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Typical representations for $\text{GL}_n(F)$
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Let $F$ be a non-discrete non-Archimedean local field with ring of integers $\mathcal{O}_F$ and residue field $k_F$ of characteristic $p$.

1. Let $s = [M, \sigma]$ be a level-zero non-cuspidal inertial class for $\text{GL}_n(F)$. If $\Gamma$ is a typical representation for the inertial class $s$ then $\Gamma$ is a sub-representation of

$$\text{ind}_{J_s}^{\text{GL}_n(F)}(\lambda_s)$$

where $(J_s, \lambda_s)$ is a Bushnell-Kutzko type for the inertial class $s$. Moreover

$$\dim_{\mathbb{C}} \text{Hom}_{\text{GL}_n(F)}(\Gamma, \text{ind}_{J_s}^{\text{GL}_n(F)}(\lambda)) = \dim_{\mathbb{C}} \text{Hom}_{\text{GL}_n(F)}(\Gamma, \text{ind}_{P}^{\text{GL}_n(F)}(\sigma))$$

for any parabolic subgroup $P$ containing $M$ as its Levi-subgroup. (Corollary 3.0.10)

2. Let $s = [T, \chi]$ be a principal series inertial class for $\text{GL}_n(F)$. If $\Gamma$ is a typical representation for the inertial class $s$ then $\Gamma$ is a sub-representation of

$$\text{ind}_{J_s}^{\text{GL}_n(F)}(\chi)$$

where $(J_s, \chi)$ is a Bushnell-Kutzko type for the inertial class $s$. Moreover

$$\dim_{\mathbb{C}} \text{Hom}_{\text{GL}_n(F)}(\Gamma, \text{ind}_{J_s}^{\text{GL}_n(F)}(\chi)) = \dim_{\mathbb{C}} \text{Hom}_{\text{GL}_n(F)}(\Gamma, \text{ind}_{B_n}^{\text{GL}_n(F)}(\chi))$$

where $B_n$ is a Borel subgroup containing $T$. (Corollary 4.0.13)

3. Let $n$ be a positive integer greater than one and $P$ be a parabolic subgroup of $\text{GL}_n(F)$ of type $(n-1, 1)$ and $M$ be the standard Levi-subgroup of the type $(n-1, 1)$. Let $s = [M, \sigma \boxtimes \chi]$ be an inertial class for $\text{GL}_{n+1}(F)$. There exists a unique typical representation $\Gamma$ occurring in the parabolic induction

$$\text{ind}_{P}^{\text{GL}_{n+1}(F)}(\sigma \boxtimes \chi)$$

Moreover $\Gamma$ occurs with multiplicity one in the above representation. (Theorem 5.3.3)

4. Let $s = [\text{GL}_2(F) \times \text{GL}_2(F), \sigma \boxtimes \sigma]$ be an inertial class for $\text{GL}_4(F)$ and $\#k_F > 3$. If $\Gamma$ is a typical representation for the inertial class $s$ then $\Gamma$ is a sub-representation of

$$\text{ind}_{J_s}^{\text{GL}_4(F)}(\lambda_s)$$

where $(J_s, \lambda_s)$ is a Bushnell-Kutzko type for the inertial class $s$. Moreover

$$\dim_{\mathbb{C}} \text{Hom}_{\text{GL}_4(F)}(\Gamma, \text{ind}_{J_s}^{\text{GL}_4(F)}(\lambda)) = \dim_{\mathbb{C}} \text{Hom}_{\text{GL}_4(F)}(\Gamma, \text{ind}_{P}^{\text{GL}_4(F)}(\sigma \boxtimes \sigma))$$

where $P$ is a parabolic sub-group of the form $(2, 2)$. (Theorem 6.3.4)

5. Let $s = [\text{GL}_2(F) \times F^\times, \sigma \boxtimes \chi]$ be an inertial class of $\text{GL}_3(F)$. We denote by $P$ the standard parabolic subgroup of the type $(2, 1)$. There exists an inertial class $s' = [\text{GL}_2(F) \times F^\times, \sigma' \boxtimes \chi']$ such that except for finite dimensional $\text{GL}_3(\mathcal{O}_F)$-sub-representations the restrictions

$$\text{res}_{\text{GL}_3(F)}(\text{ind}_{P}^{\text{GL}_3(F)}(\sigma \boxtimes \chi)) \text{ and } \text{res}_{\text{GL}_3(F)}(\text{ind}_{P}^{\text{GL}_3(F)}(\sigma \boxtimes \chi))$$

are isomorphic.
6. There exist two distinct principal series inertial classes \([T, \chi]\) and \([T, \chi']\) of \(GL_3(F)\) such that

\[
\operatorname{res}_{GL_3(O_F)}(i_{B_3}^{GL_3(F)}(\chi))/\Gamma_1 \quad \text{and} \quad \operatorname{res}_{GL_3(O_F)}(i_{B_3}^{GL_3(F)}(\chi'))/\Gamma_2
\]

are not isomorphic even for any finite dimensional \(GL_3(O_F)\)-representations \(\Gamma_1\) and \(\Gamma_2\).

7. Let \((\pi, V)\) be a cuspidal representation of \(GL_2(F)\) where \(V\) is a vector space over a finite extension \(E\) over \(\mathbb{Q}_l\). We suppose that the central character of \(\pi\) takes values in \(O_F^\times\). Let \((\pi_l, W)\) be the mod \(l\) reduction of \((\pi, V)\). If \(c(\pi)\) and \(c(\pi_l)\) are the conductors of \((\pi, V)\) and \((\pi_l, W)\) respectively then \(c(\pi) = c(\pi_l)\).

8. Let \(\#k_F = 2\) and \(\Gamma_1\) and \(\Gamma_2\) be two distinct typical representations for the inertial class \([F^\times \times F^\times, \chi_1 \boxtimes \chi_2]\), \(\operatorname{res}_{O_F^\times} \chi_1 \chi_2^{-1} \neq 1\), then the semi-simplification of the mod 2 reductions of \(\Gamma_1\) and \(\Gamma_2\) are isomorphic.