Observation of the optical analogue of the quantised conductance of a point contact

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The light power transmitted by a diffusively illuminated slit of finite thickness is observed to depend stepwise on the slit width. The steps have equal height and a width of one half the wavelength of the monochromatic light used. This novel diffraction phenomenon is the analogue of the quantization of the conductance of a point contact in a two-dimensional electron gas. In contrast to the electronic case, absorption at the walls of the slit plays an important role in determining the shape of the steps, as we show from a model calculation.

1. Introduction

Diffraction of light by an aperture is an easily observed and widely known manifestation of the wave nature of light. As a direct consequence of this diffraction, the transmission cross-section $\sigma$ of an aperture for an incident plane wave differs from its geometrical area $A$. The relation between $\sigma$ and $A$ is a function sensitive to the detailed properties of the aperture [1–4].

Recently, it was pointed out that this relation is remarkably simplified for the case of diffuse (i.e. isotropic rather than plane-wave) illumination [5]. It was predicted that $\sigma$ increases with $A$ in a series of steps of equal height. A similar simplification occurs for two-dimensionally diffuse illumination of a slit, in a plane perpendicular to the slit. The transmission cross-section per unit length of the slit, $\sigma'$, is predicted to increase stepwise as a function of its width $W$. The steps occur whenever $W = n\lambda/2$, with $n = 1, 2, 3, \ldots$, i.e. when a new mode is enabled in the slit. The diffuse illumination is required to couple equally to all modes [5].

The optical transmission characteristics of a slit have been studied extensively for plane wave illumination [6–9]. It was but recently, that the first observations of the discretised transmission cross-section for diffuse illumination were reported [10], analogously to the discretised conductance of a quantum point contact [11, 12]. In this paper we summarise our findings and discuss some (not previously published) calculations on the influence of absorption on the shape of the transmission steps.

2. The experiment

We did the experiment at a wavelength of 1.55 $\mu$m. The set-up is presented schematically in fig. 1. The device consists of two halves of an integrating sphere (40 mm diameter) made of aluminum and coated with diffusively scattering barium sulfate. The slit is at the top of the sphere where the metal is only 25 $\mu$m thick. Inside the slit, the aluminum is covered with silver to obtain a high reflection coefficient, which is required to avoid destruction of the transmission staircase by excessive absorption at the walls of the slit. The transmitted light was collected by the integrating sphere and detected. The slit width was varied by a piezo-electric transducer, and was monitored by a Michelson interferometer.

The laser beam was expanded by a microscope objective and scattered by a diffusor. Diffuse illumination in two dimensions only (no propagation in the direction parallel to the slit) is crucial to the experiment since a slit rather than an
aperture was used. Due to the large bandwidth of the laser (15 nm), the illumination was essentially incoherent.

The experimental results are presented in fig. 2, which shows the transmitted power as a function of the slit width. Trace (a) was obtained using a paper diffuser and two slits in order to make the light diffusive in a plane only. Trace (b) was obtained using a diffuser made of very many parallel glass fibres [13]. Because the latter method is intrinsically two-dimensional, it produces a higher illumination intensity, and thus a better signal-to-noise ratio. A stepwise increase of the transmitted power is clearly observed in both traces. The steps occur at $\lambda/2$ intervals in $W$, as predicted [5]. We also see that all steps have an equal height, implying that each mode transmits the same power. Because for large slit widths ($W/\lambda \to \infty$) $\sigma'$ is equal to $W$, the steps in $\sigma'$ must be equal to $\lambda/2$, the size of the intervals in $W$.

3. The shape of the steps

The steps in the transmission cross-section are not abrupt. Partly, this is caused by non-uniformities in the slit width. Another cause is the slight absorption of radiation at the walls of the slit, which remains in spite of the use of a silver coating. The resulting damping of the propagating modes [13] causes a rounding of the steps and a slight curvature of the staircase for the first few steps visible in trace (a). Rounding of the steps is also partly due to non-adiabatic coupling (with inter-mode scattering) between the narrow slit and the infinite space [14].

The polarisation (the direction of the electric field) of the (two-dimensionally) diffusive light can be chosen to be either parallel (TE mode) or perpendicular (TM mode) to the direction of the slit. The attenuation of light in the slit for the TE and TM polarisation differs significantly. This absorption results from the penetration of the electric field in the (finitely conductive) metal. For a TM mode, the field perpendicular to the slit is constant, but for a TE mode the field is (in the ideal case) a sine (see fig. 3), and thus has much more field energy in the slit than in the conductors. Hence the attenuation of the TM modes will be much larger than for the TE modes.

The effect of absorption on the shape of the steps does not play a role in the electronic
counterpart, and has therefore not been investigated previously. To study this effect we will now calculate the attenuation of the TE modes in a lossless dielectric between two infinitely large conducting plates at a distance $W$, starting from Maxwell's equations [15]. The wave equation is

$$\nabla^2 E(x, y, z) = -\mu \varepsilon \omega^2 E(x, y, z),$$

(1)

which also holds for the magnetic field $H$, and where the time dependence $\exp(i\omega t)$ has already been accounted for by insertion of $i\omega$ for the operator $\partial / \partial t$. We identify $\mu \varepsilon \omega^2 = k^2 = k_c^2 + k_v^2 + k_y^2$, with $k$ complex. With $\varepsilon_d$ and $\varepsilon_c = \varepsilon_c' + i\varepsilon_c''$ the permittivity of the dielectric and conductor, respectively, we have $\mu_0 \varepsilon_0 \varepsilon_d \omega^2 = k_d^2$ and $\mu_0 \varepsilon_0 (\varepsilon_c' + i\varepsilon_c'') \omega^2 = k_c^2$.

The propagation constants in the conductor and the dielectric should be matched, so $k_c = k_y = k_z = 0$, and because we use a plane wave propagating in the $z$-direction, $k_y - k_x \equiv k_x = 0$.

We find that $k_c^2 = k_d^2 = -k_z^2 = -k_y^2 = k_y^2 = k_v^2$.

After putting the proper boundary conditions on the interface of the conductors and dielectric at $y = \pm W/2$ we eventually find

$$\frac{\pm k_y W \exp(-ik_y W)}{[1 \pm \exp(-ik_y W)]^2} = \mu_0 \varepsilon_0 \omega^2 (\varepsilon_c' - \varepsilon_d + i\varepsilon_c''),$$

(2)

where the $\pm$ sign selects between the even and odd TE modes. In a cavity without loss we find $k_y W = n\pi$. If we now use that $k_x^2 + k_y^2 + k_z^2 = k_d^2$, with $k_x = 0$, $k_z = k_z = k', + ik''$, and $k_d^2 = \varepsilon_d k_v^2$, we find the complex wave vector in the propagation direction

$$k_z + ik'' = \sqrt{\varepsilon_d k_v^2 - k_y^2}$$

(3)

The absorption for the intensity of the $n$th mode corresponding to this wave vector in a guide of length $L$ is then given by

$$T_n = \exp(-2k'' L)$$

(4)

In Fig 4 the calculated total transmission $T = \sum_n T_n$ is shown versus the width $W$ for the TE modes of a parallel-plane waveguide. We used the dielectric constant of silver [16] at a wavelength $\lambda = 25 \mu\text{m}$ and $\lambda = 1.55 \mu\text{m}$.

The rounding of the steps visible in Fig 4 is due entirely to absorption, since the rounding due to the intermode scattering at entrance and exit of the slit [14] has been neglected in this calculation.

4. Discussion

In conclusion, we have reported the observation of the optical analogue of the conductance quantization of a point contact. We calculated
the rounding of the steps resulting from absorption in the case of TE polarisation.

It is remarkable that this optical phenomenon, with its distinctly 19th century flavour, was not noticed prior to the discovery of its electronic counterpart. There is an interesting parallel in the history of the discovery of the two phenomena. In the electronic case, the Landauer formula

\[ G = \frac{e^2}{h} \sum_{n=1}^{N} T_n, \]  

was already known before the quantised conductance of a point contact was discovered. The reason that this discovery came as a surprise, was that the relation \( G = (e^2/h)N \) (following from the Landauer formula for \( T_n = 1 \)) was regarded as an order of magnitude estimate [17]. In order to have true quantisation, the relative error in this estimate must be smaller than \( 1/N \), which at that time was not obvious.

The equivalent of the Landauer formula in optics for the transport of electromagnetic modes has been known for a long time. It is interesting to see that also in this field it was not noticed that the relation \( T = N \) holds with a better than \( 1/N \) accuracy. This is particularly apparent in, for example, a paper by Snyder and Pask [18], where they expect the relation \( T \approx N \) to hold only in the geometrical optics limit, \( \lambda \to 0 \).

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References