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**Author:** Jong, Staas de  
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6. Excursion II: The computed retinal afterimage

In this chapter, we develop an automated computational technique for displaying 2D shapes in the retinal afterimage.

The retinal afterimage is the familiar effect in the human visual system where the ongoing perception of light is influenced by the preceding exposure to it. Historically, the retinal afterimage has been used in various techniques for the visual arts, associated with Neo-impressionism, Op art, Stan Brakhage, and the Bauhaus school.

When considering techniques for the visual arts in general, the history of computer graphics shows that incorporating stages of automated computation can offer a fundamental advantage to visual artists: a control over perceived visual complexity that is otherwise unattainable.

This motivates the question whether the retinal afterimage, too, can be induced by the output from automated processes of computation. We pursue this general question for the representative case where input should specify 2D shape.

This raises a fundamental problem: How can we ensure that shape recognition by the viewer actually is due to the retinal afterimage, and is not due to normal viewing of the stimuli, which also occurs?

First, we define a general approach using visual fixation, a rasterization method, and image sequences. For this context, we then formulate a naive but formal model of induced afterimage intensity. Then, based on the model, we develop a series of rule sets for automatically computing the image sequences that serve as stimuli.

The rule sets implement different formally defined strategies toward shape display exclusive to the retinal afterimage. In ambiguous rule sets, the screen intensities that are used can each lead to multiple or all of the target afterimage intensities used. In scrambling rule sets, visual grouping underlying shape perception is actively subverted during normal viewing. In hybrid rule sets, both strategies are combined.

Two rule sets representative of the approach were tested in a pilot experiment with five subjects. When using five-letter word shapes as input, the result for both rule sets was that none of the participants recognized the shapes in the separate images of the sequence used, while all of them did so in the induced afterimage effect. This seems to indicate that the automated computational technique proposed here can be used to display shape specifically in the retinal afterimage.

In the Electronic Appendix, the following is provided: video examples referred to in the text; the image sequences used in the pilot experiment; and software implementing the approach, in source format.

Under submission.
6.1 Introduction

6.1.1 Automated computation enables human control over perceived visual complexity in the arts  
Due to the historical development of computer graphics, ever more aspects of human experience that are based on visual perception can be induced by the output from automated processes of computation. This includes, for example, the apparent presence and geometry of flat, two-dimensional (2D) shapes, as well as their colors and movements. It also includes the apparent presence and spatial geometry of three-dimensional (3D) objects, and their positions and movements.

Visual artists routinely use this, for example when operating (or writing) software based on standards like OpenGL [Woo et al. 1997], e.g. to create the 2D or 3D visuals for installations or computer games. Similarly, other standards based on automated computation like Pixar's RenderMan [Upstill 1989] are routinely used to create the images for 3D animated movies.

In such routine use, the visual artist provides input specifying desired visual results to an algorithm. The algorithm then typically executes automatically on one or more electronic digital computers. Ultimately, viewers are exposed to the output from the algorithm, via additional stages of physical transduction culminating in some type of visual display technology. These final stages typically function in such a way as to ensure that given digital output will produce similarly perceived visual effects in different individual viewers.

In many other respects, the subjective experiences of the viewers may differ, of course: Different viewers may feel quite differently about some visualized object A, all agreeing, however, that it spatially appears in front of some other object B. We will here refer to aspects of visual perception of this type – induced similarly across viewers in general, and by the output from automated processes of computation – using the term “computed aspects of visual perception”.

The use of computed aspects of visual perception in the visual arts then offers a fundamental advantage, apparent when considering real-life examples. For example, during most scenes of a typical, cinematically released 3D animated movie, the viewer, at any given moment, will be presented with the simultaneous visualization of many different 3D objects. Each of these will be characterized by a range of visual properties that may be regarded as individual to it. Also, each object may be seen as visually relating to the other objects that are present, in any of a number of ways. Moreover, over time, all of this is subject to change. For the experiences it induces, the movie relies on these types of visual complexity, offered perceptually to the viewer. Now suppose, that the visual artists who created the movie would have had to rely on manual techniques to keep track of all visual components, and to realize their display. (By manual techniques, we mean techniques not incorporating any stages of automated computation.) The artists simply would not have been able to create the movie: The personal labor required by unautomated processes achieving similar results would have been so time-consuming, as to make completion of the work (i.e., within the lifetimes of the artists) impossible.
This illustrates a more general point, that is not restricted to a specific aspect of visual perception, or to a specific type of artwork: Given a type of visual effect, by using techniques for its production that are partly automated, a visual artist may create works based on the effect that offer a controlled visual complexity which would otherwise be unrealizable.

Here, we will consider the question whether this fundamental advantage might also be extended to the type of visual effect that is often called the retinal afterimage.

6.1.2 The retinal afterimage The retinal afterimage is the familiar effect in the human visual system where the ongoing perception of light is influenced by the preceding exposure to it.

Inside the eye, images of the outside world are projected onto the retina, a layer containing light-sensitive cells. Of these, the “rods” respond to night-time light levels, while the “cones” respond to daytime light levels (see e.g. [Angel 2000]). Usually, cone cells are present in three types, each responding to activity within a different frequency range of the incoming light. This trichromatic response, for the frequency ranges associated with red, green, and blue light, enables human color perception. Afferent neural connections link the rod and cone cells of the retina to the rest of the central nervous system (see e.g. [Kalat 2004]). Further processing of color signals is thought to occur according to the opponent-process model: neural connections signal the red versus green; blue versus yellow; and bright versus dark intensities of incoming light (see e.g. [Rathus 2012]). The cells involved in this continuously adapt to the light levels they are exposed to, and this underlies a range of effects in human visual perception.

One such effect is the well-known phenomenon of “peripheral fading”. When looking at an unchanging scene, in which some central point is surrounded by low-contrast shapes, the latter may seem to disappear completely into the background, after staring at the central point for a while [Troxler 1804]. In fact, if there were no eye movement at all, the whole scene would vanish from perception, as the retinal cells would completely adapt to the stable image projected in such a situation, and cease to respond [Martinez-Conde et al. 2004]. This means that for normal vision, eye movement must always be present: even while staring, there will be involuntary, small saccades. (These can be observed directly using Figure 6.1, which also induces a retinal afterimage.) However, for areas where low-contrast shapes are projected onto the retina, small eye movements will not vary detected light levels much, and adaptation, and the consequent fade from perception, can still occur.

Like peripheral fading, the retinal afterimage is thought to occur because of adaptation. When staring at a red figure on a white background, the retinal cells under the influence of the figure shape will adapt to its incoming light, with cells responding to blue and green light adapting to relatively lower levels of intensity. If, after a while, vision is then suddenly directed to a completely white area, the retinal cells adapted to the figure shape will respond to light levels which have increased in the blue and green frequency ranges, while staying constant in the red frequency range. This can then
result in the repeated perception of the figure shape, but in a color complementary to red. This demonstrates the classic afterimage effect, which is thought to happen according to the opponent-process model of human color perception [Hering 1964] [Hurvich and Jameson 1957] [Rathus 2012].

**Figure 6.1** The retinal afterimage, and the presence of small involuntary eye movements. Please look at the black dot for one minute; then look at the white dot. Even while staring, the afterimage of the crossing lines will seem to move around. This is due to small eye movements. (Image reproduced from [Martinez-Conde et al. 2004] [Verheijen 1961].)

### 6.1.3 Use of the retinal afterimage in techniques for the visual arts

Historically, the retinal afterimage has been consciously used in various visual techniques, associated with different groups of visual artists. These groups include the artists of the Neo-impressionism and Op art movements, and visual artists part of, or influenced by, the Bauhaus school.

In [Chevreul 1839], translated in [Chevreul 1855], the 19\textsuperscript{th}-century scientist Michel Eugène Chevreul discussed different forms of contrast between colors, and their occurrence and use in the arts. This included the retinal afterimage, discussed as “successive contrast” and “mixed contrast”. Decades later, the painter Georges Seurat became the seminal figure in the movement of Neo-impressionism, and also for the related techniques of divisionism and pointillism. Seurat based his work on the color theories publicized by Chevreul and other scientists of his time [Poplawski 2003] [Gardner and Kleiner 2010]. Neo-impressionism, with the goal of maximizing the brilliance of color, rejected mixing paint on the palette, and instead relied on mixing colors during the process of viewing [Signac 1899]. This was done using the technique of divisionism: paint is applied dot by dot, with adjacent dots colored according to pairs of complementary colors. In recent teaching materials, it is assumed that when viewing painted areas containing such pairs of dots, a given color may be perceived as
brighter or more intense due to the retinal afterimages induced by its complementary color [Scholastic Inc. 2008].

Starting in the second half of the 20th century, the Optical art or Op art movement [Houston and Hickey 2007] [Museum of Modern Art and Seitz 1965] also emphasized the process of viewing in art, making heavy use of illusory effects in the human visual system. This included the retinal afterimage, e.g. in works by the painters Bridget Riley [Sylvester and De Saussure 2012], Richard Anuszkiewicz [Madden et al. 2010], and Larry Poons [Morgan 2007]. More generally, the retinal afterimage is part of the techniques associated with Op art [Parola 1969].

Painters, in the traditional sense, have not been the only visual artists to consciously make use of the retinal afterimage. The influential experimental film maker Stan Brakhage, for example, felt at one point that afterimage colors were the only true colors [Brakhage 1967]. Brakhage produced some of his work by directly painting on the successive frames of analog film strips, then shown using a movie projector.

Josef Albers was one of the teachers at the original Bauhaus in Germany. In the course of the 20th century, his work, both as an artist and as an educator, became very influential in visual art and design (see e.g. [Chilvers 2009]). In his teaching, Albers presented the retinal afterimage as a fundamental aspect of human color perception, generally to be taken into account when using color in visual art and design [Albers 2006].

6.1.4 The computed retinal afterimage: 2D shape Historically, the visual artists and movements discussed in Section 6.1.3 regarded the retinal afterimage as a significant effect, studied it, and incorporated it into the visual techniques they used. To us, this motivates the question whether techniques for producing the retinal afterimage, like those for producing other visual effects, could be partly automated. This could increase the scope of the personal labor of visual artists interested in using the retinal afterimage: By using partially automated techniques, controlled visual complexity could be arrived at in less time, both for preliminary studies and for finished works – just as is already possible for other aspects of human visual perception, e.g. those discussed in Section 6.1.1.

Like other computed aspects of visual perception, forms of the computed retinal afterimage would include a specification stage, followed by automated computation, output display, and viewer perception. A fundamental choice before implementing such stages is what aspects of the retinal afterimage to make subject to specification by the visual artist. Candidates for this include color, textural effects, and shape.

Color specification would enable the artist to automatically induce specific recognizable afterimage colors, similar to those identified by Chevreul and Albers. Textural specifications would enable the artist to automatically induce specific recognizable patterned afterimage effects across areas of display, similar to effects obtained by divisionists using dot patterns. Shape specification would enable the artist to automatically induce specific recognizable 2D shapes in the afterimage. In Op art,
afterimage shape has often been used to present abstract forms, while figurative use has occurred in countless demonstrations of the retinal afterimage, from [Goethe 1795] to [Jenkins and Wiseman 2009].

Here, we will aim to implement the computed retinal afterimage based on shape specification. This because shape seems to be a basic aspect of afterimage perception, suitable for demonstrating feasibility of the computed retinal afterimage in general.

6.1.5 The problem of shape recognition outside the afterimage  Our current goal can be described as finding a method for automatically inducing specific recognizable 2D shapes in the retinal afterimage. However, if the output display stage of such a method will present the viewer with images, there is a problem to be aware of, clarified by Figures 6.2a and 6.2b.

Both of these figures can be used to induce a retinal afterimage. Before this is done, a pattern of bird shapes can be recognized in Figure 6.2a, and the image of a face in Figure 6.2b. When trying out Figure 6.2a, due to its peculiar symmetry, the afterimage will contain what seems like an almost identical, shifted copy of the bird shapes already seen. When trying out Figure 6.2b, the afterimage will briefly show the positive of a face, having a much less obvious likeness to its negative predecessor. This is reflected in the greater element of surprise associated with seeing the second afterimage.

![Figure 6.2a](image_url)  Example afterimage shapes. First, please enlarge this image, and increase its visual contrast, as much as is possible and comfortable. Then, from close by, focus on the crosshair in the middle for about one minute. After this, close your eyes. Briefly, an afterimage will appear. Subsequent blinking may bring it back, as it becomes less and less distinct. (Image: detail from a work by M.C. Escher.)
These examples demonstrate how, to a varying extent, the contents of the retinal afterimage may also be recognized while looking in an ordinary fashion at the imagery used to induce the effect. When considering a method for automatically inducing shapes in the retinal afterimage, this poses a problem: How can we be sure that recognition of shapes by the viewer is actually due to the afterimage effect, and not due to ordinary viewing – clearly also a possibility?

Here, to avoid such false positives, we will require explicitly negative results for shape recognition outside of the afterimage effect. Furthermore, we will limit our automated approach to the greyscale case, and not use different perceived light intensities for different types of retinal color receptors. Also, we will induce the retinal afterimage using the minimum amount of images: one bias image for retinal adaptation, followed by one trigger image triggering the effect. (In Figures 6.2a and 6.2b, these images correspond to the image looked at initially, and then closing one's eyes, respectively.) Having two such greyscale images produce an afterimage showing the specified 2D shape, while this shape is not recognizable in either of the images separately, then corresponds to visualization exclusive to the retinal afterimage.

![Figure 6.2b](image)

*Figure 6.2b  Another example afterimage shape. (See Figure 6.2a for viewing directions.)*

### 6.2 Computing shapes in the retinal afterimage

Below, the construction of a method for computing 2D shapes in the retinal afterimage will be discussed. This will be illustrated by figures in the text, and by video examples which can be found in the Electronic Appendix. All development was done using the

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1 The Electronic Appendix can be retrieved via [http://staas.home.xs4all.nl](http://staas.home.xs4all.nl).
TFT LCD display of a Packard Bell R3450 laptop computer, which has a resolution of 98 dpi. This display was used during daytime (set to maximum brightness), in an indoors setting otherwise without artificial lighting. Image sequences were viewed from a distance of approximately 60 cm, chosen as a typical and comfortable viewing distance. Bias images were displayed for 20 seconds, and immediately followed by their trigger images.

In the Electronic Appendix, software implementing the method can also be found, allowing direct experimentation with various aspects of the method. These include input patterns, visualization parameters, and rule sets, discussed below. It may be necessary to use the software to recreate video examples, for playback on other display devices than the one used here: Differences in color gamut between various display types and technologies can be considerable, and may initially prohibit an effective reproduction of the greyscale intensities used. The solution in such cases would be to repeat construction, as it is described in the text, for the specific display in question.

6.2.1 Controlling induced afterimage intensity: visual fixation We will use visual fixation in order to cause predictable and distinct retinal adaptation in the viewer. To enable visual fixation, we will use a crosshair: the type of shape already used in Figures 6.2a and 6.2b. The crosshair that will be used here is shown in Figure 6.3, and consists of two hairlines of one pixel wide, intersecting in a precisely defined area. The hairlines are 40 pixels long, and colored green in order to stand out in the greyscale imagery they will be used with. Thick black edges have been added in order to ensure that the crosshair's center will be easy to focus on, regardless of the contents of the surrounding image. Since we are using an electronic display, this surrounding image can be replaced seamlessly, so that triggering the afterimage effect no longer requires eye movement by the viewer.

![Figure 6.3](image) The crosshair for visual fixation.

During a typical image sequence, the viewer will be asked to focus constantly on the crosshair’s center, which in practice will probably mean constantly correcting for small fixational errors. This can be facilitated by supporting the head with both hands while the elbows are resting on some surface, as suggested in [Verheijen 1961].

6.2.2 Simultaneously inducing different afterimage intensities: a rasterization method To display 2D shapes, the retinal afterimage will have to create the simultaneous perception of different light intensities at different locations. To do this, we will simply divide the bias and trigger images into square areas of \(m \times m\) pixels. Through visual fixation, each square in a bias image will correspond to a square in the following trigger image, creating a separate sequence of greyscale intensities. We will use a total image size of \(800 \times 600\) pixels, which allows full-size playback on many different types of displays.
If we try this out for $m = 50$ however, using a chequered black and white pattern (see Figure 6.4, to the left) followed by a similar image where black is replaced by a light grey (of 90% intensity), we find that unintended lighter and darker shades appear along the square edges in the trigger image. This can be seen in the first sequence of Video Example I.

A possible explanation is that we do not succeed in projecting the shapes in the afterimage precisely over those in the trigger image. This is not surprising when recalling the afterimages produced by Figures 6.2a and 6.2b, which were not exact inversions, but blurred variations of the original images. In addition, the already blurred pattern of retinal adaptation cannot be placed exactly and steadily over the trigger image, because of the small eye movements illustrated in Figure 6.1.

We may, however, try to alleviate the effects of this, by blurring the edges in the trigger image. The idea here is that the middle sections of squares, which do overlap, produce the desired afterimage intensities; while the borders in between those intensities, although unstable, can be made into smooth transitions, by having the light differences resulting from small eye movement be more gradual.

The contents of a pixel matrix, such as a trigger image, can be blurred by convolving them with the contents of another matrix, which specifies how each pixel’s greyscale value is to be recomputed as a weighted sum of the values of itself and its neighbours. We will use $n \times n$ convolution matrices, with all elements equal to $1 / n^2$, so that in general the replacement of the value $p_{x, y}$ of a pixel at location $(x, y)$ will be the mean

$$\frac{1}{n^2} \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} P_{x-(n/2)+i, y-(n/2)+j}.$$  

(Here, pixels lying outside the pixel matrix are assigned the value of the nearest pixel inside the matrix.) For small $n$, this results in bands of $n - 1$ pixels wide between the square areas, linearly traversing the difference in greyscale intensity. For odd $n$, the middle of such a band will be at the location of the original sharp edge (see Figure 6.4 for an example). Now, as $n$ increases from $n = 1$, the perception of unintended shades along the square edges in the trigger image decreases; around $n = 13$ it may seem largely gone, and replaced by an impression of squares of even intensity, with fuzzy borders between them. This can be seen in the second sequence of Video Example I, where the trigger image has been convolved for $n = 13$.

This sequence also shows another effect, however: squares in the afterimage may sometimes seem to join their neighbours, resulting in areas of even intensity which break the regular chequering pattern. We might have anticipated that different sequences of greyscale intensities could be used to create similar afterimage intensities, but these irregularities seem surprising in that they show that apparently similar causes – the regular structures in the bias and trigger images – do not always lead to similar results in the afterimage. In [Wade 1978], a range of potential factors influencing the
complete or partial disappearance and reappearance of structured afterimages is examined. In [Lou 2001], the influence of selective attention in this is highlighted and studied, including an effect of filling-in of enclosed regions. The irregularities that can be observed in the chequering pattern might provide an example of related effects.

Still, it seems possible that these squares with fuzzy borders could be used to construct the display of 2D shapes in the retinal afterimage. For this, it would be desirable to increase shape resolution by decreasing square size \( m \). A minimum for this would be \( m = n \), since below this value, convolution would not leave the original greyscale intensities of trigger image squares present. Going down from \( m = 50 \), there initially \( (m = 38, m = 32, m = 25) \) is the impression of fuzzy squares as described before – imperfect, but apparently similarly so. For lower values \( (m = 22, m = 19, m = 13) \) the squares give an increasingly unstable impression, distorted by large diagonal patterns. The third sequence of Video Example I again shows the chequered sequence, now with square size decreased to \( m = 25 \) (and \( n = 13 \) as before).

Above, we have determined \( n \) and \( m \) only tentatively, and the software in the Electronic Appendix allows free experimentation with both parameters.

![Figure 6.4 Example sequence. Left: middle section, surrounding the crosshair, of a bias image. Middle: the corresponding section of a trigger image. Right: a more detailed comparison, illustrating convolution of the trigger image (\( n = 13 \)).](image)

### 6.2.3 A naive but formal model of induced afterimage intensity

In the previous section, the retinal afterimage of Figure 6.4 and Video Example I seemed to give an overall impression according with the discussion of the afterimage in Section 6.1.2: The light grey squares preceded by black ones appeared to light up, which can be explained by retinal adaptation to lower light levels. In order to realize a general method for 2D shape display, we would like to explore these effects of adaptation in a formal way. We will do this by making a number of naive assumptions, giving us a simple model to work with – explicitly not intended as a model for the retinal afterimage in any general sense. Factors influencing afterimage color perception which will not be explicitly taken into account, here, e.g. include post-adaptation contour alignment [Daw 1962] and the presence of induced contrast, both during and after adaptation [Anstis et al. 1978]. This will be further discussed in Section 6.4.

Before formulating assumptions, we have to make explicit a distinction between bias and trigger intensity on the one hand, and afterimage intensity on the other. The
former will mean actual light levels produced by screen pixels, corresponding to greyscale values stored in display memory. The latter, a perceived intensity, will indicate a scale between dark and light based on the subjective impression created in the viewer.

The first naive assumption we will use, then, is that using one particular sequence of bias and trigger intensities will normally result in the perception of one particular afterimage intensity. Denoting the set of possible pixel display intensities as a finite subset \( I \subset [0, 1] \) (where 0 means black and 1 means white), we model afterimage intensity as a real-valued function \( f_a : I \times I \rightarrow R \) of pixel display intensities (while assuming environment light, exposure times and retinal sensitivity to be constant).

Then, if bias intensity \( b \) and trigger intensity \( t \) both have the same value for a sequence (as e.g. for the white squares in Figure 6.4), we denote the resulting afterimage intensity by this value, and assume that changing the bias intensity would cause it to change as well:

\[
 b = t \iff f_a(b, t) = t
\]

From the discussion of retinal adaptation in Section 6.1.2, we would expect such a change in bias intensity to be associated with either a decrease or an increase in afterimage intensity (as e.g. for the light grey squares in Figure 6.4):

\[
 b > t \iff f_a(b, t) < t
\]

\[
 b < t \iff f_a(b, t) > t
\]

Our final assumption (which will be exemplified by Figure 6.5 in Section 6.2.4.1) is that one particular trigger intensity will give a darker impression in the afterimage if and only if there has been retinal adaptation to a lighter bias intensity:

\[
 b_1 > b_2 \iff f_a(b_1, t) < f_a(b_2, t)
\]

6.2.4 Visualization exclusive to the retinal afterimage: rule sets Having made these assumptions, suppose now we would want to produce afterimages using some set \( A = \{a_1, a_2, \ldots, a_n\} \subset R \) of \( n > 1 \) afterimage intensities, with \( a_1 < a_2 < \ldots < a_n \). There would probably be various ways to arrive at such a set, but in any case, we would have to select pairs of bias and trigger intensities with which to produce it. We define an afterimage rule set for producing \( A \) as a partial function \( f_r : I \times I \rightarrow A \) which is surjective, so that it produces all of \( A \):

\[
 \forall a \in A \ \exists b, t \in I : f_r(b, t) = a
\]

and which respects the afterimage function \( f_a \):

\[
 \forall a \in A \ : \ f_r(b, t) = a \ \Rightarrow \ f_a(b, t) = a
\]

(2)

Now, given a rule set and a particular pattern of afterimage intensities to be produced, we can determine the contents for corresponding bias and trigger images by applying the rule set to the pattern. In the next sections, we will define various types of rule sets (accompanied by concrete examples), with properties in favor of shape display exclusive to the afterimage. For this, it is important to mention first that rule sets will be used non-deterministically: If more than one \((b, t)\) pair may produce a given \( a = f_r(b, t) \), one of them will be chosen at random.

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6.2.4.1 Ambiguous rule sets

First, we will focus on the subgoal that a pattern of afterimage intensities should not be recognizable in the trigger image used to produce it. A certain way to achieve this would be to use a rule set in which any trigger intensity can lead to any afterimage intensity: We could then freely choose our pattern of trigger intensities, regardless of the afterimage that is to be produced. We say a rule set \( f \) is trigger-ambiguous if:

\[
f_r(b, t) = a \Rightarrow \forall a' \in A \ \exists b' \in I : f_r(b', t) = a'
\]

The simplest examples of such rule sets would use a single trigger intensity to produce the minimum two afterimage intensities. Consider for example the rule set \( f_1 \), shown in the diagram of Figure 6.5. (In the diagram, \( B \) and \( T \) indicate the subsets of \( I \) containing \( f_1 \)'s bias and trigger intensities, respectively. These subsets are formally defined below.) Rule set \( f_1 \) is based on a dark grey trigger intensity, which is either darkened by a preceding white intensity, giving \( a_1 = f_1(1, 0.25) \), or lit up by a preceding black intensity, giving \( a_2 = f_1(0, 0.25) \). This will not allow a choice in trigger intensity, but the ambiguity requirement still guarantees that shapes in the afterimage will not be recognizable in the always evenly grey trigger images.

This is illustrated by the image sequence depicted in Figure 6.5, and demonstrated in Video Example II, which is the result of applying \( f_1 \) to an example afterimage pattern consisting of a symbol and a simple regular pattern. This sequence also shows how, by generating negative bias images and neutral trigger images, rule set \( f_1 \) implements the classic afterimage effect.

\[1.00\]
\[B\]
\[a_1\]
\[T\]
\[a_2\]
\[0.00\]
\[A\]

**Figure 6.5** Trigger ambiguity. Left: diagram of rule set \( f_1 \), with arrows linking the bias and trigger intensities that are used to their target afterimage intensities. Above, left: the bias image from an image sequence generated by rule set \( f_1 \). Above, middle: the evenly grey trigger image. Above, right: the target afterimage pattern, with \( a_1 \) shown as black, and \( a_2 \) shown as white.

Since we would like shapes in the afterimage to go unrecognized in the bias image also, we introduce a second concept, analogous to trigger ambiguity. We say a rule set \( f \) is bias-ambiguous if:

\[
f_r(b, t) = a \Rightarrow \forall a' \in A \ \exists t' \in I : f_r(b, t') = a'
\]
Our example for this type of rule set will introduce the simultaneous use of different bias-trigger intensity pairs to produce the same afterimage intensity. Starting point here will be the fact that black and white bias intensities can lead to two distinct afterimage intensities by simply remaining constant. This corresponds to a rule set \( f_2 \), shown in the diagram of Figure 6.6, of which the first half is given by \( a_1 = f_2 (0, 0) \) and \( a_2 = f_2 (1, 1) \). To satisfy the ambiguity requirement, the second half will then have to allow each bias intensity to lead to both of the afterimage intensities.

In the case of black, for example, we have to find a \( t \) for which \( a_2 = f_2 (0, t) \). This can be done by varying \( t \) and judging how well the combination of \( f_2 (0, t) \) and \( f_2 (1, 1) \), applied to an all-\( a_2 \) afterimage pattern, succeeds in creating an even impression in the afterimage. Three ranges will then be distinguishable for \( t \): a high range where \( f_2 (0, t) \) will seem lighter than \( f_2 (1, 1) \); a low range where the opposite is true; and a range inbetween, where the afterimage intensities will seem similar, but still may flicker or otherwise remain visually separate. The most stable results in this middle range seemed to be around \( t = 0.87 \), so that we will use \( a_2 \approx f_2 (0, 0.87) \).

For the white case, a similar procedure can be followed, using an all-\( a_1 \) afterimage pattern. This resulted in an \( a_1 \approx f_2 (1, 0.15) \).

Applying our now-complete rule set \( f_2 \) to the same afterimage pattern that we have used before has resulted in the image sequence shown in Figure 6.6, and demonstrated in Video Example III. Here, for each instance of an afterimage intensity, the choice between applicable bias-trigger intensity pairs has been arbitrary, so that the bias image has become a random pattern of black-and-white, which is reflected in slight distortions in the trigger image.

**Figure 6.6 Bias ambiguity.** Left: diagram of rule set \( f_2 \), with arrows linking the intensities that are used. Above, left: the random bias image from an image sequence generated by rule set \( f_2 \). Above, middle: the slightly distorted trigger image. Above, right: the target afterimage pattern, with \( a_1 \) and \( a_2 \) shown as before.

Both of the previous examples have shown in practice how an ambiguity requirement guarantees the unrecognizability of afterimage shapes, in either the bias image or the trigger image. Neither example did however conceal the afterimage.
pattern in the image not subject to ambiguity. We would like to have a rule set \(f\), which is fully ambiguous: both bias-ambiguous and trigger-ambiguous. Unfortunately, such a rule set cannot exist, which we can prove within our framework.

**Proof.** Suppose a rule set \(f\) is bias-ambiguous.

Define the sets \(B\) and \(T\) of \(f\)’s bias and trigger intensities:

\[
B = \{ b \mid b \in I \land (\exists t \in I, a \in A : f_r(b, t) = a) \}
\]

\[
T = \{ t \mid t \in I \land (\exists b \in I, a \in A : f_r(b, t) = a) \}
\]

Then choose

\[
b_{\text{max}} \in B \text{ so that } \forall b \in B : b \leq b_{\text{max}}
\]

\[
a_{\text{max}} \in A \text{ so that } \forall a \in A : a \leq a_{\text{max}}
\]

\[
a_{\text{min}} \in A \text{ so that } \forall a \in A : a \geq a_{\text{min}}.
\]

There must exist a \(t' \in T\) with \(f_r(b_{\text{max}}, t') = a_{\text{max}}\) (bias ambiguity).

However, for this \(t'\), trigger ambiguity will not hold, because

\[
\neg \exists b' \in B : f_r(b', t') = a_{\text{min}}.
\]

**Proof.** Suppose \(\exists b' \in B : f_r(b', t') = a_{\text{min}}\).

Then \(f_a(b', t') < f_a(b_{\text{max}}, t')\) by Prop. (2),

since \(n > 1\) guarantees \(a_{\text{min}} < a_{\text{max}}\).

It then follows by Prop. (1) that \(b' > b_{\text{max}}\),

which is impossible by definition.

Therefore \(f\) cannot be both bias- and trigger-ambiguous. 

This does not have to mean that ambiguity is completely useless to our purposes however: we may still realize a decrease in recognizability in both the bias and the trigger image by informally relaxing requirements to the level of a partial ambiguity, where it suffices that each bias or trigger intensity can lead to more than one of the afterimage intensities.

This is demonstrated by rule set \(f_3\), which again will use black and white as its bias intensities, but this time to produce three afterimage intensities. The construction of \(f_3\), shown in the diagram of Figure 6.7, starts by searching for a \(t_1\) and \(t_2\) for which \(f_s(0, t_1) = f_s(1, t_2)\), and using the result as the middle afterimage intensity \(a_2\). This may be done starting out from a medium grey trigger intensity, as in \(t_1 = 0.5 - d\) and \(t_2 = 0.5 + d\), with \(d\) increasing from 0. Varying \(d\), we can then search for matching afterimage intensities in a process similar to that described for rule set \(f_2\). This seemed to give the most stable impression around \(d = 0.13\), so that \(a_2 \approx f_3(0, 0.37)\) and \(a_2 \approx f_3(1, 0.63)\).

We quickly obtain additional brighter and darker afterimage intensities, in a way that satisfies partial ambiguity, by choosing \(a_1 = f_3(0, 0.63)\) and \(a_1 = f_3(1, 0.37)\).

The rule set defined above will create image sequences in which the parts of the target afterimage pattern using \(a_1\) or \(a_3\) will be unambiguously repeated in the bias and trigger images, using separate greyscale intensities. However, the recognition of parts will be hampered, because the remaining image space, which leads to \(a_2\), uses the same
greyscale values randomly. This is demonstrated in Video Example IV and illustrated in Figure 6.7, using the example target afterimage pattern, which this time has been adapted to use three intensities.

Figure 6.7 Partial ambiguity. Left: diagram of rule set \( f_3 \), with arrows linking the intensities that are used. Above, left and middle: the randomly distorted bias and trigger images from an image sequence generated by rule set \( f_3 \). Above, right: the target afterimage pattern, with \( a_1 \), \( a_2 \) and \( a_3 \) shown as increasingly bright greys.

6.2.4.2 Scrambling rule sets A basic property of human visual perception is that adjacent areas of a similar shade tend to be grouped together and perceived as a shape. We will now introduce another approach to defining rule sets, using this tendency in a subversive manner. As before, the goal is to have shapes recognized in the afterimage not be recognized by normal viewing of the image sequence producing the afterimage.

First, we will need a tool to look at how rule sets reorder intensities, when comparing the bias images they generate to the target afterimage patterns. Given a rule set \( f_r \), we can enumerate the set \( B \) of its bias intensities (defined in Section 6.2.4.1) according to \( b_1 < ... < b_{|B|} \) – just as we have done for the set \( A \) of afterimage intensities from the outset. We then define \( f_r \)'s mapping scheme as a tuple of \( |B| \) subsets from the set \( \{1, ... , |A|\} \), where the \( i \)-th subset consists of all \( j \) for which \( \exists t \in I : f_r(b_i, t) = a_j \).

This means that, reading a mapping scheme from left to right, we find for each bias intensity, from dark to light, the ranks of the afterimage intensities to which it is linked. For example, the mapping scheme for rule set \( f_1 \) is given by (\{2\}, \{1\}), meaning that firstly, its darkest bias intensity is mapped to its lightest afterimage intensity; and that secondly, its lightest bias intensity is mapped to its darkest afterimage intensity (see the diagram of Figure 6.5). As another example, the mapping scheme for rule set \( f_2 \) is (\{1, 2\}, \{1, 2\}); just as it will be for any other bias-ambiguous rule set which uses two bias and two afterimage intensities (see the diagram of Figure 6.6).

Now, suppose we have a target afterimage pattern displaying a shape on a background, with each in one intensity. Both will be repeated in the bias image, in bias intensities according to the rule set used. Arbitrarily choosing one of the bias intensities present for the shape, suppose that it is nearer in brightness to one of the bias
intensities present for the background than to all other intensities also present for the shape. Where adjacent, these two bias intensities will tend to visually group together, distorting the perception of the original shape. A rule set \( f_r \) will have this property for all possible combinations of uniformly tinted shapes if:

\[
f_r(b, t) = a \Rightarrow \forall a' \in A, a' \neq a : \exists b', t' \in I : \[
\begin{align*}
&f_r(b', t') = a' \land \forall b'', t'' \in I, b'' \neq b :
&f_r(b'', t'') = a' \Rightarrow |b - b'| < |b - b''| 
\end{align*}
\tag{3}
\]

However, given a rule set \( f_r \), it may be more intuitive to look at its mapping scheme: if in it, multiple occurrences of the same afterimage intensity are always separated by occurrences of all other afterimage intensities lying strictly inbetween, the above property will hold. Consider for example the mapping schemes \( \{1\}, \{2\}, \{1\} \) and \( \{1\}, \{2\}, \{1\}, \{2\} \). (For an example illustrating the use of \( \{1\}, \{2\}, \{1\}, \{2\} \), please preview how the rule set shown in the diagram of Figure 6.9 links its trigger intensities to its afterimage intensities.) Both of the above mapping schemes correspond to rule sets satisfying the property, and both have a scrambling effect on the shapes of the afterimage, as is illustrated in Figure 6.8. However, the effect of the first mapping scheme is crippled by the fact that it has afterimage intensity \( a_2 \) preceded by bias intensity \( b_2 \) only, which greatly aids shape recognition. We still need to make explicit that a rule set \( f_r \) should produce each afterimage intensity using multiple bias intensities:

\[
f_r(b, t) = a \Rightarrow \exists b', t' \in I, b' \neq b : f_r(b', t') = a
\tag{4}
\]

Figure 6.8 Scrambling effects. Right: the target afterimage pattern. Left: a corresponding bias image using the mapping scheme \( \{1\}, \{2\}, \{1\} \). Middle: a stronger result using mapping scheme \( \{1\}, \{2\}, \{1\}, \{2\} \).

Now, we say a rule set \( f_r \) is bias-scrambling if it has both properties defined in Props. (3) and (4). Also, by repeating the argument for trigger images instead of bias images, we get analogous definitions for trigger intensity mapping schemes and trigger-scrambling rule sets.

Example rule set \( f_r \) will be bias-scrambling as well as trigger-scrambling, using four bias and four trigger intensities. Its construction starts by combining the outer bias intensities, black and white, with the inner trigger intensities, two greys both near medium grey, but still separately distinguishable (this is shown in the diagram of Figure 6.9). More precisely, we choose \( a_1 = f_r(1, 0.52) \) and \( a_2 = f_r(0, 0.48) \), so that each trigger intensity will produce an afterimage intensity on the opposite side of an
intermediate grey. We then finish construction by looking for pairs of identical bias and trigger intensities which also produce $a_1$ and $a_2$. This can be done in a process similar to that described for rule set $f_2$ in Section 6.2.4.1, using all-$a_1$ and all-$a_2$ afterimage patterns. The pairs producing the most stable impressions seemed to be $a_1 \approx f_4 (0.39, 0.39)$ and $a_2 \approx f_4 (0.62, 0.62)$.

This means that both 0.39 and 0.62 will play double roles in $f_4$: on the one hand, as the outer trigger intensities, resulting in the trigger intensity mapping scheme ({1}, {2}, {1}, {2}); and on the other hand, as the inner bias intensities, resulting in the bias intensity mapping scheme ({2}, {1}, {2}, {1}).

The result of this is demonstrated in Video Example V, of which the image sequence is shown in Figure 6.9. We can see that the trigger image has a weakened scrambling effect, due to the relative closeness of its middle two intensities: an outer trigger intensity will visually group almost equally well with both of these intensities. Increasing the difference in brightness between the middle trigger intensities involves a trade-off however, as it means decreasing the difference between afterimage intensities, thus reducing contrast of the afterimage.

![Figure 6.9 Bias-scrambling, trigger-scrambling. Left: diagram of rule set $f_4$, with arrows linking the intensities that are used. Above, left and middle: the scrambled bias and trigger images from an image sequence generated by rule set $f_4$. Above, right: the target afterimage pattern, with $a_1$ shown as black and $a_2$ shown as white.](image)

6.2.4.3 Hybrid rule sets When comparing the ambiguity requirement to the scrambling requirement, it is obvious that the former is stronger in that it really guarantees unrecognizability of the afterimage shape, although this is limited to either the bias image or the trigger image; while the latter, although we have seen it used in both images simultaneously, cannot rule out recognizability. It therefore becomes worthwhile to look at ways of combining both approaches in a single rule set, and we will now discuss two examples of this.

Our first example, rule set $f_5$, will be bias-scrambling and trigger-ambiguous, using four bias and two trigger intensities (shown in the diagram of Figure 6.10). It was found that, when starting out similarly to rule set $f_4$ of Section 6.2.4.2, but with the
trigger intensities near medium grey somewhat further apart, as in $a_1 = f_5(1, 0.57)$, $a_2 = f_5(0, 0.43)$, matching pairs of identical bias and trigger intensities were given by $a_1 \approx f_5(0.43, 0.43)$ and $a_2 \approx f_5(0.57, 0.57)$. This results in a rule set with trigger ambiguity and a bias intensity mapping scheme given by $\{(2), \{1\}, \{2\}, \{1\}$, just as in rule set $f_5$. An image sequence created by this rule set is demonstrated in Video Example VI, and depicted in Figure 6.10.

As it turns out, in the afterimages produced by rule set $f_5$, shape and background are hard to distinguish, seeming of an almost even grey. The afterimage intensities $a_1$ and $a_2$ apparently lie too close to each other to get a good contrast. Instead of trying to improve the current rule set, we will move on to the next, however: It will swap the ambiguity and scrambling requirements, while making sure from the outset that we will get an afterimage contrast similar to that of rule set $f_4$.

Example rule set $f_6$ will be bias-ambiguous and trigger-scrambling, using two bias and four trigger intensities. The first part of its construction is the same as for rule set $f_5$, choosing $a_1 = f_6(1, 0.52)$ and $a_2 = f_6(0, 0.48)$ (shown in the diagram of Figure 6.11). We continue by searching for a $t_1$ with $a_1 \approx f_6(0, t_1)$ and a $t_4$ with $a_2 \approx f_6(1, t_4)$. Here, testing with all-$a_1$ and all-$a_2$ patterns seemed to give the most stable results for $a_1 \approx f_6(0, 0.25)$ and for $a_2 \approx f_6(1, 0.74)$. This results in a bias-ambiguous rule set, with its trigger intensity mapping scheme given by $\{(1), \{2\}, \{1\}, \{2\}$, again as in rule set $f_4$. Figure 6.11 shows an image sequence generated by the rule set, which is demonstrated in Video Example VII.

Although the afterimage intensities of rule set $f_6$ give a better contrast than those of $f_5$, its scrambling suffers from the same weakening effect mentioned for rule set $f_4$, only more so, as the outer trigger intensities now lie further apart. The scrambling effect created by rule set $f_5$ will probably be better, as its middle bias intensities lie relatively further apart.
6.2.4.4 Multi-trigger sequences

Bias-ambiguous rule sets have the property that, once a random bias image is chosen, it may still lead to any possible afterimage pattern, depending on the contents of the trigger image. Using a rule set which is both bias-ambiguous and trigger-scrambling, this can be tried out in practice: Unlike in the similar case for trigger ambiguity, it may be possible to combine a strong bias impression with multiple different trigger images, thereby inducing multiple different afterimage patterns in a single run.

In Video Example VIII, this idea has been implemented by replacing a single trigger image with two images shown in quick succession. The two corresponding target afterimage patterns show the words “hello” and “world”, respectively. (See Figure 6.12.) Here, bias duration has been extended from 20 to 30 seconds, to improve contrast in the second afterimage. Extending duration further seemed to lead to an uncomfortable viewing experience with unstable afterimages. Choosing the duration of the first trigger image also involved a trade-off, between smaller values giving little time for recognition of the first afterimage, and greater values deteriorating the quality of the second. This resulted in a tentative choice for 1.5 seconds. As before, the software in the Electronic Appendix allows free experimentation.

Figure 6.12 Multiple trigger images for a “hello world” example. A sequence generated by rule set $f_6$. Left: the middle section of the bias image (shown twice). Middle: the corresponding sections of the first and second trigger images. Right: the target afterimage patterns ($a_1$, $a_2$ as before).
6.3 Evaluation: pilot experiment

6.3.1 Overview We tested the approach described in Section 6.2, in order to verify that it indeed can be used to automatically display specified 2D shapes in the retinal afterimage. This included verifying that ordinary viewing of the images used does not result in recognition of the specified shapes. To this end, a pilot experiment was conducted, with its participants naive to the visualization method and without prior knowledge of the target afterimage patterns used.

In the experiment, the parameters for visual fixation and rasterization were set to the tentative defaults identified in Sections 6.2.1 and 6.2.2. Then, to determine representative rule sets for use in the experiment, first trigger-ambiguous rule set \( f_1 \) and bias-ambiguous rule set \( f_2 \) (see Section 6.2.4.1) were dropped from consideration, since both trivially allow shape recognition in the image not subject to ambiguity. Of the remaining rule sets, bias-ambiguous, trigger-scrambling rule set \( f_6 \) (see Section 6.2.4.3) seemed a better candidate than bias-scrambling, trigger-scrambling rule set \( f_4 \), since \( f_6 \)'s bias ambiguity guarantees unrecognizability, where \( f_4 \)'s bias scrambling does not. Rule set \( f_6 \) also seemed a better candidate than bias-scrambling, trigger-ambiguous rule set \( f_5 \), since \( f_6 \) was designed to give a stronger afterimage contrast than \( f_5 \). Rule set \( f_6 \) did not seem a necessarily better candidate than the remaining, partially ambiguous rule set \( f_3 \) (see Section 6.2.4.1), however. Therefore, both \( f_6 \) and \( f_3 \) were chosen to represent the approach in the experiment.

Regarding target afterimage patterns, a choice was made to use word shapes, on an even background, as quickly recognizable everyday shapes, presented without distraction. The 2D word shapes were then placed centered, near the crosshair. This reflected the fact that in regular vision, the recognition of letter strings as words happens more quickly when the letters are placed in the center, rather than in the periphery, of visual fixation [Lee et al. 2003].

When using a line thickness of 1 square unit (see Section 6.2.2), it turned out that individual letters in the afterimage sometimes would appear so distorted as to be unreadable. For this reason, mostly, a line thickness of 2 square units was used.

In order to have naive test subjects get used to the visualization method, an introductory sequence, serving as a dry-run, was included. Preliminary testing of rule sets \( f_3 \) and \( f_6 \) using two image sequences based on 3-letter word shapes showed improving afterimage recognition results when extended with two sequences based on 5-letter word shapes. This resulted in the choice to use two sequences based on 5-letter word shapes, preceded by two sequences based on 3-letter word shapes for the pilot experiment.

6.3.2 Procedure The experiment was performed separately with 5 unpaid volunteers between 20 and 24 years old. All test subjects were new to the visualization method. Three of them were female, with two having normal vision, and one being somewhat far-sighted. Of the two males, one had normal vision, and one was near-sighted (with
correction). None of the participants was dyslexic, and all were both willing and able to fully concentrate on the images and image sequences presented to them.

Conditions for the experiment were as described in Section 6.2: Image sequences were viewed on the TFT LCD display of a Packard Bell R3450 laptop computer (resolution 98 dpi), from a distance of approximately 60 cm. The experiment was performed during daytime, in an indoors setting, with indirect daylight and without artificial lighting. When images were shown separately, they were displayed for 20 seconds; when they were shown in sequence, the bias image was displayed for 20 seconds and immediately followed by the trigger image, displayed for 5 seconds. The specific images and image sequences used in the experiment can be found and played back in the Electronic Appendix, which is accessible using any standard web browser.

The experiment started with the chequered, third sequence of Video Example I (see also Figure 6.4), as a dry-run to get used to the visualization method. After test subjects were seated in front of the screen, they were shown the bias image, then asked whether they had recognized a pattern in it, and if so, what pattern. This was repeated for the trigger image. The test subjects were then told they would now see the first image switch to the second, and were asked to concentrate on the moment of the switch. Instructions were given to constantly focus on the crosshair’s center throughout the sequence, while resting one's head on both hands, with the elbows placed on armrests. After the sequence had completed, the same question asked for the separate images was asked again. The attention of the test subjects was then pointed to how the light grey fuzzy squares seemed to light up in the sequence, in which they were preceded by black squares. The sequence was then repeated a few times until the subjects indicated being accustomed to this effect.

After this, four sequences were presented using the same procedure, only now the subjects wrote down their answers in a form, and did not receive feedback anymore. They could however indicate uncertainty about their response, in which case a sequence was repeated until a final answer was settled upon. The first sequence to be presented was generated by rule set $f_3$, based on a 3-letter pattern spelling “red”. The second sequence was generated by rule set $f_6$, based on a 3-letter pattern spelling “low”. After these were completed, the participants had the opportunity to write down any general remarks, and there was a short break. Then, the third sequence was shown, generated by rule set $f_3$, based on a 5-letter pattern spelling “light”. This was followed by the fourth sequence, generated by rule set $f_6$, based on a 5-letter pattern spelling “hello”. Finally, there was another opportunity to write down general remarks.

6.3.3 Results For all test subjects and all image sequences, none of the target afterimage patterns were ever recognized in the separate images. For the scrambled images, no responses came near word shape recognition, apart from one remark for the fourth sequence about “dark letter-like shapes in the center”. For the partially ambiguous images, what came nearest recognition were remarks, sometimes made for the third sequence, about the separate images having a distinctly brighter or darker region in the center. For the ambiguous (i.e. random) images, the responses, although
sometimes richly imaginative (e.g. “a hurricane in some kind of frozen circular motion”) most often were a simple “no”.

As expected from preliminary testing, the results for shape recognition in the afterimage were mixed for the 3-letter sequences. Participants often expressed doubt, and often needed multiple iterations to settle on their answers. In the first sequence, “red” was recognized 3 out of 5 times, and in the second sequence, “low” was recognized 2 out of 5 times.

For the 5-letter sequences, results improved considerably. All test subjects recognized the target words, in both sequences, mostly immediately: In the third sequence, “light” was once reported after four runs (at approximately 50 instead of 60 cm viewing distance); in all other cases, it was reported on the first run. In the fourth sequence, with a different participant being the exception, “hello” was once reported after two runs (and written down with the first letter capitalized); in all other cases, it was reported on the first run. In Figure 6.13, the experimental results for word shape recognition outside and within the afterimage effect are summarized.

![Figure 6.13](image)

**Figure 6.13** Results of the pilot experiment: word shape recognition outside and within the retinal afterimage.

### 6.4 Conclusion

In the pilot experiment, rule sets \( f_3 \) and \( f_6 \) were each used to generate an image sequence based on a 5-letter target afterimage word shape specified in square units. Showing these image sequences to 5 test subjects resulted in none of them recognizing the 5-letter word shapes in the separate bias and trigger images, and all of them doing so in the afterimage effect (see Figure 6.13).

As expected, a similar procedure for 3-letter word shapes showed weaker performance, with word shapes being less often recognized successfully within the afterimage effect. One possible explanation for this, is that the afterimage display of 3-letter word shapes might be less robust than that of 5-letter word shapes in the face of the type of irregularities first identified in Section 6.2.2. If this is the case, it could be a reason for increasing the sophistication of the rasterization method, and its explicitly
naive formal model of induced afterimage intensities. This could be informed by the examination of a range of potential factors influencing the complete or partial disappearance and reappearance of structured afterimages in [Wade 1978]; and also by the study of the influence of selective attention in this, including an effect of filling-in of enclosed regions, in [Lou 2001]; as well as by the study of the influence of post-adaptation contour alignment, where an afterimage may appear with much more stability and intensity if the geometry of contours still present for a previously induced afterimage aligns retinally with the geometry of contours present in the image being viewed, as in [Daw 1962]; and also by the study of how post-adaptation contours influence, and indeed can be used to control, the way in which color filling-in and spreading effects occur in the retinal afterimage [Van Lier et al. 2009].

In general, and also when developing partly automated techniques for other aspects of the retinal afterimage than 2D shape, e.g. textural effects and color (see Section 6.1.4), another important factor to be explicitly taken into account is the presence of induced simultaneous contrast in colors, both during and after adaptation, and the influence this has on the appearance of colors and greys in the retinal afterimage [Anstis et al. 1978]. Also more generally, the use of display technology with integrated eye tracking might support more natural eye movement during retinal adaptation; and the latter could be made to occur faster, by using darker surroundings and brighter stimuli than we have done here.

In any case, the results of the pilot experiment indicate that we have developed an automated method which, for a subset of all specifiable, recognizable 2D shapes, can successfully induce these specifically via the retinal afterimage. Thereby, and in this sense, our method realizes a form of the computed retinal afterimage (see Section 6.1). This also demonstrates feasibility of the computed retinal afterimage in general. Like other visual effects (see Section 6.1.1) the retinal afterimage, too, can be a computed aspect of human visual perception. This gives visual artists interested in the retinal afterimage the advantage of being able to create controlled visual complexity that uses the effect, in less time (see Section 6.1.4).

In the Electronic Appendix (see Section 6.2), we have included a software toolkit allowing complete reproduction of and free experimentation with the approach. The toolkit, released in source format under a public license, has been implemented for the Max programming environment widely used by contemporary artists [Puckette 2002].