THREE-MODE CORRESPONDENCE ANALYSIS

Leiden electorate study

- abstentions
- VVD
- PvdA
- 1981 national
- 1982 provincial
- 1982 municipal
- 89 ward number

56 55
32 16
40 30 50 3
33 32 14 5
49 37 11
29 57
36 37 35 28 33 25 17 19 22 23 24 26 27 42 15
48 54

15
15.1 INTRODUCTION

Contingency tables turn up in many research projects in many contexts, and there exists an extensive collection of techniques for their analysis. Especially in recent years the development of loglinear models for contingency table analysis has enabled researchers to make more detailed statements about association in multi-way tables than just reporting descriptive levels of significance or p-levels. Notwithstanding, or because of, the refined machinery connected with loglinear models there are serious problems with their application to large tables, and/or to higher dimensional tables. These problems centre around (1) the null distribution and power of test statistic when the numbers of observations per cell is low, (2) the difficulty of interpreting the interaction terms when there are very many of them (as is the case with large tables), and (3) the complexity of interpreting high-order interaction terms, especially if there are a lot of observations.

Here we will specifically pay attention to interactions of large three-way contingency tables.

15.2 LOGLINEAR MODELS, INTERACTIONS, AND CHI-TERMS

A saturated model for two-way contingency tables is a model which completely accounts for the data by specifying all effects and interactions. It has the form

$$\log f_{ij} = \ell_{++} + (\ell_{i+} - \ell_{++}) + (\ell_{+j} - \ell_{++}) + \log r_{ij},$$

where $i=1, \ldots, k; j=1, \ldots, m$.
with \( f_{ij} \) the observed cell count; \( \ell_{ij} = \log f_{ij} \); \( r_{ij} \) is the residual and the "+" indicates summation over the index it replaces.

Using Bishop, Fienberg & Holland's (1975) notation, this can be written as:

\[
\log f_{ij} = u + u_1(i) + u_2(j) + u_{12}(ij) + \log r_{ij}.
\]

There are two main effect vectors \( u_1 \) and \( u_2 \), and one two-way interaction matrix \( u_{12} \). The formula for a non-saturated model has a combination of one or more of these terms on the right hand side.

The most common model for a two-way table is the model of independence between rows and columns:

\[
\log e_{ij} = u + u_1(i) + u_2(j).
\]

This model may be tested against the data by assessing the size of the residuals \( r_{ij} = f_{ij} - e_{ij} \) via Pearson's \( \chi^2 \)-test,

\[
\chi^2 = \sum (f_{ij} - e_{ij})^2/e_{ij},
\]

or the -2loglikelihood ratio,

\[
\text{LR} = -2\log(f_{ij}/e_{ij}).
\]

The values of the test statistics are evaluated against percentage points of the \( \chi^2 \)-distribution with \((k-1)(m-1)\) degrees of freedom. Given non-independence, one can inspect the residuals for specific patterns. While these patterns are easily visible in small tables, visually analysing more or less subtle relationships from a large table can become too difficult. In addition, the residuals themselves suffer from differences in size due to original differences in size of the frequencies, and for comparing the residuals it is more appropriate to standardize them in some way. One obvious way is to use standardized residuals or as we, somewhat inappropriately, will call them chi-terms:

\[
X_{ij} = (f_{ij} - e_{ij})/e_{ij}^b.
\]

A more subtle kind of standardization leads to Haberman's adjusted residuals (Haberman, 1976). Here we will deal exclusively with the chi-terms, or \( X_{ij} \).

When confronted with a two-way contingency table a reasonable procedure for analysis can be summarized as follows:
a) construct a model;
b) test for appropriateness of the model (optional);
c) interpret the terms of the model;
d) compute the chi-terms;
e) analyse the chi-terms for specific patterns.

For three-way tables the procedure is essentially the same, but the situation is more complex as there are now three main effects, three two-way interactions and one three-way interaction:

\[ \log f_{ijk} = u_{1(i)} + u_{2(j)} + u_{3(k)} + u_{12(ij)} + u_{13(ik)} + u_{23(jk)} + u_{123(ijk)} \]

Again a model consists of a subset of terms from the right hand side. A simple model is the three-way independence model consisting of \( u, u_1, u_2, \) and \( u_3 \). In this case we obtain the chi-terms:

\[ X_{ij} = \frac{(f_{ijk} - e_{ijk})}{e_{ijk}}, \text{ with} \]

\[ \log e_{ijk} = u_{1(i)} + u_{2(j)} + u_{3(k)} \]

or \[ e_{ijk} = \frac{f_{i++}f_{j++}f_{+++}}{f_{i+j+k}} \]

Looking at these chi-terms or standardized residuals implies inspecting all two-way and three-way interactions simultaneously. One can also use the chi-terms from other loglinear models which include more \( u \)-terms, and we will do so in our example.

15.3 CORRESPONDENCE ANALYSIS FOR CONTINGENCY TABLES

In this section we will first give a short summary of correspondence analysis of two-way contingency tables, and then show how it can be generalized to three-way tables. Further extensions to higher-way tables are possible but will not be discussed here.

Two-way tables. The aim of correspondence analysis (Benzécri, 1976; Gifi, 1981, Ch. 3) for two-way contingency tables is to quantify or scale the column and row categories in such a way that the nature of the interaction both within and between rows and columns becomes directly visible, for example via a combined plot.
of the row and column categories. The way to arrive at such a combined plot is via the singular value decomposition $G \Lambda H'$, (see section 2.2), of the matrix $X$ with chi-terms $X_{ij} = (f_{ij} - e_{ij})/e_{ij}^2$. This procedure has close links with the joint plots considered in section 6.10. The columns of $G$ are the quantifications (or scalings) of the rows, and columns of $H$ are the quantifications of the columns of the table. The combined configuration may be interpreted as follows (see e.g. Israëls, et al., 1981):

a. the origin represents the marginal distributions for both rows and columns, and thus has the same meaning for both, i.e. it is the centre of gravity for each row (column);

b. the distance of a row (column) point to the origin shows to what extent the distribution within that row (column) deviates from the marginal distribution. Thus the origin is the point indicating independence between the row and column variables;

c. the direction emanating from the origin indicates the kind of deviation; when row (column) categories are close together their conditional distributions resemble each other;

d. when rows and columns deviate in opposite directions they are negatively related;

e. the size of the deviation from independence is indicated by the distance from the origin.

Central to these interpretations are the ideas of distance, direction, and the role of the centre of the configuration. For instance, Gifi (1981, p. 134-137) gives precise definitions of the distances both in terms of deviations from the marginal proportions, and in terms of "$\chi^2$-distances" between rows (columns) of the contingency table. The role of the origin, as the point representing the marginal distributions and as the point indicating independence, can be seen directly from the fact that the analysis is performed on the matrix of chi-terms rather than on the original frequencies.

Three-way tables. The generalization to three-way tables is based on the observation in section 2.2 that three-mode principal
component analysis is a generalization of singular value decompo-
sition. A three-mode principal component analysis on a matrix of
chi-terms derived from some three-way contingency table is, there-
tore, a three-way analogon of correspondence analysis, as was
pointed out by de Leeuw (1981, pers. comm.). Another kind of appli-
cation of multi-mode models to contingency tables can be found in
Carroll (1975), Green, Carmone, & Wachspress (1976), Carroll,
Pruzansky, & Green (1977). They used the CANDECOMP procedure (Car-
roll & Chang, 1970; see also section 3.3) to estimate the param-
ters of Lazarsfeld's latent class model (Lazarsfeld & Henry, 1968).
In Carroll et al. (1977) it was shown that their procedure is also
a generalization of correspondence analysis.

It is possible to make another fruitful generalization within
the context of correspondence analysis to three-way tables. One may
analyse a table of chi-terms not only from an independence model,
but from any model. Deviations from the origin are then interpreted
as deviations from this model.

It is not the intention to develop the mathematics of this
proposal. Instead, we will show via an example of the voting beha-
vior in Leiden (the Netherlands) during three elections how the
procedure works.

15.4 LEIDEN ELECTORATE STUDY: DATA

In the Netherlands there are three kinds of elections: na-
tional, provincial and municipal, which follow each other mostly at
irregular intervals. Although different issues play different roles
in these elections, each has a national overtone. Popularity and
impopularity of political parties at the national level invariably
have their impact on the local elections, and on the other hand,
results from local elections are seen as a measure for the popula-
arity of parties on the national level.

The data for the present study consist of the results for the
wards of Leiden in these three different kinds of elections: the
1981 national parliamentary elections, the March 1982 provincial
elections, and the June 1982 municipal elections. Data are avail-
able for the 58 wards or precincts of the city and 9 parties or combinations of parties who participated in all 3 elections. Van der Heijden (1982) analysed the first two elections (with 10 rather then 9 parties) using correspondence analysis as implemented in ANACOR (Gifi, 1981 (Ch.3), 1982). In particular, he performed regular correspondence analysis on the three-way table by rearranging the table as an (58+58)×10 table, a 58×(10+10) table, and an (incomplete) (58+10+2)×(58+10+2) bi-marginal table. See also Gifi (1981, section 4.4.4) for similar correspondence analyses on three-way tables. Here we will analyse the 58×9×2 table in its three-way form. The three zero cells of the 1982 municipal elections were set equal to one to facilitate comparison with other analyses.

15.5 LOGLINEAR ANALYSIS

Before embarking on a three-mode correspondence analysis, it is useful to investigate first a three-way contingency table with loglinear models. This kind of analysis gives insight in the relevance of the various interactions present in the data, and makes it possible to decide which interactions should be investigated further.

Only those models are appropriate or permissible for the Leiden electorate data which include the ward, election, and ward × election effects, as their marginal distributions are fixed by the design (see e.g. Fienberg, 1980, p.95,96 for a discussion of margins fixed by the design), as the size of the wards and the total number of eligible voters in each election are known a priori. As, in addition, the sizes of the wards in each election were almost the same, the variables ward and election are independent as well. According to Theorem 2.4-1 of Bishop, Fienberg & Holland (1975, p.39; see also Fienberg, 1980, p.49) this implies (together with the negligible three-way interaction; see below) that we may collapse either over wards to investigate the party×election interaction, or over elections to investigate the party×ward interaction. In other words, inspecting and interpreting these two-dimensional margins is all that is necessary for these data, possibly
with a quick look at the three-way interaction to make sure everything is in order there.

All possible models were generated by BMDP4F (Dixon, 1981, p.143ff), and for each the -2loglikelihood ratio statistic with the appropriate degrees of freedom was determined. In Fig. 15.1 we have summarized the results of all possible hierarchical models, and in Table 15.1 we have given some details for the permissible models.

Fig. 15.1 Leiden electorate study: Fit of loglinear model

The huge values of the test statistic ensure the significance of each and every model, i.e. no model fits the data well. With a total of roughly 232 thousand observations and 1566 cells this is hardly surprising. The large number of observations even ensures that uninteresting small differences between models lead to significant test results, especially between the levels indicated in
Table 15.1  Leiden electorate study: permissible loglinear models

<table>
<thead>
<tr>
<th>models</th>
<th>-2 loglikelihood ratio (in thousands)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>degrees of freedom</td>
</tr>
<tr>
<td>W+E</td>
<td>1480</td>
</tr>
<tr>
<td>WE</td>
<td>1368</td>
</tr>
<tr>
<td>P+E+WE</td>
<td>1360</td>
</tr>
<tr>
<td>PE+WE</td>
<td>1344</td>
</tr>
<tr>
<td>PW+WE</td>
<td>912</td>
</tr>
<tr>
<td>PW+PE+WE</td>
<td>896</td>
</tr>
<tr>
<td>PWE</td>
<td>0</td>
</tr>
</tbody>
</table>

Note: The models are designated by the highest order interactions present. Implicit in the notation is the inclusion of the hierarchical lower-order interactions, e.g. WE indicates the model W + E + WE.

Fig.15.1. In other words testing is an uninteresting, but especially unnecessary exercise in this case. What is interesting, is the relative magnitudes of the -2loglikelihood ratios themselves.

Fig. 15.1 shows that the major effects are party, party\xward, and party\xelection. It also shows that the three-way interaction can be neglected, i.e. set to zero, for most practical purposes. In other words, we may here assume that the three-way interaction reflects random variation. After all, it takes 896 degrees of freedom to reduce the test statistic from 1264 to zero, when going from the model including all two-way interactions to the model which also includes the three-way interactions, i.e. the saturated model.

Inspecting the ward\xparty (58\times9) two-dimensional margin is clearly something that cannot be done properly without an adequate graphical representation. In fact an ordinary two-way correspondence analysis on the party\xward margin is already sufficient (see Van der Heijden, 1982). A visual inspection of a table of the party\xelection interaction is, on the other hand, quite feasible as Table 15.2 shows. The most salient features are the decline of the PvdA (and the CDA and D'66 to a lesser extent), the relative stability of the VVD, and the other smaller parties, and the 150%...
Table 15.2 Leiden electorate study: Model III: party-election interaction

<table>
<thead>
<tr>
<th>party</th>
<th>Average number of votes per party over all wards</th>
<th>standardized residuals (obs-exp)/sqrt(exp)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PvdA</td>
<td>404</td>
<td>245</td>
</tr>
<tr>
<td>CDA</td>
<td>221</td>
<td>180</td>
</tr>
<tr>
<td>VVD</td>
<td>211</td>
<td>207</td>
</tr>
<tr>
<td>Abstentions</td>
<td>187</td>
<td>468</td>
</tr>
<tr>
<td>D'66</td>
<td>146</td>
<td>93</td>
</tr>
<tr>
<td>Small left</td>
<td>114</td>
<td>113</td>
</tr>
<tr>
<td>Small right</td>
<td>20</td>
<td>21</td>
</tr>
<tr>
<td>Invalid</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>Soc. Party</td>
<td>8</td>
<td>10</td>
</tr>
</tbody>
</table>

Note: small left: CPN, PSP; small right: BP, GPV, SGP

increase in abstentions. After the provincial and municipal elections the abstainers were by far the largest party, and the Socialist P arty also increased its support considerably. Furthermore, in absence of a sizeable three-way interaction, the conclusion may be drawn that the abstainers came from especially the PvdA wards. The great decline of the PvdA in the provincial, and the slight increase in the municipal elections follow the popularity of this party on the national level quite closely.

For the purpose of illustrating three-mode correspondence analysis, and in order to compare its results with the loglinear results we will investigate the chi-terms (or standardized residuals) of a number of models via this technique. This does not imply that for these data all the models should be investigated in this way, if it was the subject matter which was of primary importance. We will, however, investigate the chi-terms from the models I, II, and III in Table 15.1.

Determining an appropriate model for the data, and inspecting the chi-terms or standardized residuals from such a model is, by the way, directly analogous to the problem of determining the appropriate centering for numerical data treated in detail in Chapter 6.
15.6 MODEL III: POLITICAL ALIGNMENT OF THE CITY OF LEIDEN

In this section we will look especially at the party×ward interaction. In other words, we want to find out how people in the various wards voted. This can be accomplished by looking at the party×ward margin, i.e. summed over elections (external averaging), or by using Model III:

\[
\log e_{ijk} = u + u_{W(i)} + u_{P(j)} + u_{WE(ik)} + u_{PE(jk)}, \text{ or}
\]

\[
e_{ijk} = \frac{f_{i+k} f_{j+k}}{f_{++k}},
\]

with \(W=\) ward, \(P=\) party, and \(E=\) election. In this case only the party×ward interaction, \(u_{WP(ij)}\), (i.e. which parties acquire their votes from which wards) and the three-way interaction \(u_{WP_E(ijk)}\) remain to be analysed via a correspondence analysis of the chi-terms. By looking at the joint plot from a Tucker2 analysis using the average

Fig. 15.2 Leiden electorate study: Model III - party×ward interactions (labelled by ward number)
core plane (internal averaging) we can display the relationships between the wards and the parties (Fig. 15.2). This plot can be interpreted using the guidelines given in section 15.3.

Invalid votes occur apparently randomly over the city, and are therefore located near the zero point of the plot. Wards 55 and 56 both vote predominantly VVD, the right-wing conservative party, more than the Leiden average. The inner city (wards 1 through 8) votes more than other wards for the small left-wing parties, but, in addition, the VVD receives an excess of votes in ward 1, the PvdA in ward 3. Similarly, in a typical labour district such as 'De Kooi' (wards 18 to 26) the labour party PvdA receives more votes than its marginal distribution would predict. A number of interesting details can be discerned, but these are probably only meaningful for people familiar with Leiden itself, and they will not be covered here.

Fig. 15.3 Leiden electorate study: Model III - real estate values and party alliances (in thousands of guilders; aug. 1982)
In Fig. 15.3 the wards are labelled according to the estimated average value of the real estate in these wards. From this plot it is clear that average real estate value is a reasonable global predictor of voting behaviour, but it should be kept in mind that in some wards the variance of real estate values is very large.

So far we have concentrated on the party×ward interaction, and have disregarded the three-way interaction on the assumption that it was not overly large. From the TUCKALS2 core matrix (Table 15.3) we are able to assess, whether this is indeed a valid assumption. In all elections the ward×party relationships turn out to be virtually the same, as do the relative SS(Fit) of each election, and thus the lack of a substantial three-way interaction is confirmed.

Table 15.3 Leiden electorate study: T2 core matrix for Model III

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>components for:</td>
<td>1 2</td>
<td>1 2</td>
<td>1 2</td>
<td>parties 1 2</td>
</tr>
<tr>
<td>wards 1</td>
<td>-18 0</td>
<td>-17 1</td>
<td>-18 1</td>
<td>-18 0</td>
</tr>
<tr>
<td>wards 2</td>
<td>0 11</td>
<td>-1 10</td>
<td>1 10</td>
<td>0 10</td>
</tr>
<tr>
<td>Relative SS (Fit)</td>
<td>.78</td>
<td>.79</td>
<td>.79</td>
<td>Overall=.79</td>
</tr>
</tbody>
</table>

15.7 MODEL II: DECLINE OF SUPPORT FOR THE PvdA

As mentioned before, the party×election interaction can easily be inspected from the two-dimensional margin as we did in Table 15.2. For illustrative purposes, however, we very briefly report the analysis of chi-terms from Model II:

\[ \log e_{ijk} = u + u_W(i) + u_P(j) + u_E(k) + u_{WP(ij)} + u_{WE(ik)} \], or

\[ e_{ijk} = f_{ij} f_{i+k} / f_{i+} \]

with again W=ward, P=party, and E=election. The chi-terms contain the two-way interaction between party and election, and the three-
way interaction, as they are the only interactions not included in the model. In a TUCKALS2 analysis, the unimportance of the three-way interaction now should follow in a TUCKALS2 analysis from the unidimensionality of the ward space, with all entries approximately equal to $1/\sqrt{58} = 0.13$. The real average of the first component was also 0.13 with a standard deviation of 0.03. All wards were within ± 2 standard deviations from the average. The overall SS(Fit) was 0.940, and that of the first ward component 0.933, confirming the uni-dimensionality of the ward space.

Before reporting the results for the party×election interaction we will show the consequence of the uni-dimensionality of the ward space (G), and the equality of all $g_i$:

$$
\hat{z}_{ijk} = \sum_{p=1}^{s} \sum_{q=1}^{t} g_{ipjq} = \sum_{q=1}^{t} \sum_{p=1}^{s} g_{ipjq} = \sum_{q=1}^{t} \sum_{j=1}^{h} \sum_{q=1}^{t} \sum_{p=1}^{s} g_{ipjq} \Rightarrow \alpha_{ijk} = \alpha_{ijq} = \alpha_{ijqk}
$$

in other words, for each ward $i$ we obtain the same values for the party×election interaction. Adjusted with a constant scaling factor these values are given in Table 15.4.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>PvdA</td>
<td>6.3</td>
<td>-3.4</td>
<td>-2.9</td>
</tr>
<tr>
<td>CDA</td>
<td>2.2</td>
<td>-0.9</td>
<td>-1.3</td>
</tr>
<tr>
<td>VVD</td>
<td>0.7</td>
<td>0.2</td>
<td>-0.8</td>
</tr>
<tr>
<td>Abstentions</td>
<td>-9.9</td>
<td>3.9</td>
<td>6.0</td>
</tr>
<tr>
<td>D66</td>
<td>5.1</td>
<td>-0.7</td>
<td>-4.4</td>
</tr>
<tr>
<td>Small left</td>
<td>0.3</td>
<td>0.0</td>
<td>-0.3</td>
</tr>
<tr>
<td>Small right</td>
<td>0.1</td>
<td>0.4</td>
<td>-0.5</td>
</tr>
<tr>
<td>Invalid</td>
<td>0.2</td>
<td>0.4</td>
<td>-0.6</td>
</tr>
<tr>
<td>Soc. Party</td>
<td>-1.7</td>
<td>-1.2</td>
<td>2.8</td>
</tr>
</tbody>
</table>

Note: The values supplied by the program were divided by 2.3.

Comparison with the standardized residuals in Table 15.2 shows that the two are identical.
15.8 MODEL I: SIMULTANEOUS ANALYSIS OF ALL NON-FIXED INTERACTIONS

Model I is the independence model, i.e. it postulates that the expected values of the cell counts can be reconstructed from the one-dimensional margins, and all fixed margins. This results in independence between party and ward x election. Model I has the form:

\[ \log e_{ijk} = u + w_{i} + u_{P(j)} + u_{E(k)} + u_{WE(ijk)}, \]

or

\[ e_{ijk} = e_{i+k+j+k}^{i-j+k} \]

with W=ward, P=party, and E=election. The chi-terms for this model consist of the ward x party and the party x election, and the three-way interactions. If we neglect the three-way interaction, the correspondence analysis of the chi-terms allows us a simultaneous analysis of the most important interactions in the data.

Let us first inspect the party space (Fig. 15.4) independently of the wards and the elections, as a proper interpretation of its structure is crucial for the remainder. The space is dominated by the position of the VVD, the PvdA, and the abstentions. In other words these are the 'parties' with conditional distributions substantially deviating from those expected under Model I. From the previous analyses we know the party space to be a resultant of the party x ward and the party x election interactions. However, from this party space alone we may conclude that the VVD, PvdA, and the abstentions are all negatively related and in the same way, as the angles between their vectors emanating from the origin are all roughly equal, i.e. 120°. In Fig. 15.4 the first component of the party space for Model II and Model III are drawn on the basis of a pro-custes rotation (see e.g. Gower, 1975).

### Table 15.5

<table>
<thead>
<tr>
<th>Leiden electorat study: TUCKALS2 core planes</th>
<th>elections</th>
</tr>
</thead>
<tbody>
<tr>
<td>components for:</td>
<td>1</td>
</tr>
<tr>
<td>wards</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
</tbody>
</table>
Fig. 15.4 Leiden electorate study: Model I - party space

The angles between the first components of Models I, II, and III (computed from separate procrustes rotations) are $\alpha_{1,II} = 29^\circ 6'; \alpha_{1,III} = 31^\circ 8'; \alpha_{II,III} = 49^\circ 9'$. The correlations between the first components after the separate rotations are $r_{1,II} = .77; r_{1,III} = .82; r_{II,III} = .45$.

The TUCKALS2 core matrix (Table 15.5) shows how for each election the configurations of parties and wards are connected. Note that the minus signs indicate that the components of the wards and parties are inversely related.

If we neglect the two smaller values in each frontal plane, we can see the dramatic change which took place between the national elections of 1981 and the provincial elections in 1982. Whereas in 1981 party component 1(2) was related to ward component 2(1), in the 1982 elections the situation is completely reversed. To see what this means we will produce a combined joint plot for the three
elections (Fig. 15.5). In this plot the most influential parties are shown as directions in the ward space. The directions were produced by postmultiplying the party space with each core planes in turn, and interpreting the resulting coordinates as vectors in the ward space.

Fig. 15.5 Leiden electorate study: Model I – changing party alliances
The conclusions from Model I are the same as those from Models II and III combined, but the graphical representation in Fig. 15.5 makes it easier to formulate certain conclusions. It can be seen that the VVD has a stable electorate, e.g. in wards 56, 33, 55, 16, 1 etc. the VVD scored above expectation in all three elections, and in 20, 24, 21, etc. it scored below expectation.

Abstention in the national elections of 1981 is almost unrelated with party preference in the wards for the VVD and PvdA, as its direction in 1981 is nearly orthogonal to the main axis of the wards. In addition, abstention was below expectation. This reflects the very high abstention rate in the other two elections; after all the interaction effects should sum to zero.

Abstention in the provincial elections of 1982 is very strongly related to PvdA-voting in 1981. In other words, a high proportion of the wards which scored above expectation for the PvdA in 1981, abstained above expectation for the provincial elections in 1982. The same is true for the municipal elections, only less so. There is a considerable projection of the absentions in the municipal elections in the direction perpendicular to the main ward axis, indicating that also for other than PvdA wards abstention was above expectation in that election, but unrelated to the party preference of the wards. Note, by the way, that no statements can be made in terms of individual voters, as we have only the data available aggregated at the ward level.

The projections of all wards on the common PvdA82 direction are small, but generally higher for those wards which voted PvdA in 1981. Note, however, that only thirteen wards scored above expectation on this vector, so that the most stable support for the PvdA can be found there.

15.9 CONCLUSION

From the above discussion and example it can be seen that three-mode principal component analysis can be fruitfully used to study interactions in large three-way contingency tables. At the same time it is clear that it is helpful, and may be essential, to
precede the analysis by a loglinear analysis to gain insight in the importance of the various interaction effects, and to decide for which model the chi-terms have to be evaluated. Without such insight it is difficult to attribute the observed interactions to particular effects.

In a sense the example is easier to interpret than data with a sizeable three-way interaction. On the other hand, particularly in the case of three-way interactions a three-mode correspondence analysis has much to offer that is not easily available in other techniques.