}} \right)^{1/[L:K]},$$
$$\lambda_1 \lambda_2 \leqslant e^{(n+1)\#S} \left(\prod_{\omega \in T} \max_{1 \leqslant i < j \leqslant n} \frac{|\det(l_i, l_j)|_\omega}{A_{i\omega} A_{j\omega}} \right)^{1/[L:K]}.$$
(Lemma 4.3.2)

4. For a polynomial P with coefficients in K, we define $H^*(P) := \prod_{\nu \in M_K} \max(1, |p_1|_\nu, \ldots, |p_t|_\nu)$, where $p_1, \ldots, p_t$ are the non-zero coefficients of P. Call two binary forms $F, F^* \in \mathcal{O}_S[X, Y]$ $\mathrm{GL}(2, \mathcal{O}_S)$-equivalent if $F^*(X, Y) = uF(aX + bY, cX + dY)$ for some $u \in \mathcal{O}_S^*$, $\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \in \mathrm{GL}(2, \mathcal{O}_S)$.
Let $F \in \mathcal{O}_S[X, Y]$ be a binary cubic form of non-zero discriminant $D(F)$. Then F is $\mathrm{GL}(2, \mathcal{O}_S)$-equivalent to a binary form F^* such that
$$H^*(F^*) \leqslant e^{12\#S} |D(F)|_S$$
(Corollary 4.3.7).

5. Let $F \in \mathcal{O}_S[X,Y]$ be a binary form of degree $n \geqslant 4$ with non-zero discriminant. Then F is $\mathrm{GL}(2,\mathcal{O}_S)$-equivalent to a binary form F^* such that

$$H^*(F^*) \leqslant e^{(n-1)\left(\#S(n+11)-5\right)} |D(F)|_S^{20+\frac{1}{n}}$$

(Main Theorem of Chapter 5).

6. Under the assumption of the abc-conjecture over number fields the following can be proved. Let $F \in \mathbb{Z}[X,Y]$ be a binary form of degree $n \geqslant 4$ with non-zero discriminant. Then F is $\mathrm{GL}(2,\mathbb{Z})$-equivalent to a binary form F^* of height

$$H^*(F^*) \leqslant c_1(n)|D(F)|^{c_2(n)},$$

where $H^*(F^*)$ is the maximum of the absolute values of the coefficients of F^*.

7. Let $n \geqslant 4$. Let $f \in k[t][x]$ be a polynomial of degree n with distinct roots $\gamma_1, \ldots, \gamma_n \in \overline{k(t)}$. Choose for every $\nu \in M_K$ an extension of $|\cdot|_\nu$ to $\overline{k(t)}$. Then for every finite subset S of M_K,

$$\prod_{\nu \in S} \min_{1 \leqslant i < j \leqslant n} |\gamma_i - \gamma_j|_\nu \geqslant c(n)^{-1} H^*(f)^{-n+1+\frac{n}{40n+2}},$$

where $c(n) = \exp\left(\frac{(n-1)\left((n+11)\#S-5\right)}{20+1/n}\right)$.

8. Let $f \in k[t][x]$ be a cubic polynomial with distinct roots $\gamma_1, \gamma_2, \gamma_3 \in \overline{k(t)}$ and $\nu = \nu_\infty$. Then

$$\min_{1 \leqslant i < j \leqslant 3} |\gamma_i - \gamma_j|_\nu \geqslant H^*(f)^{-2}.$$

On the other hand, there exists $c > 0$ such that for every $H > 0$ there exists a cubic polynomial $f \in k[t][x]$ with

$$\min_{1 \leqslant i < j \leqslant 3} |\gamma_i - \gamma_j|_\nu \leqslant c H^*(f)^{-2}, \quad H^*(f^*) \geqslant H.$$

9. It is not knowledge but the act of learning, not possession but the act of getting there, which grants the greatest enjoyment. - Carl Friedrich Gauss

10. If you have a good theory, forget about the reality. - Slavoj Žižek