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Curriculum Vitae

Weidong Zhuang was born on November 23, 1983. He enrolled as a student in mathematics in University of Science and Technology of China in 2005. He obtained his bachelor's degree in 2009 and was admitted in the Erasmus-Mundus ALGANT joint master program. He studied in Université Paris-Sud 11, Orsay for the first year and went to Leiden for his second year. He wrote his master thesis "Hasse-Weil Zeta-Function in a Special Case" under the supervision of Professor M. Harris. He received his master's degree in 2011 and started his PhD project, supported by NWO, under the supervision of Dr. Jan-Hendrik Evertse in Leiden University.

Weidong Zhuang married Yana Zuo in 2012. Their son Zhida was born in 2014.

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