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Stellingen

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Typical representations for $\mathrm{GL}_n(F)$

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Let F be a non-discrete non-Archimedean local field with ring of integers $\mathcal{O}_F$ and residue field k_F of characteristic p.

1. Let $s = [M, \sigma]$ be a level-zero non-cuspidal inertial class for $\mathrm{GL}_n(F)$. If Γ is a typical representation for the inertial class s then Γ is a sub-representation of
$$\mathrm{ind}_{J_s}^{\mathrm{GL}_n(\mathcal{O}_F)}(\lambda_s)$$
where (J_s, λ_s) is a Bushnell-Kutzko type for the inertial class s. Moreover
$$\dim_{\mathbb{C}} \mathrm{Hom}_{\mathrm{GL}_n(\mathcal{O}_F)}(\Gamma, \mathrm{ind}_{J_s}^{\mathrm{GL}_n(\mathcal{O}_F)}(\lambda)) = \dim_{\mathbb{C}} \mathrm{Hom}_{\mathrm{GL}_n(\mathcal{O}_F)}(\Gamma, i_P^{\mathrm{GL}_n(F)}(\sigma))$$
for any parabolic subgroup P containing M as its Levi-subgroup. (Corollary 3.0.10)

2. Let $s = [T, \chi]$ be a principal series inertial class for $\mathrm{GL}_n(F)$. If Γ is a typical representation for the inertial class s then Γ is a sub-representation of
$$\mathrm{ind}_{J_s}^{\mathrm{GL}_n(\mathcal{O}_F)}(\chi)$$
where (J_s, χ) is a Bushnell-Kutzko type for the inertial class s. Moreover
$$\dim_{\mathbb{C}} \mathrm{Hom}_{\mathrm{GL}_n(\mathcal{O}_F)}(\Gamma, \mathrm{ind}_{J_s}^{\mathrm{GL}_n(\mathcal{O}_F)}(\chi)) = \dim_{\mathbb{C}} \mathrm{Hom}_{\mathrm{GL}_n(\mathcal{O}_F)}(\Gamma, i_{B_n}^{\mathrm{GL}_n(F)}(\chi))$$
where B_n is a Borel subgroup containing T. (Corollary 4.0.13)

3. Let n be a positive integer greater than one and P be a parabolic subgroup of $\mathrm{GL}_n(F)$ of type $(n-1, 1)$ and M be the standard Levi-subgroup of the type $(n-1, 1)$. Let $s = [M, \sigma \boxtimes \chi]$ be an inertial class for $\mathrm{GL}_{n+1}(F)$. There exists a unique typical representation Γ occurring in the parabolic induction
$$i_P^{\mathrm{GL}_n(F)}(\sigma \boxtimes \chi)$$
Moreover Γ occurs with multiplicity one in the above representation. (Theorem 5.3.3)

4. Let $s = [\mathrm{GL}_2(F) \times \mathrm{GL}_2(F), \sigma \boxtimes \sigma]$ be an inertial class for $\mathrm{GL}_4(F)$ and $\#k_F > 3$. If Γ is a typical representation for the inertial class s then Γ is a sub-representation of
$$\mathrm{ind}_{J_s}^{\mathrm{GL}_4(\mathcal{O}_F)}(\lambda_s)$$
where (J_s, λ_s) is a Bushnell-Kutzko type for the inertial class s. Moreover
$$\dim_{\mathbb{C}} \mathrm{Hom}_{\mathrm{GL}_4(\mathcal{O}_F)}(\Gamma, \mathrm{ind}_{J_s}^{\mathrm{GL}_4(\mathcal{O}_F)}(\lambda)) = \dim_{\mathbb{C}} \mathrm{Hom}_{\mathrm{GL}_4(\mathcal{O}_F)}(\Gamma, i_P^{\mathrm{GL}_4(F)}(\sigma \boxtimes \sigma))$$
where P is a parabolic sub-group of the form $(2, 2)$. (Theorem 6.3.4)

5. Let $s = [\mathrm{GL}_2(F) \times F^\times, \sigma \boxtimes \chi]$ be an inertial class of $\mathrm{GL}_3(F)$. We denote by P the standard parabolic subgroup of the type $(2, 1)$. There exists an inertial class $s' = [\mathrm{GL}_2(F) \times F^\times, \sigma' \boxtimes \chi']$ such that except for finite dimensional $\mathrm{GL}_3(\mathcal{O}_F)$-sub-representations the restrictions
$$\mathrm{res}_{\mathrm{GL}_3(\mathcal{O}_F)}(i_P^{\mathrm{GL}_3(F)}(\sigma \boxtimes \chi)) \quad \text{and} \quad \mathrm{res}_{\mathrm{GL}_3(\mathcal{O}_F)}(i_P^{\mathrm{GL}_3(F)}(\sigma \boxtimes \chi))$$
are isomorphic.

6. There exist two distinct principal series inertial classes $[T, \chi]$ and $[T, \chi']$ of $\mathrm{GL}_3(F)$ such that

$$\mathrm{res}_{\mathrm{GL}_3(\mathcal{O}_F)}(i_{B_3}^{\mathrm{GL}_3(F)}(\chi))/\Gamma_1 \quad \text{and} \quad \mathrm{res}_{\mathrm{GL}_3(\mathcal{O}_F)}(i_{B_3}^{\mathrm{GL}_3(F)}(\chi'))/\Gamma_2$$

are not isomorphic even for any finite dimensional $\mathrm{GL}_3(\mathcal{O}_F)$-representations Γ_1 and Γ_2.

7. Let (π, V) be a cuspidal representation of $\mathrm{GL}_2(F)$ where V is a vector space over a finite extension E over $\mathbb{Q}_l$. We suppose that the central character of π takes values in $\mathcal{O}_E^\times$. Let (π_l, W) be the mod l reduction of (π, V). If $c(\pi)$ and $c(\pi_l)$ are the conductors of (π, V) and (π_l, W) respectively then $c(\pi) = c(\pi_l)$.

8. Let $\#k_F = 2$ and Γ_1 and Γ_2 be two distinct typical representations for the inertial class $[F^\times \times F^\times, \chi_1 \boxtimes \chi_2]$, $\mathrm{res}_{\mathcal{O}_F^\times} \chi_1\chi_2^{-1} \neq 1$, then the semi-simplification of the mod 2 reductions of Γ_1 and Γ_2 are isomorphic.