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*Inverse problems for universal deformation rings of group representations*

van Krzysztof Dorobisz

In the below propositions $k$ is a finite field and $\hat{C}$ is the category of all complete noetherian local rings with residue field $k$, as defined in the thesis.

1. (Theorem 5.1) Let $R$ be an object of $\hat{C}$, $n \geq 2$ be a natural number and consider the representation $\bar{\rho}: \text{SL}_n(R) \to \text{GL}_n(k)$ induced by the canonical reduction $R \to k$. Then $R$ is the universal deformation ring of $\bar{\rho}$ if and only if

$$ (n,k) \notin \{(2,2), (2,3), (2,5), (3,2)\}.$$

2. (Theorem 6.31) Denote by $W(k)$ the ring of Witt vectors over $k$, by $K$ its field of fractions and suppose $R \in \text{Ob}(\hat{C})$ can be obtained as a universal deformation ring of a representation of some finite group. If $R$ has characteristic zero, then $R \otimes_{W(k)} K$ is a finite étale $K$-algebra. In particular, for every positive integer $m$, the power series ring $W(k)[[X_1, \ldots, X_m]]$ can not be obtained as a universal deformation ring of a finite group representation.

3. (Corollary 3.14) If $k \neq \mathbb{F}_2, \mathbb{F}_3$ then for every positive integer $m$ the ring $k[[X_1, \ldots, X_m]]$ can be obtained as a universal deformation ring of a finite group representation.

4. (Proposition 4.3) Given a positive integer $n$ and $i,j \in \{1, \ldots, n\}$, we denote by $I_n$ the $n \times n$ identity matrix and by $e_{ij}$ the $n \times n$ matrix having all entries equal to zero, except of the $(i,j)$-th entry, equal to one.

Suppose $G$ is a profinite group and $\bar{\rho} : G \to \text{GL}_n(k)$ is a continuous representation having a universal deformation ring $R_{\bar{\rho}}$. Let $R \in \text{Ob}(\hat{C})$ and $\rho \in \text{Lift}_{\bar{\rho}}(R)$ be given and assume that for some $i,j \in \{1, \ldots, n\}$ we have $I_n + R e_{ij} \subseteq \text{im} \rho$. Then:

- there exists a universal deformation ring $R_k$ of the representation $\text{im} \bar{\rho} \hookrightarrow \text{GL}_n(k)$.
- the fiber product $R \times_k R_k$ is a quotient of $R_{\bar{\rho}}$ and a necessary condition for $R_{\bar{\rho}} \cong R$ is $R_k \cong k$.

5. Suppose $k = \mathbb{F}_5$, $n = 2$ and consider the representation $\bar{\rho}$ defined as in the first proposition. Then $R \times_k \mathbb{Z}_5[\sqrt{5}]$ is the universal deformation ring of $\bar{\rho}$.

6. Suppose $k \neq \mathbb{F}_2$ and let $R \in \text{Ob}(\hat{C})$ be given. We denote by $\mu_R$ the set of the multiplicative representatives of the non-zero residue classes in $k$ (the image of the Teichmüller lift of $k^\times$), define

$$ G := \begin{pmatrix} \mu_R & R & R \\ 0 & \mu_R & R \\ 0 & 0 & \mu_R \end{pmatrix} $$

and let $\bar{\rho}$ be the composition of the inclusion $G \hookrightarrow \text{GL}_3(R)$ with the canonical reduction $\text{GL}_3(R) \to \text{GL}_3(k)$. Then $R$ is the universal deformation ring of $\bar{\rho}$. 
7. The groups $\text{SL}_2(\mathbb{F}_3)$ and $\text{PSL}_2(\mathbb{F}_7)$ are isomorphic.

8. Not every profinite group is isomorphic to its own profinite completion.

9. Every natural number is interesting.

10. The supreme command of the English language shown by the Dutch and their readiness to help foreigners by automatically switching to this language, though admirable, have at least one downside: that of making it difficult to practice the Dutch language, especially in its spoken form.