QUANTUM BALLISTIC ELECTRON TRANSPORT IN A
CONSTRICTED TWO-DIMENSIONAL ELECTRON GAS

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We present an experimental and theoretical study of electron transport in constricted geometries, defined in a high mobility two-dimensional electron gas (2DEG). In zero magnetic field, the conductance of single point contacts, defined by a lateral depletion technique, changes in quantized steps of $2e^2/h$, when the width is varied. This quantization, which persists in a magnetic field, is shown to result from the ballistic transport through the point contact, in which one-dimensional subbands are formed. Electron focusing has been observed in a double point contact geometry, showing ballistic and phase coherent transport along the boundary of a 2DEG. A description of the focusing is given in terms of a non-local voltage measurement.

INTRODUCTION

Electron transport in low dimensional systems has been studied predominantly in the diffusive regime, where the elastic mean free path $l_e$ is smaller than the dimensions of the system. In this diffusive regime, quantum effects in the conductance at low temperatures may manifest themselves as localization, the constructive interference of back scattered electron waves, or as aperiodic oscillations as a function of magnetic field, known as universal conductance fluctuations. The observation of these effects is directly related to the fact that phase coherence is not destroyed by elastic scattering, and may extend on a scale far beyond the mean free path $l_e$ between impurity scattering.

We have studied electron transport in the ballistic regime, where electrons are scattered (or reflected) at the boundaries of the conductor only. This has been achieved by defining submicron geometries in the two-dimensional electron gas of high-mobility ($l_e=8.5 \mu m$) GaAs/AlGaAs hetero structures. The two-dimensionality of the electron transport in hetero structures makes these systems ideal starting points for the study of electron transport in constricted geometries. Confinement in two directions can be obtained by fabricating narrow wires in a 2DEG, either with etching techniques, or by lateral depletion, using a split-gate on top of the hetero structure. The large Fermi wave length
\( \lambda_f \) (typically 40 nm) makes these systems attractive for the study of quantum transport. Because of the lack of impurity scattering in ballistic systems, localization and universal conductance fluctuations are suppressed. The quantum size effects, arising from the lateral confinement of electrons, are therefore preferably studied in a ballistic system.

We give a survey of our experimental and theoretical study of electron transport in narrow and short constrictions, through which quantum ballistic transport occurs. With a double point contact geometry, the coherent ballistic electron transport along the 2DEG boundary has been investigated.

**QUANTUM BALLISTIC TRANSPORT IN SINGLE POINT CONTACTS.**

The inset of Fig. 1 shows a schematic layout of the samples used to study electron transport in narrow and short constrictions. By means of electron beam and optical lithography a split gate is fabricated on top of the hetero structure, having a width of 250 nm between the gate electrodes. The point contact is defined by depleting the electron gas underneath the gate. At \( V_g = -0.6 \text{V} \) the electron gas underneath the gate is fully depleted and a constriction with \( W \approx 250 \text{ nm} \) is formed. A further reduction of the gate voltage narrows the constriction until it is fully pinched off at \( V_g = -2.2 \text{V} \). Ohmic contacts, between which the resistance can be measured, are attached to the wide regions.

![Fig. 1 Conductance quantization in zero magnetic field. The inset shows a schematic layout of the sample. The depletion regions around the gates are indicated.](image)

In the experiments the resistance between the ohmic contacts is measured as a function of gate voltage, both in the absence and presence of a magnetic field. The result for zero magnetic field\(^3\) is given in Fig. 1. Similar results have been obtained independently by
Wharam et al. The conductance of the point contact is shown, obtained from the measurement after subtraction of a constant series resistance. A sequence of plateaux is observed, where the conductance is quantized in multiples of \(2e^2/h\).

To interpret these results, we model the constriction region as a narrow channel, in which the electrons are confined by an electrostatic potential \(eV(x) = \frac{1}{2} m \omega_0^2 x^2 + eV_0\). As a result of this confinement, one-dimensional subbands are formed with subband separation \(\hbar \omega_0\). The narrow channel may now be envisaged as an electron wave guide with dispersion relation:

\[
E_n = \left(n - \frac{1}{2}\right) \hbar \omega_0 + \frac{\hbar^2 k_y^2}{2m} + eV_0
\]

The modes, or 1D subbands, in which the electron waves propagate are indexed with \(n\) \((=1,2,3,...)\) and \(k_y\) denotes the wave vector along the channel. In Fig. 2 the subband occupation is shown for two different values of the gate voltage.

![Subband occupation for two different gate voltages. A decrease in gate voltage reduces the number of occupied subbands.](image)

In the wide 2DEG regions the electron states are occupied to the Fermi level \(E_{FB}\) (=12.5 meV). A reduction of the gate voltage increases both \(\omega_0\), which is a measure of the lateral confinement, as well as \(eV_0\), the electrostatic energy in the constriction. As shown in Fig. 2, both result in a reduction of the number of occupied subbands \(N_c\).

By applying a voltage \(V\) over the constriction, the right and left-going states are populated to different electrochemical potentials \(\mu_1\) and \(\mu_2\). As is clear from fig. 2, the net current \(I\) results from the energy interval \(eV = \mu_1 - \mu_2\), in which only right-going states are occupied. The conductance can now be evaluated:
Essential for the quantization is the energy and subband index independence of the product of spin-degenerate 1D density of states \( N_n(E) = \frac{2}{\pi} \frac{dE_n}{dk_y} \) and the velocity \( v_n(E) = \frac{1}{h} \frac{dE_n}{dk_y} \), which gives the result:

\[
G_c = \frac{N_c}{\sum_{n=1}^{N_c} \frac{2e^2}{h} = \frac{2e^2}{h} \text{Int} \left( \frac{E_F}{\hbar \omega_0} + \frac{1}{2} \right)}
\]

In the classical limit \( N_c \gg 1 \), this equation describes a Sharvin contact resistance. Eq. 3 can also be viewed as a direct consequence of the quantum mechanical Landauer formula, applied to the case of a perfect conductor. The resulting finite conductance was first identified as a quantum contact resistance by Imry. Eq. 3 predicts a contact resistance of \( 2e^2/h \) per occupied quantum channel. The conductance increases stepwise whenever the Fermi level \( E_F \), controlled by the gate voltage, reaches a new 1D subband. It must be noted, however, that Eq. 3 has been derived for an infinitely long channel. The actual constrictions are not only narrow but also short (the length \( L \) may be estimated from the geometry of the depletion regions, shown in Fig. 1). Also Eq. 3 only holds if no electron states with negative velocity are occupied in the energy range \( \mu_1 - \mu_2 \), which requires the absence of back scattering in or near the constriction. Although impurity scattering may probably be neglected (\( l_c \gg W, l_c \gg L \)), quantum mechanical reflection of electron waves may occur as a result of the relatively abrupt widening at the ends of the constriction. We surmise that the transition regions in between the quantized plateaux may be explained by the partial reflection of electron waves.

In a perpendicular magnetic field hybrid magneto-electric subbands are formed as a result of both electric and magnetic confinement. Because of the translational invariance along the narrow channel, the electron transport may still be envisaged as propagation of electron waves, which now have a different dispersion:

\[
E_n = \left(n - \frac{1}{2}\right)\hbar \omega + \frac{\hbar^2 k_y^2}{2m^*} + eV_0, \quad \text{with } m^* = m \frac{\omega^2}{\omega_0^2}, \quad \text{and } \omega = \sqrt{\omega_0^2 + \left(\frac{eB}{m}\right)^2}
\]

The magnetic fields increases the subband separation which is already present in a narrow channel. Despite the different dispersion relations the relation \( N_n(E) v_n(E) = 4/h \) still holds in a magnetic field and the conductance is:

\[
G_c = \sum_{n=1}^{N_c} \frac{2e^2}{h} = \frac{2e^2}{h} \text{Int} \left( \frac{E_F}{\hbar \omega_0} + \frac{1}{2} \right)
\]
Eqs. 4 and 5 show a gradual transition between the quantization in zero field, determined by $\omega_c$, and the quantization in high magnetic fields, determined by $\omega_c=\frac{eB}{m}$. Experimentally this is observed in Fig. 3, which shows the conductance of a point contact, obtained from the measured resistance for several values of the magnetic field, after subtraction of a gate voltage independent series resistance.

Fig. 3 Conductance quantization in a magnetic field. A magnetic field increases the subband separation, which leads to broadening of the plateaux.

As in the zero field case, a sequence of quantized plateaux is observed. The effect of the magnetic field is to reduce the number of plateaux in a given gate voltage interval. This clearly shows magnetic depopulation of subbands. At high fields plateaux at odd multiples of $e^2/h$ are beginning to be resolved as a result of the spin-splitting of the 1D subbands in the constriction. Spin-splitting in a parallel magnetic field has been studied by Wiaram et al.

COHERENT ELECTRON FOCUSING

In the previous sections the electron motion was restricted laterally by the electric field of the split-gate. However, as we will discuss below, electrons can also be confined to the boundary of the 2DEG by the application of a magnetic field. Classically, electrons propagate along the boundary in skipping orbits (see inset Fig. 5) with cyclotron radius $l_c=\frac{mv_F}{eB}$, with multiple specular reflections at the 2DEG boundary. As a result of the
translational invariance in the direction along the boundary, the quantum mechanical transport may be treated as the propagation of electron waves along the 2DEG boundary. The quantization of the periodic motion perpendicular to the boundary, associated with the skipping orbits, leads to discrete modes in which these waves can propagate. These modes, or magnetic edges states, also play a role in the (quantum) Hall effect in narrow wires\textsuperscript{1,10}. Experiments have been performed\textsuperscript{11}, in which electrons are injected into the 2DEG by means of an injector point contact, and are collected in a second point contact, with separation $L=3\mu m$, after deflection by the magnetic field (see Fig.4).

Fig. 4 Experimental set up for the electron focusing experiment. A gate on top of a Hall bar defines the injector and collector point contacts. The inset shows the double point contacts.

Fig.5 Electron focusing spectra at several temperatures. At 4 K focusing peaks at the classical positions (arrows) are seen, at lower temperatures a fine structure is resolved.
The classical propagation in skipping orbits gives rise to electron focusing. Peaks are observed in the collector voltage for fields $B_f = n \frac{2m v_F}{(eL)}$, (indicated by arrows in Fig.5) where an integer number $n$ of cyclotron diameters fits in between the point contacts. At 4 K up to 8 peaks are observed, which illustrates the high degree of specularity of the reflections at the 2DEG boundary. At low temperatures fine structure develops in the collector signal. This can be understood by the coherent exitation of a number of edges states by the injector. At the collector point contact interference occurs, depending on the relative phases of the waves. The observation of this interference shows the phase-coherent propagation of electron waves along the 2DEG boundary.

The electron focusing experiment may be described as a non-local voltage measurement. In a three-terminal setup, shown in Fig.5, the collector signal for reverse fields is a measure of the longitudinal resistance, whereas for positive fields a Hall resistance $R_{xy}$ is superimposed. Alternatively the experiment has been performed as a four-terminal Hall measurement (Fig.6).

![Fig. 6 Electron focusing in a four-terminal Hall geometry.](image)

For reverse fields the classical Hall resistance $R_{xy} = B/(ne)$ is observed, for positive fields the electron focusing gives rise to a modulation around the average Hall voltage.

CONCLUSIONS

We have observed a number of new phenomena associated with quantum ballistic transport. The possibility to make constrictions which resemble an electron waveguide, by means of lateral confinement from a split-gate, has led to the observation of conductance quantization in the absence of a magnetic field. The propagation of electron waves along a 2DEG boundary has been studied with an electron focusing experiment.
The results show that quantum transport can be controlled on a microscopic scale, which leads us to expect more fascinating developments in this new field.

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