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Abstract

In this thesis we classify typical representations for certain non-cuspidal Bernstein components of GL_n over a non-Archimedean local field. Following the work of Henniart in the case of GL_2 and Paskunas for the cuspidal Bernstein components, we classify typical representations for Bernstein components of level-zero for GL_n for $n \geq 3$, principal series components, components with Levi subgroup of the form $(n, 1)$ for $n > 1$ and certain components with Levi subgroup of the form $(2, 2)$.

Each of the above component is treated in a separate chapter. The classification uses the theory of types developed by Bushnell and Kutzko in a significant way. We will give the classification in terms of Bushnell-Kutzko types for a given inertial class.

Samenvatting

In deze scriptie classificeren wij typische representaties voor bepaalde niet-cuspidale Bernstein componenten van GL_n over een niet Archimedisch lokaal lichaam. Gebruik makend van het werk van Henniart in het geval van GL_2 voor de cuspidale Bernstein componenten, classificeren wij representaties voor Bernstein componenten van niveau nul voor GL_n voor $n \geq 3$, voor principal series components, voor componenten met Levi ondergroep van de vorm $(n, 1)$ met $n > 1$ en sommige componenten met Levi ondergroep van de vorm $(2, 2)$.

Alle bovenstaande componenten worden in hun eigen hoofdstuk behandeld. De classificatie berust zwaar op de theorie van typen ontwikkeld door Bushnell en Kutzko en deze zal ook gegeven worden in de termen van de Bushnell-Kutzko typen behorend bij een gegeven inertie klasse.

Résumé

Dans cette thèse, nous classifions les représentations typiques pour certaines composantes de Bernstein de GL_n sur un corps localement compact non Archimédien. Suite aux travaux de Henniart dans le cas de GL_2 et de Paskunas pour les composantes de Bernstein cuspidales, nous classifions les représentations typiques pour les composants de Bernstein de niveau zéro pour $n \geq 3$, les composantes de séries principales, les composantes dont le sous-groupe de Levi est de forme $(n, 1)$ pour $n > 1$ et certaines composantes dont le sous-groupe de Levi est de la forme $(2, 2)$.

Chacune des composantes ci-dessus est traitée dans un chapitre distinct. La classification utilise d'une manière significative la théorie des types développée par Bushnell-Kutzko, et elle est établie en termes de tels types.

Curriculum Vitae

Santosh Nadimpalli was born on 4th-January 1990 in Vijayawada, India. He attended Visakha valley school in Visakhapatnam until his 10th class. He completed his intermediate education in Hyderabad aiming to be an engineer. He got admission in Indian Statistical Institute to pursue bachelors of mathematics in 2007.

In 2010 he was offered Alcant Masters scholarship to study in University of Padova and Université Paris Sud. He spent the first year of masters in Padova (2010 – 2011) and the second year (2011 – 2012) in Orsay. As a second year masters student he approached Prof. Guy Henniart for Masters thesis. He wrote his masters thesis with Henniart and completed his masters in 2012.

With Alcant Doctorate scholarship, in 2012 he began his Phd jointly hosted by University of Leiden and Université de Paris Sud. Following the suggestion of Henniart he is working on the topic of typical representations for the Phd.

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